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A STUDY OF THE DISTRIBUTION OF CURVATURE GIVEN BY AN EXPONENTIAL LAW

The article investigates the distribution of the curvature given by the exponential law for further application in geometric modeling of plane curves. The work is a continuation of the research conducted by the team of the Department of Computer Engineering within the framework of the research topic. An algorithm is proposed for forming a section of the curve of a line on the basis of the curvature distribution law in general form. Curves of the distribution of the curvature and the slopes of the tangents to the curve are constructed. The problem of constructing the dependences of the curvature distribution crossing the abscissa is solved.

***Ключові слова**: exponential curvature distribution, geometric modeling, flat curve line.*

Formulation of the problem

Curved lines occupy a special position in various fields of science and technology. They are used in solving various scientific and engineering problems, in geometric modelling of various technical objects. With their help, you can visually track the trajectory of the movement of objects, the course of a process, display the results of experimental studies or theoretical calculations.

Despite the fact that scientists in geometry and other applied fields of science have developed quite different methods of modelling curves, there are a number of problems for the solution of which it is important to develop new approaches to the shaping of curves.

The curve must be a smooth line, have continuous derivatives (in many practical applications, the continuity of the first and second derivatives is sufficient, ensuring a satisfactory distribution of curvature).

Analysis of recent research and publications

In the modern literature [1-6, 10], various ways of analytic determination of plane curves can be found. In this case, there are methods that uniquely determine the curve. This includes algebraic and various transcendental curves, that is, curves that have a definite analytic dependence in an explicit or implicit form, and parametric curves.

The Hermite method [5], the Bezier method [5], the Bol method [8], the beta spline method [2], the Catmull-Rom method [9], the B splines method [10] are widely used in geometric modelling of the curve lines. A certain distribution, especially in computer graphics, is the so-called NURBS-curves, which are inhomogeneous rational Bezier splines and are given by the coordinates of the initial and final points and the set of intermediate points [10].

When modelling curves, their parametric equations are used, in which the length of the arc of the curve (natural equations) is taken as a parameter. But such curves are very few and not all of them can be useful in practical applications.

One of the first papers dealing with the modelling of curves on the basis of the linear law of curvature distribution is work [7]. Various laws of curvature distribution for plane curves with respect to profiles of tur-

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bine blades and gas turbine engine compressors were investigated in [4].

Research Objective

The aim of this article is to study the distribution of the curvature given by the exponential law for further application in geometric modelling of plane curved lines. The work is a continuation of the research conducted by the team of the Department of Computer Engineering within the framework of the research topic.

Basic material

To form the line curve section based on the law of curvature distribution, it is necessary to perform the following actions:

1. Specify the form of the law of the distribution of curvature from the length of the arc in general form.
2. By integration, determine the distribution function of the slope angle of the tangent to the simulated curve (in general form).
3. Determine the parametric equations of the plane curve of the line.
4. Using the boundary conditions, determine all possible unknown parameters.
5. Form a system of nonlinear integral equations and solve one of the optimization methods or a numerical method of solving a system of nonlinear equations.
6. The unknown coefficients obtained are substituted into the parametric equations of plane curves.

This algorithm was developed as a result of long researches of the authors of the article (since 2004).

In this paper, only the first two points of the algorithm will be considered.

1. As the law of distribution of curvature, the exponential distribution is taken as a function of the arc length of the curve in the form:

$$k(s) = ae^{bs}, \quad (1)$$

where a, b are unknown distribution parameters.

Let us investigate this dependence – for this we construct curvature graphs depending on unknown parameters.

In fig. 1-5 shows the results of constructing exponential curvature distributions with a change in the coefficient a from –1 to 1 in increments of 0.25. In all the graphs below, the arc length of the curve changes from 0 to 10.

Analysis of the graphs shows that they depend on the signs of the unknowns: for $a < 0$, the graphs are concave, and for $a > 0$ they are convex, and the larger the value of a, the greater the curvature of the graph, and for 0 – turns into a straight line; for $ab < 0$, the graphs decrease, and for $ab > 0$ – they increase, with the equality 0 – they turn into straight lines (fig. 3).

In fig. 6-7 show the dependence of the curvature as the coefficient b varies from –1 to 1 in increments of 0.25 (the arc length of the curve varies from 0 to 1).

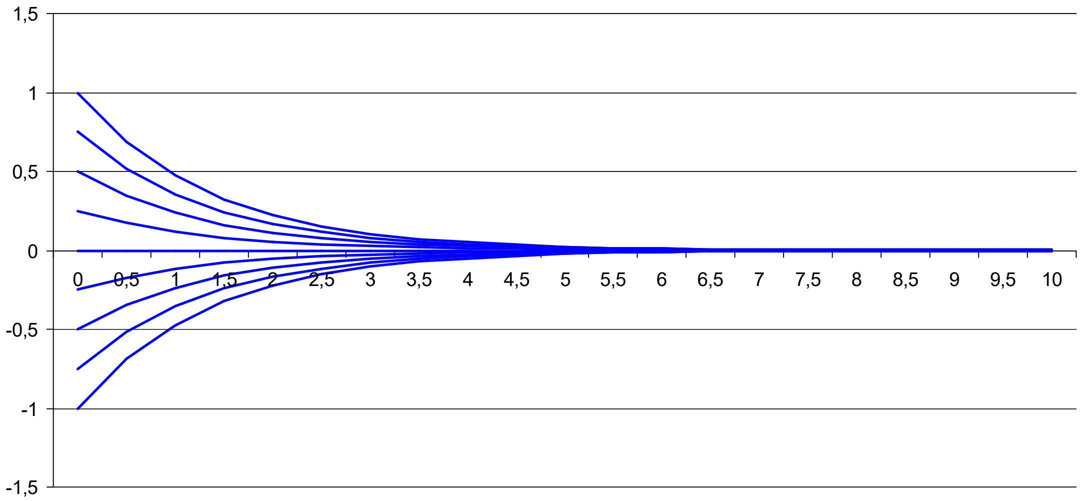

Fig. 1. Curvature distribution under $b = –0,75$

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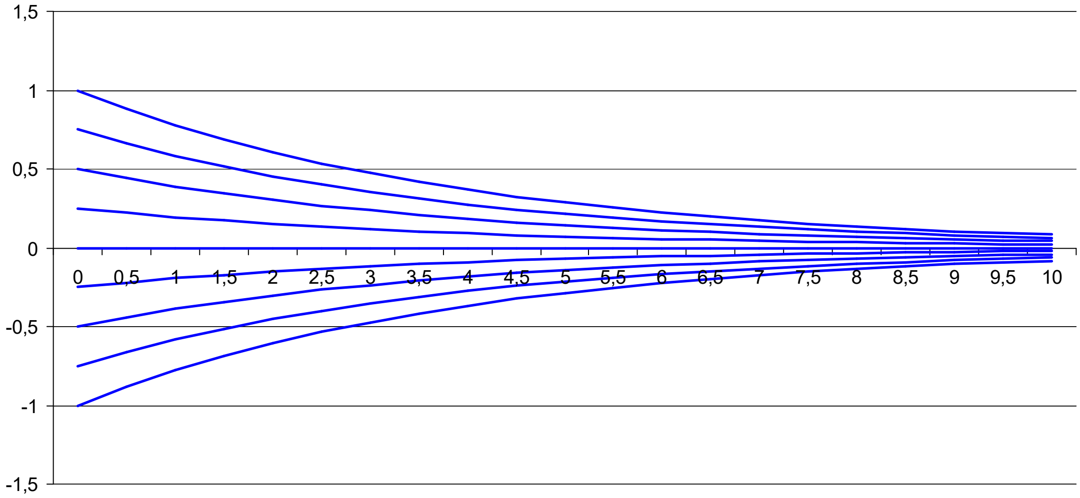

Fig. 2. Curvature distribution under $b = -0{,}25$

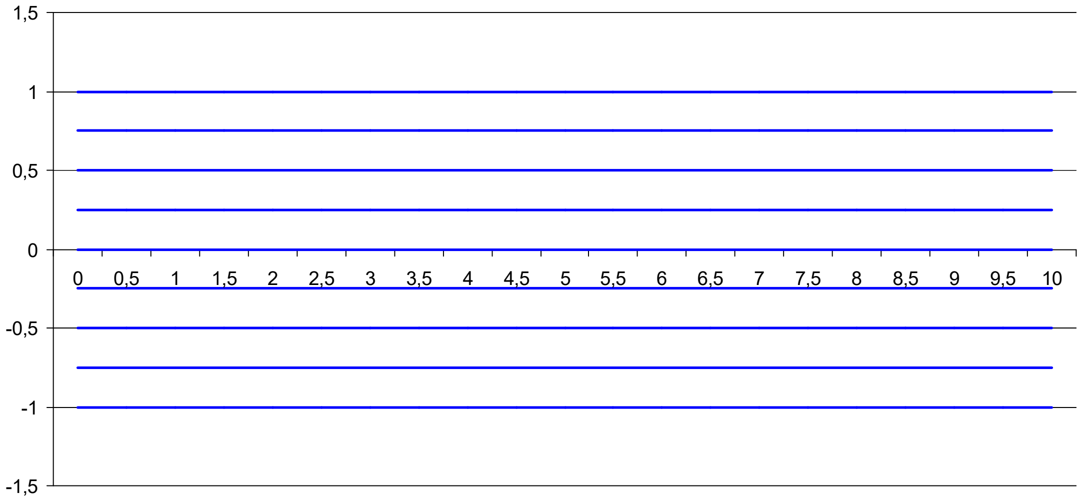

Fig. 3. Curvature distribution under $b = 0$

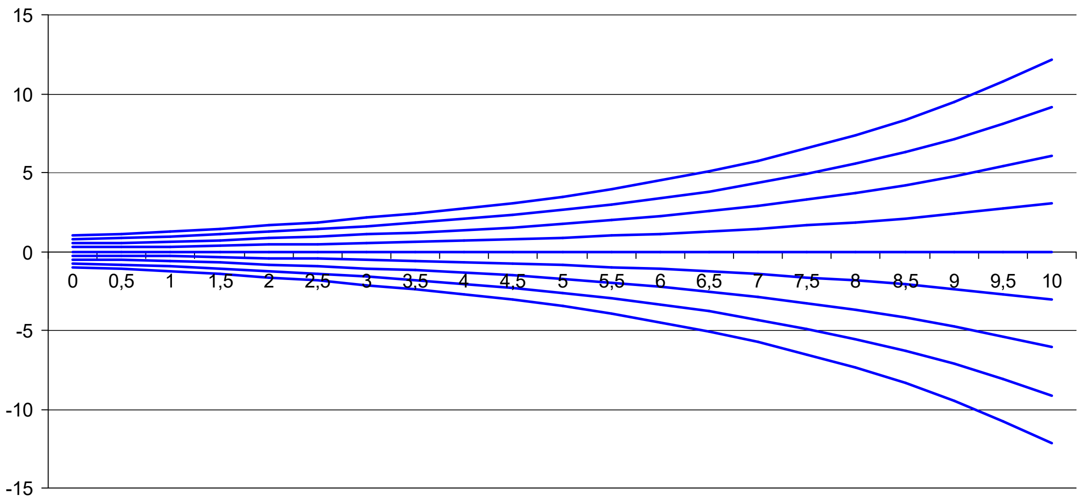

Fig. 4. Curvature distribution under $b = 0{,}25$

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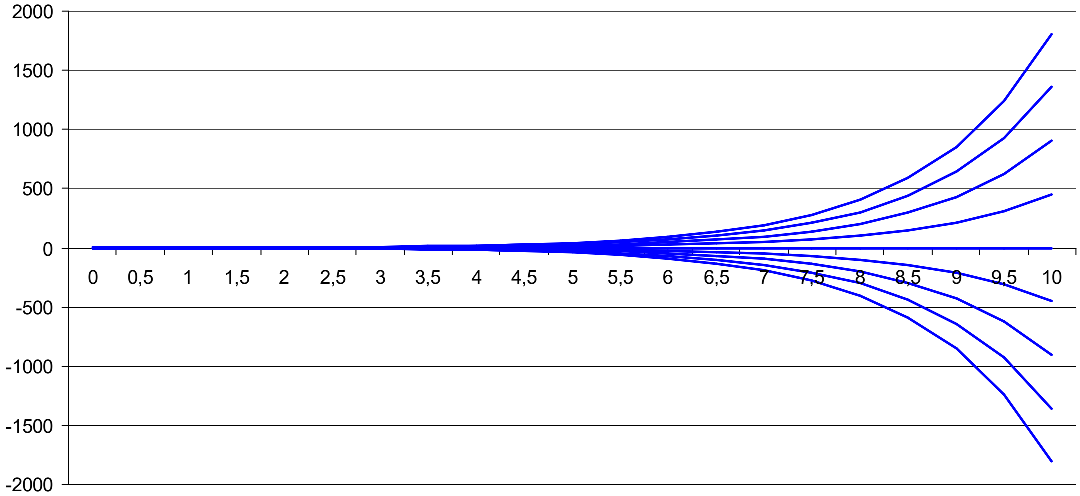

Fig. 5. Curvature distribution under *b* = 0,75

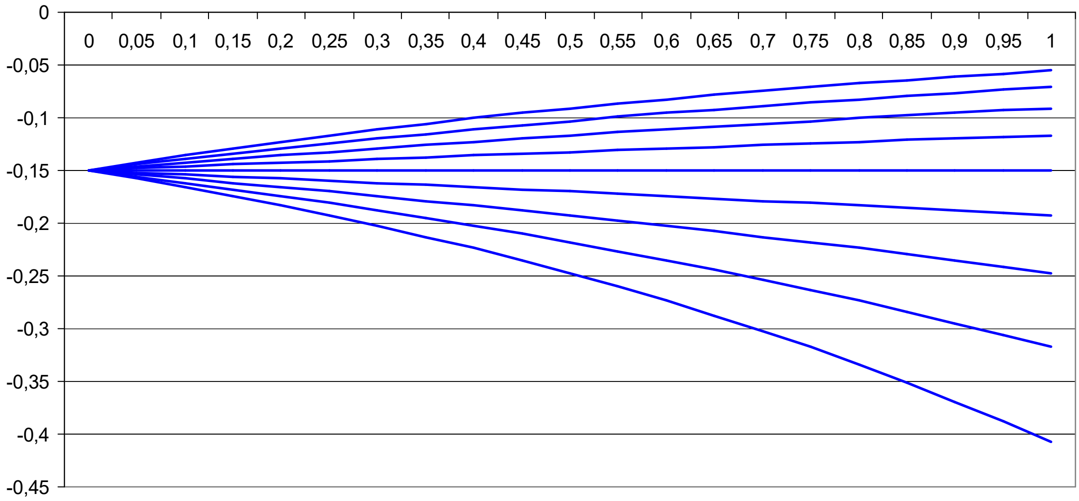

Fig. 6. Curvature distribution under *a* = –0,15

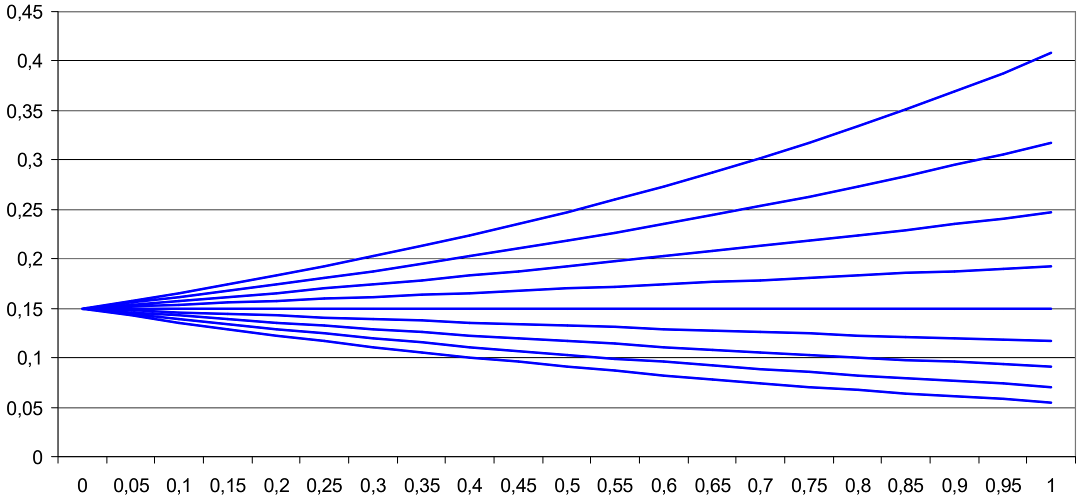

Fig. 7. Curvature distribution under *a* = 0,15

Consider the case where the initial k_1 and the finite k_2 curvature values and the length of the curve S are given. We substitute them in the expression (1):

$$k_1 = k(0) = ae^{b0} = a;$$

$$k_2 = k(S) = ae^{bS} = k_1 e^{bS};$$

from here

$$b = \frac{\ln \frac{k_2}{k_1}}{S}. \qquad (2)$$

In fig. 8-12 show the curvature regions obtained for different values of k_1 and k_2 (arc length of the curve $S = 1$).

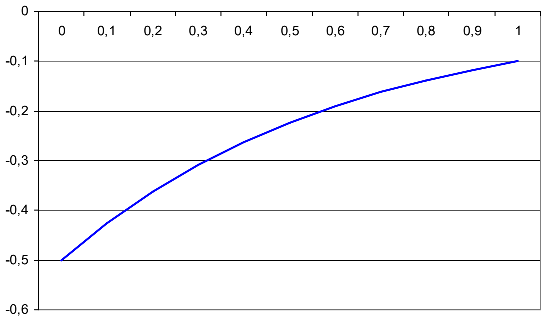

Fig. 8. The section of curvature at $k_1 = -0{,}5$; $k_2 = -0{,}1$

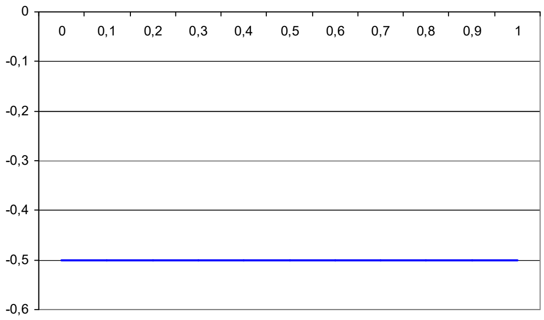

Fig. 9. The section of curvature at $k_1 = -0{,}5$; $k_2 = -0{,}5$

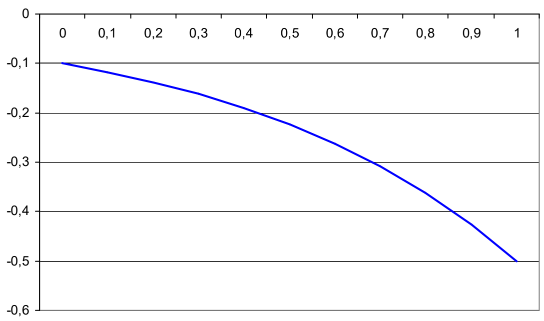

Fig. 10. The section of curvature at $k_1 = -0{,}1$; $k_2 = -0{,}5$

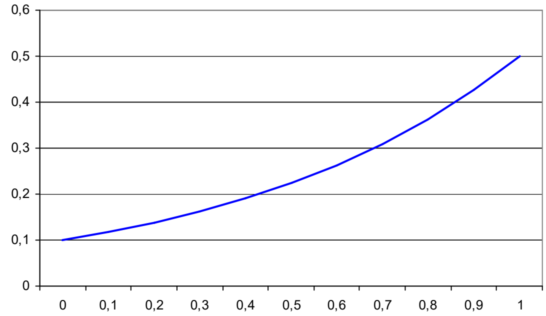

Fig. 11. The section of curvature at $k_1 = 0{,}1$; $k_2 = 0{,}5$

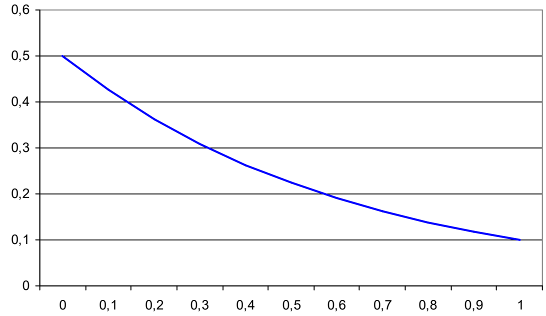

Fig. 12. The section of curvature at $k_1 = 0{,}5$; $k_2 = 0{,}1$

As can be seen from expression (2), the values of k_1 and k_2 must be of the same sign and not equal to 0. Due to this circumstance, one can not construct the dependence of the distribution of curvature intersecting the abscissa axis.

To solve this problem, perform the following transformations:

$$a = 1 + 2sign(\Delta k),$$

where $\Delta k = k_2 - k_1$; sign() – sign definition function; the multiplier 2 is necessary if $\Delta k = 0$;

$$b = \frac{\ln\left(1 + \frac{\Delta k}{a}\right)}{S}.$$

And, as a consequence, the equation of the general form of the curvature distribution will have the following form:

$$k(s) = a\left(e^{bs} - 1\right) + k_1.$$

2. The distribution of the slope angle of the tangent to the simulated curve (in general form) is obtained by integrating the dependence (1):

$$\varphi(s)=\varphi(0)+\int_0^s k(s)ds=\varphi(0)+\int_0^s ae^{bs}ds=$$
$$=\varphi(0)+\frac{a}{b}e^{bs}\Big|_0^s=\varphi(0)+\frac{a}{b}\left(e^{bs}-1\right),$$

where $\varphi(0)$ – angle of inclination of the tangent to the curve at the starting point.

We will construct graphs of the slope angles of the tangent plane curve of the line as a function of unknown parameters.

In fig. 13 shows the results of plotting the slope of the tangents for the data used in the construction of fig. 1, and the angle at the starting point is 45°.

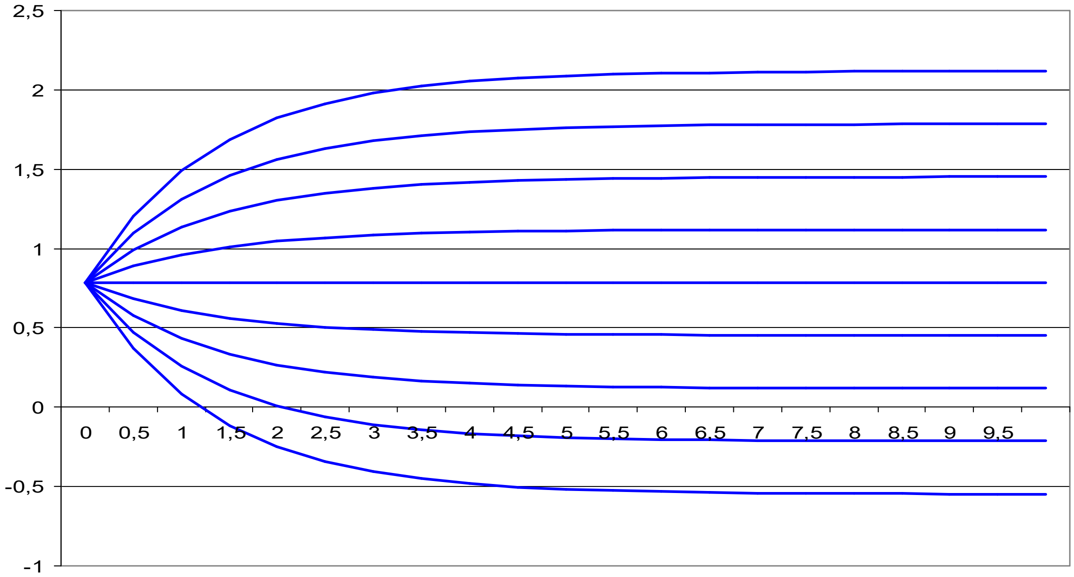

Fig. 13. The resulting angle of inclination of the tangent

Conclusions

Thus, the feasibility of applying the exponential law of curvature distribution for geometric modeling of plane curved lines is investigated.

In the future, it is planned to develop a method for geometric modeling of plane curves with a given exponential law of curvature distribution, realizing the remaining points of the algorithm.

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ДОСЛІДЖЕННЯ РОЗПОДІЛУ КРИВИНИ, ЗАДАНОГО ЕКСПОНЕНЦІЙНИМ ЗАКОНОМ

У статті досліджено розподіл кривини, заданий експоненційним законом, для подальшого застосування в геометричному моделюванні плоских кривих ліній. Робота є продовженням досліджень, що проводяться колективом кафедри комп'ютерної інженерії в рамках науково-дослідної теми. Запропоновано алгоритм формування ділянки кривої лінії на основі закону розподілу кривини в загальному вигляді. Побудовано графіки розподілу кривини і кутів нахилу дотичних до кривої. Вирішено проблему побудови залежностей розподілу кривини, які перетинають вісь абсцис.

***Keywords**: експоненційний розподіл кривини, геометричне моделювання, плоска крива лінія.*

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ИССЛЕДОВАНИЕ РАСПРЕДЕЛЕНИЯ КРИВИЗНЫ, ЗАДАННОГО ЭКСПОНЕНЦИАЛЬНЫМ ЗАКОНОМ

В статье исследовано распределения кривизны, заданного экспоненциальным законом, для дальнейшего применения в геометрическом моделировании плоских кривых линий. Работа является продолжением исследований, проводимых коллективом кафедры компьютерной инженерии в рамках научно-исследовательской темы. Предложен алгоритм формирования участка кривой линии на основе закона распределения кривизны в общем виде. Построены графики распределения кривизны и углов наклона касательных к кривой. Решена проблема построения зависимостей распределения кривизны, пересекающих ось абсцисс.

***Ключевые слова**: экспоненциальное распределение кривизны, геометрическое моделирование, плоская кривая линия.*

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