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VOLTERRA NEURAL NETWORK CONSTRUCTION IN THE NONLINEAR DYNAMIC SYSTEMS MODELING PROBLEM

Abstract. *The features of using the theory of Volterra series and neural networks in problems of nonlinear dynamic systems modeling are considered. A comparative analysis of methods for constructing models of nonlinear dynamic systems based on the theory of Volterra series and neural networks is carried out; areas of effective application of each method are indicated. The problem statement is formulated, consisting in the creation of a mathematical apparatus for transforming models of nonlinear dynamic systems derived from the Volterra series apparatus into an artificial neural network of a certain structure. The three-layer structure of a direct signal propagation neural network has been substantiated and investigated for represent nonlinear dynamic systems. It is outlined a class of systems that can be efficiently approximated by this network. The dependence of the Volterra kernels coefficients and the weighting coefficients of the hidden layer of the three-layer forward-propagation neural network is established. An algorithm for constructing an artificial neural network based on the Volterra series is given. The results of computer simulation of nonlinear dynamic systems using the Volterra neural network and direct signal propagation neural network are presented. The analysis of experimental data confirms the effectiveness of using Volterra neural networks in problems of modeling nonlinear dynamic systems. Conclusions and recommendations on the effective use of Volterra neural networks for modeling nonlinear dynamic systems are made.*

Keywords: *neural networks; Volterra series; nonlinear dynamical systems*

Introduction

Modeling of nonlinear dynamical systems is widely used in investigations of complex objects and systems of the surrounding world. An effective method of such studies is the use of the theory of Volterra series to describe the nonlinear and dynamic properties of the control object (CO) according to the input and output data.

As disadvantages of this method, it is worth noting that models in the form of the Volterra series are well suited for modeling an CO with weak inertial nonlinearities, as well as a high computational complexity of calculations of the Volterra kernels of higher order.

Another approach to solving this problem is the use of artificial neural networks of direct propagation and, in particular, three-layer perceptions.

The neuronal artificial neural networks (NN) are used in various applications with great success because they are versatile and capable of generalization. But on the other hand, the use of NN is associated with certain problems, most importantly - there is no guarantee that a fixed model will work well to solve an application problem.

The lack of effective methods for constructing models of nonlinear inertial objects possessing predominantly Volterra and NN series simultaneously induces further research in this area.

Problem statement. A comparative analysis of the considered methods of constructing models of non-linear inertial objects is difficult due to the use of fundamentally different modeling methodologies.

Volterra series are analytic expansion expressions for nonlinear functional, while NN are structural imitative models.

To date, there is no universal mathematical apparatus for pre-braining models in the form of Volterra series in the NN. In this paper, the task of searching for such a mathematical transformation is proposed, which would link the models in the form of the Volterra series in the NN with the preservation of nonlinear and inertial properties of the CO.

Analysis of recent research and publications The widespread use of both methods in practice determines the actual task of conducting a comparative study of both methods with a view to their further development.

For a universal description of an CO of unknown structure, it is advisable to use nonlinear nonparametric dynamical models based on integral-power Volterra series, the main feature of which is simultaneous and compact accounting of nonlinear and dynamic properties of CO in the form of multidimensional weight functions – Volterra's kernels [1; 2]. Calculation of the coefficients of the kernel, in the general case, is a problem requiring a large amount of computation [3]. Some authors proposed a method for extracting the Volterra kernel of any order as a function of the weights of a direct propagation of a neural network with a delayed time with one hidden layer [4]. Determination of the coefficients of kernels of various orders using other topologies of neural networks is given in [5-7], also in the electronics field [8; 9].

All these approaches relate to inputs of time series of one variable. In practice, not only the rows of time series of one variable are of interest, but functions that depend on many variables [10; 11].

An analysis of recent studies and publications showed insufficient coverage of the problem of the creation of a mathematical model for transforming models in the form of Volterra series in the NN.

The purpose of the study. Improve the accuracy of simulation of nonlinear dynamic CO by developing a combined method based on the Volterra series and the direct propagation of the NN.

To achieve this goal, the following tasks are set.

1. Selection and research of the structure of the artificial neural network for the representation of nonlinear dynamical systems.

2. Determination of the dependence of Volterra kernel coefficients and weight coefficients of the corresponding neural network.

3. Development of an algorithm for constructing an artificial neural network based on the Volterra series.

4. Research and analysis of the proposed algorithm with the help of computer modeling of the test nonlinear dynamical object with Volterra NN and direct signal propagation.

Research methods. Theoretical studies are based on the theory of nonparametric identification and modeling of nonlinear inertial systems for the construction of integral information models based on multidimensional weight functions, the theory of functional analysis and the theory of artificial neural networks for the construction of information models.

The elements of the theory of computational experiments, as well as the means of simulation simulation, are used to solve test and applied problems and to analyze the accuracy and noise immunity of information models.

The method of constructing an artificial neural network based on the Volterra series. Volterra NN is a network with polynomial nonlinearity, which allows us to construct models for identifying nonlinear objects, eliminating interference noise, and also for forecasting variables in time of unsteady signals, for example, forecasting of transport flows [10; 11].

The Volterra expansion for describing the nonlinear dynamic systems with many inputs and many outputs in a discrete form for the implementation of the NN can be presented in the following form:

$$\begin{aligned} y_j(t) &= \sum_{i_1=1}^{v}\int_0^t w_{i_1}^j(\tau)x_{i_1}(t-\tau)d\tau + \\ &\sum_{i_1=1}^{v}\sum_{i_2=1}^{v}\int_0^t\int_0^t w_{i_1i_2}^j(\tau_1,\tau_2)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)d\tau_1\,d\tau_2 + \\ &+\sum_{i_1=1}^{v}\sum_{i_2=1}^{v}\sum_{i_3=1}^{v}\int_0^t\int_0^t\int_0^t w_{i_1i_2i_3}^j(\tau_1,\tau_2,\tau_3)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)\times \\ &\times x_{i_3}(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \ldots, \end{aligned} \quad (1)$$

where: $y_j(t)$ – CO response on output j at the current time t under zero initial conditions; $x_1(t),\ldots,x_v(t)$ – input signals; $w_{i_1\ldots i_n}^j(\tau_1,\ldots,\tau_n)$ – Volterra kernel of order n by $i_1,\ldots, i_n$ outputs and output j, the functions symmetric relative to real variables $\tau_1,\ldots,\tau_n$; v, μ – number of CO inputs and outputs, respectively.

Each term here is a co-linear linear first-order filter, where the corresponding weights represent the impulse response of another linear filter of the next level.

The following notation has been adopted:

$\mathbf{X} = (x_1, x_{t-1},\ldots,x_{t-L})^{\mathrm{T}}$ – input vector; L – number of single delays; w_i , w_{ij}, w_{ijk} – weights of NN, named Volterra kernels coefficients, corresponding to the reactions of higher orders; K – levels count.

Volterra NN training, presented in Fig. 1, it is proposed to carry out with the help of a stochastic analog of an adaptive learning algorithm [13], which is based on the idea of the methods of conjugate gradients. Details of Volterra NN training algorithms can be found in [10; 12; 14].

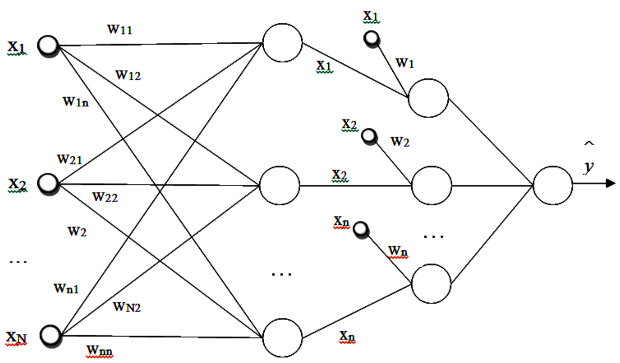

Fig. 1. Structure of the NN on the basis of the Volterra series at $K=2$

The algorithm for the construction of Volterra NN is considered in [15]. For the simulation of nonlinear dynamic objects, the considered algorithm was further developed.

Step 1. Determinate the number of input neurons based on the input vector **X**, the number of levels K, on which the filters are defined to be equal to 1.

Step 2. Carry out preliminary normalization of the data according to the formula:

$$x`(t) = [(x(t) - \mathrm{mean}(x(t)))/(\max(x(t)) - \min(x(t))]. \quad (2)$$

Step 3. Initialize the weights of the NN using Volterra coefficients determined by the expression (1).

Step 4. Calculate the learning error function by the formula:

$$E = 1/2\sum\nolimits_{t=1,\ldots,N}(y(t) - y`(t)). \quad (3)$$

We set the maximum error E_{max}, if $E < E_{max}$ and remember the weights **W**, else passing to the *Step 5*.

Step 5. Use an adaptive learning algorithm [17].

Step 6. Appreciate the complexity of the network. We consider that the complexity of the current network does not correspond to the given problem if the error E with the number of training periods decreases not fast enough or does not decrease at all. Reducing the speed below a given threshold signals that in order to improve the results when the error rate falls, one more level must be added, the alternate term in the Volterra series.

For a short time, the error rate can be determined as follows:

$$\Omega > (E_t - E_{t-\delta}) / E_{t0}. \qquad (4)$$

There E_{t0} – error value at the moment t_0;

t_0 – the moment of adding the previous level;

Ω` – some given minimum speed of the error change, in case of which the decision to change the network is taken;

δ – the number of epochs – the training cycles of the network before the product of the error rate estimation.

Step 7. Calculate y_n and E, repeating the training as long as there is a mistake $E < E_{max}$.

Remember **W**.

Note that the number of neurons in each hidden layer for the given model is the same and coincides with the number of input neurons. The learning process begins with one hidden layer if the required approximation accuracy is not reached, and then the next layer is added. Since, with the addition of a layer, the accuracy of the approximation increases, then ultimately the traceable accuracy is reached.

Experimental part. To test the program realization of the method of constructing Volterra NN for modeling nonlinear inertial CO, it is necessary to conduct an experiment on real data with the calculation of quality metrics.

Consider the test nonlinear dynamical object (Fig. 2).

Let $W_1(t)$ and $F(y)$ determinates by expressions:

$$W_1(t)=e^{-\alpha t},\ F(y)=\beta y^2(t). \qquad (5)$$

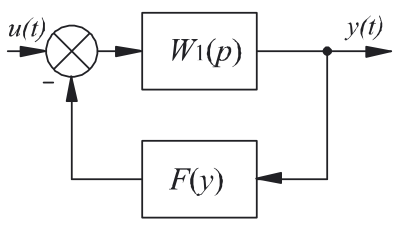

Fig. 2. The structure of the test nonlinear dynamic object

We consider α and β constants that are not available for measurements.

Analytic expressions for the first order Volterra kernel and the diagonal sequences of the Volterra kernels of the second and third order:

$$\begin{aligned} &w_1(\tau_1) = e^{-\alpha\tau}; \\ &w_2(t,t) = \frac{\beta}{\alpha}(e^{-2\alpha t} - e^{-\alpha t}), \\ &w_3(t,t,t) = 2\left(\frac{\beta}{\alpha}\right)^2 \cdot (e^{-3\alpha t} - 2e^{-2\alpha t} + e^{-\alpha t}) \end{aligned} \qquad (6)$$

After determining the weights of the NN, based on the coefficients of the Volterra series, graphs are constructed that reflect the analytical reactions of the CO and the responses obtained with the Volterra NN to the input test response in the form of the δ-function (Fig. 3). The given graph demonstrates rather close CO responses for both models: analytical and models in the form of the NN.

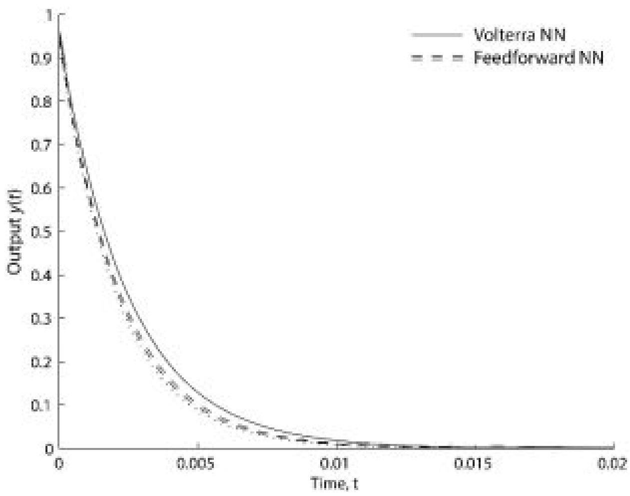

Fig. 3. Dependence of the response $y(t)$ with different values of the parameters α and β

The researches of the quality of the test CO model from depending from the structure of the NN and the training algorithm settings were performed. Volterra NN and NN of direct signal propagation parameters in modeling the test nonlinear dynamic CO are given in Table 1.

Table 1. Results of NN training

Parameter	NN	
	Volterra	Direct signal propagation
Desired accuracy	0,001	0,001
Errors	0,02	0,05
Learning speed	0,01	0,004

The table below shows the advantage of the Volterra neural network in front of the neural network of the direct distribution of signal in the learning speed and modeling accuracy.

Conclusions and perspectives of further research. Volterra models are a canonical representation of a wide class of nonlinear inertial systems. The task of creating neural networks for the modeling of nonlinear dynamic CO based on models in the form of Volterra series was successfully rescued.

The three-layer structure of the artificial neural network of the direct propagation of the signal for the representation of nonlinear dynamical systems was substantiated and investigated. A class of systems is outlined that can be effectively approximated by this network.

The dependence of Volterra coefficients and the weight coefficients of the latent layer of a three-layer NN of direct propagation of the signal is established.

An algorithm for constructing an artificial neural network based on the Volterra series has been developed.

The results of computer simulation of nonlinear dynamic systems with Volterra neural network and direct signal propagation are presented. The analysis of experimental data confirms the efficiency of using Volterra NN in the problems of simulation of nonlinear dynamical systems.

A generalized mathematical framework for evaluating the efficiency of various topologies of neural networks for nonlinear mapping of input vectors into weekly scalars (or vectors) is proposed.

Computational experiments demonstrate the advantage of the Volterra NN to the neural network of direct propagation of the signal at the learning speed and modeling accuracy in the simulation of nonlinear dynamic CO.

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ПОБУДОВА НЕЙРОННОЇ МЕРЕЖІ ВОЛЬТЕРРА В ЗАДАЧАХ МОДЕЛЮВАННЯ НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМ

Анотація. *Розглядаються особливості використання теорії рядів Вольтерра і нейронних мереж в задачах моделювання нелінійних динамічних систем. Проведено порівняльний аналіз методів побудови моделей нелінійних динамічних систем на основі теорії рядів Вольтерра і нейронних мереж, позначені області ефективного застосування кожного з методів. Сформульовано постановку задачі, яка полягає у створенні математичного апарату перетворення моделей нелінійних динамічних систем, отриманих на основі апарату рядів Вольтерра в штучну нейронну мережу певної структури. Обгрунтовано і досліджена тришарова структура штучної нейронної мережі прямого поширення сигналу для подання нелінійних динамічних систем. Окреслено клас систем, які можуть бути ефективно апроксимувати цією мережею. Встановлено залежність коефіцієнтів ядер Вольтерра і вагових коефіцієнтів прихованого шару тришарової нейрон-ної мережі прямого поширення сигналу. Наводиться алгоритм побудови штучної нейронної мережі на основі ряду Вольтерра. Представлені результати комп'ютерного моделювання нелінійних динамічних систем за допомогою нейронної мережі Вольтерра і прямого поширення сигналу. Аналіз експериментальних даних підтверджує ефективність викорис-ня нейронних мереж Вольтерра в задачах моделювання нелінійних динамічних систем. Зроблено висновки та надано рекомендації щодо ефективного застосування штучних нейронних мереж Вольтерра для моді-вання нелінійних динамічних систем.*

Ключові слова: *нейронні мережі; ряди Вольтерра; нелінійні динамічні системи*

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ПОСТРОЕНИЕ НЕЙРОННОЙ СЕТИ ВОЛЬТЕРРА В ЗАДАЧАХ МОДЕЛИРОВАНИЯ НЕЛИНЕЙНЫХ ДИНАМИЧЕСКИХ СИСТЕМ

Аннотация. *Рассматриваются особенности использования теории рядов Вольтерра и нейронных сетей в задачах моделирования нелинейных динамических систем. Проведен сравнительный анализ методов построения моделей нелинейных динамических систем на основе теории рядов Вольтерра и нейронных сетей, обозначены области эффективного применения каждого из методов. Сформулирована постановка задачи, состоящая в создании математического аппарата преобразования моделей нелинейных динамических систем, полученных на основе аппарата рядов Вольтерра в искусственную нейронную сеть определенной структуры. Обоснована и исследована трехслойная структура искусственной нейронной сети прямого распространения сигнала для представления нелинейных динамических систем. Очерчен класс систем, которые могут быть эффективно аппроксимированы этой сетью. Установлена зависимость коэффициентов ядер Вольтерра и весовых коэффициентов скрытого слоя трехслойной нейронной сети прямого распространения сигнала. Приводится алгоритм построения искусственной нейронной сети на основе ряда Вольтерра. Представлены результаты компьютерного моделирования нелинейных динамических систем при помощи нейронной сети Вольтерра и прямого распространения сигнала. Анализ экспериментальных данных подтверждает эффективность использования нейронных сетей Вольтерра в задачах моделирования нелинейных динамических систем. Сделаны выводы и даны рекомендации по эффективному применению искусственных нейронных сетей Вольтерра для моделирования нелинейных динамических систем.*

Ключевые слова: *нейронные сети; ряды Вольтерра; нелинейные динамические системы*