

# Methodological Problems in Solvency Assessment of an Insurance Company<sup>1</sup>

Rosa Coccozza<sup>2</sup>, Emilia Di Lorenzo<sup>3</sup>, Marilena Sibillo<sup>4</sup>

## Abstract

The recent wide development and changes in insurance markets highlighted the necessity to map out the solvency analysis in a more complete framework. The approach we present in the paper comes up with an integrated analysis of the risk profile of an insurance business, taking into account the actual European directives about solvency assessment. The aim of the paper is to construct a methodology apt to incorporate properly the effect of the risk sources in calculating mathematical provisions related to a portfolio of insurance policies.

**Jel classification:** G22, G28, G13

**Key words:** Life insurance, financial risk, demographic risk, capital adequacy, reserves, conditional random processes.

## 1. Life insurance business: a risk management approach

An insurance company is solvent “if it is able to fulfil its obligations under all contracts under all reasonably foreseeable circumstances” (IAIS 2002). Nevertheless, in order to come to a practicable definition, it is necessary to make clear under which situation the appropriateness of the assets to cover claims is to be considered.

The question, referring to the evaluation purposes, does not have a comprehensive answer, since it depends on many issues. Indeed, it is relevant whether the company is deemed as a closed operation or a going concern, thus including only written business (run-off basis) or also future new business (going-concern basis). Additionally, it depends on the aim of the evaluation that is the mere financial progress of the company or its ability to meet claims and other obligations under all but the most extreme circumstances. The first topic defines the relevant risk factors: on a going concern basis also fluctuations around the expected value of the new business will be taken into account (thus allowing for netting within different pools), while on a runoff basis only fluctuations within the single pool will be considered (thus excluding clearing). The second one defines the amplitude of the evaluation, which in probabilistic terms would correspond to the confidence level choice (from intermediate levels to extreme events). Therefore, solvency evaluation is a process, whose logical paradigm should sequentially consist of three main steps: relevant risk recognition, risk measurement and definition of capital requirements to absorb potential losses.

In general, the main risk for a firm is that revenues prove to be unable to cover expenses and, regard the valuation also the shareholders, to remunerate adequately capital invested. This is namely business risk. This very broad definition does embrace all risks and provide for measuring them through the variance of the expected profit over a time bucket (risk horizon, accounting period etc.) or also over the entire duration of the business. Premiums and claims respectively are typical insurance revenues and expenses: therefore business risk stems from the potential inequality among these elements, with the further difficulty that revenues have to be estimated before expenses because of the inverted cycle. Therefore, all the factors that can induce the inequality – and in fact to a loss – are relevant and define the whole risk subsystem. This approach does imply that

---

<sup>1</sup> This research was partially supported by Italian MIUR (Project: PRIN 2002 Metodi e strumenti per l’analisi e la gestione dei rischi nei settori delle assicurazioni di persone e previdenziale. Although the paper is the result of a common study, the first section is written by R. Coccozza and the rest two are by E. Di Lorenzo and M. Sibillo.

<sup>2</sup> Ph.D., Associate professor of financial intermediaries at the University of Naples, Italy.

<sup>3</sup> Professor of financial mathematics at the University of Naples, Italy.

<sup>4</sup> Professor of financial mathematics at the University of Salerno, Italy.

the capability of covering expenses by means of premiums depends on the design of relevant cost drivers. For life business, the main puzzles are mortality and interest rates<sup>1</sup>, which could impact of the single period performance (income), by the modification they produce on relevant costs and revenues. Therefore, concentrating on the pure premium components and performing an analysis on a run-off basis, the risk impact evaluation can start off on the breakdown of the income components over a single accounting period.

Profit ( $\pi_t$ ) can be outlined as the algebraic sum of: provisions at the beginning of the year ( $R_{t-1}$ ), earned premiums ( $P_t$ )<sup>2</sup>, investment income ( $(R_{t-1}+P_t)(e^\delta - 1)$ ), payments to policyholders ( $S_t$ ) and provisions at the end of the year ( $R_t$ ), that is

$$\pi_t = (R_{t-1} + P_t)e^\delta - [S_t + R_t]. \quad (1)$$

For an immediate temporary ( $n$ ) unitary annuity, equation (1) can be specified as<sup>3</sup>

$$\begin{aligned} \pi_t &= N_{t-1}^x r_{t-1}^x e^\delta - N_t^x (1 + r_t^x) = \\ &= N_{t-1}^x [r_{t-1}^x e^\delta - p_{x+t-1} (1 + r_t^x)] \end{aligned} \quad (2)$$

where  $N_t^x$  is the actual number of survivors at age  $x+t$  and  $r_t^x$  is the expected single reserve for each existing policy at the end of period  $t$ .

In this perspective accrued income can be seen as a function of the force of the total rate of return  $\delta$  for the period  $(t-1, t)$  and of the actual number of survivors  $N_{t-1}^x$  and of the probability  $p_{x+t-1}$ , and

$$\frac{\partial \pi_t}{\partial N_{t-1}^x} = r_{t-1}^x e^\delta - p_{x+t-1} (1 + r_t^x) > 0, \quad (3)$$

$$\frac{\partial \pi_t}{\partial p_{x+t-1}} = -N_{t-1}^x (1 + r_t^x) < 0, \quad (4)$$

$$\frac{\partial \pi_t}{\partial \delta} = N_{t-1}^x r_{t-1}^x e^\delta > 0. \quad (5)$$

The first derivative (equation 3) gives the filter of the *insurance risk*, that is a risk indicator for the event of an actual number of survivors diverse from the expected one. The indicator is always positive because a higher number of survivors in the preceding period forces the insurer to increase the corresponding provision, giving rise to a capitalised value larger than expected expenses. This benefit, although always positive, shows a dynamics connected to the age ( $x$ ) and the evaluation time ( $t$ ) as shown by Figure 1.

<sup>1</sup> For example, for temporary annuities, the minimal equilibrium is directly dependent on the single period differentials between the integral of the instantaneous total return on assets purchased with written premium and that of the original interest rate applied in premium rating and the logarithm of the actual number of survivors and the expected number of survivors (Cocozza et al., 2003b).

<sup>2</sup> For the sake of clarity, premiums are assumed to be earned at the beginning of the accounting period and payments to policyholders are assumed to be performed immediately before the end of the period.

<sup>3</sup> The aim of the paper is to outline a valuation methodology: we have decided to work all the examples through an annuity but any other kind of policy could serve our purpose.

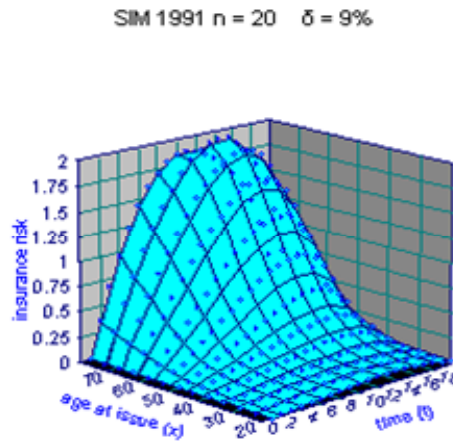


Fig. 1. Dynamics connected to the age and the evaluation time

The second derivative (equation 4) gives the *longevity risk* indicator, that is a proxy for the event that there is a shock in the mortality function. The negative sign accounts for the expense increase because of a larger number of survivors for the current period. The impact of the longevity risk is proportional to the size of the portfolio and to the reserve value, thus increasing not only for younger policyholders and/or for evaluation time closer to the issue (as shown by Figure 2) but also for larger size portfolios.

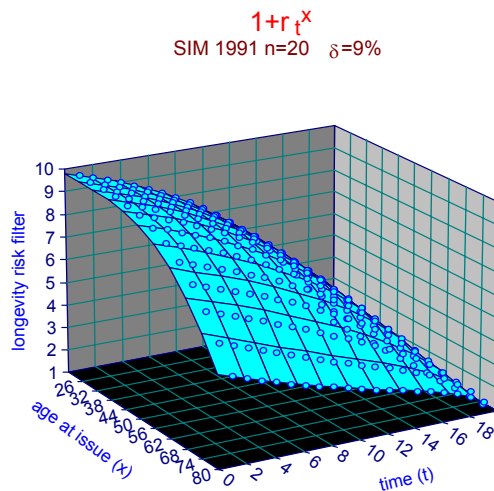


Fig. 2. The proportion of longevity risk impact th the size of the portfolio

The third derivative (equation 5) gives the *investment risk*, since it gives the multiplier of the rate shock. The positive sign of the indicator accounts for the direct proportionality between the rate shock and the income over period  $t$ ; naturally the higher the value of the reserve is, the stronger the impact appears to be (Figure 3).

SIM 1991  $n=20$   $\delta=9\%$

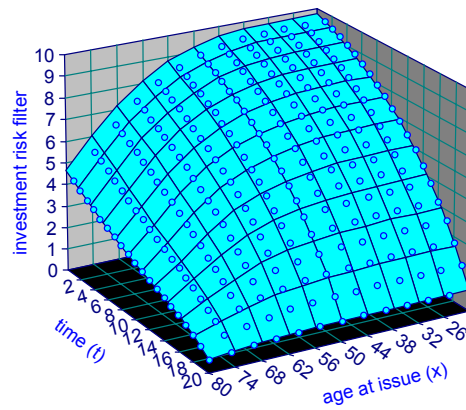


Fig. 3. The value of the reserve and the impact ratio

It can be easily shown that the impact of financial and longevity risk is far larger than insurance risk since their relevance is filtered through the reserve value connected to the portfolio size. This implies that their impact is directly proportional to the actual number of survivors  $N_{t,i}^x$  and, in a sense, to the number of issued policies  $c$ . In conclusion, the variation velocity of the profit is measured by means of the gradient

$$\nabla \pi_t = \left( \frac{\partial \pi_t}{\partial N_{t-1}^x}, \frac{\partial \pi_t}{\partial p_{x+t-1}}, \frac{\partial \pi_t}{\partial \delta} \right)$$

since

$$\Delta \pi_t \cong \nabla \pi_t (\Delta N_{t-1}^x, \Delta p_{x+t-1}, \Delta \delta). \tag{6}$$

Therefore, the total variability of the profit over the single time bucket can be divided into the main three components, thus giving the opportunity to separately identify the impact of a specific risk factor, also considering the time evolution of the indicators.

## 2. The mathematical representation

As already pointed out in the previous sections, in riskiness valuation concerning the liability components, the joint effect of demographic and financial factors plays a fundamental role. In this sense it is necessary to correctly evaluate the impact of such factors and their interactions on the mathematical reserve. On the basis of the balance equation, we can argue that fluctuations of the rate of return are particularly relevant for the assets, while on the liabilities the interactions between demographic and financial components have a more and more marked effect.

As a first step in solving the problem, we focus on the riskiness analysis concerning the mathematical reserve, for which we introduce a simple and a suitable measure, apt to quantify the synergy produced by the random fluctuation of interest and mortality (survival).

Let us consider a portfolio of  $c$  identical  $n$ -year temporary life annuity-immediate, each policy being of 1 unit payable at the end of each year while the life aged  $(x)$  survives. Let us denote the random variable representing the future lifetime of the  $i$ -th insured (for each  $i$ ) by  $T_i(x)$

and the curtate future lifetime of  $(x)$  by  $K_i(x)$ , the is the number of complete future years lived by  $(x)$  in  $[0, n-x]$ .

The *prospective loss* at time  $t$ , for the  $i$ -th policy, is defined as the difference  ${}_tL^{(i)}$  at time  $t$  between the present value of future benefit payments and the present value of future premium payments, assuming that  $T_i(x) > t$ :

$${}_tL^{(i)} = \sum_{j=1}^{K_i(x+t)} e^{-\int_t^{t+j} \delta(s) ds}, \quad (7)$$

where  $\delta(s)$  is the force of interest.

The *net premium reserve* at time  $t$  is defined as the conditional expectation of  ${}_tL^{(i)}$ , given that  $T_i(x) > t$ .

Let us denote by  ${}_tL$  the prospective loss for the entire portfolio, according to the following notation:

$${}_tL = N^x(t) \sum_{r=1}^{n-t} {}_r p_{x+t} e^{-\int_t^{t+r} \delta(s) ds}, \quad (8)$$

where  $N^x(t)$  is the number of survivors at time  $t$ .

Since in our analysis  ${}_tL$  is affected by two risk sources,  $\delta(s)$  and  $K_i(x)$ , the regression function  $[E[{}_tL | \delta(s)]]$  (Frees, 1998) provides an average of  ${}_tL$  over all values of  $\delta(s)$ . In this way we consider a conditional mean value with respect to the fluctuations of the insured's lifetime into the residual policy duration  $[t, n]$ . In this framework  $\text{var}[E[{}_tL | \delta(s)]]$  represents a measure of the volatility of  ${}_tL$  arising from the uncertain behaviour of the interest rate.

By means of this measure we quantify the financial risk, taking into account also the presence, even though averaged, of the demographic component.

We observe that:

$$\begin{aligned} \text{var}[E[{}_tL | \delta(s)]] &= \text{var} \left[ E \left[ N^x(t) \sum_{r=1}^{n-t} {}_r p_{x+t} e^{-\int_t^{t+r} \delta(s) ds} \mid \delta(s) \right] \right] = \\ &= \text{var} \left[ (c_t p_x) \sum_{r=1}^{n-t} {}_r p_{x+t} e^{-\int_t^{t+r} \delta(s) ds} \right] = \\ &= (c_t p_x)^2 \sum_{r=1}^{n-t} \sum_{s=1}^{n-t} {}_r p_{x+t} {}_s p_{x+t} \text{cov} \left( e^{-\int_t^{t+r} \delta(u) du}, e^{-\int_t^{t+s} \delta(u) du} \right). \end{aligned} \quad (9)$$

We can also observe that the residual risk component is quantified by the obvious difference because of the uncertainty of life durations

$$\text{var}[{}_tL] - \text{var}[E[{}_tL | \delta(s)]]. \quad (10)$$

### 3. How to quantify the uncertainty impact

Let us assume that the force of interest in the period under consideration is governed by a deterministic low,  $r(t)$ , deducible on the basis of the current relevant rates, corrected by a stochastic process,  $X(t)$ , apt to summarize the randomness affecting the rate evolution in time and to capture all its possible deviations from the deterministic pattern. So, it is natural to set (Di Lorenzo et al., 1999)

$$\delta(t) = r(t) + X(t). \quad (11)$$

In our example  $X(t)$  is an Ornstein-Uhlenbeck process, with parameters  $\beta > 0$  and  $\sigma > 0$ , and initial position  $X(0) = 0$ , involved by the following stochastic differential equation

$$dX(t) = -\beta X(t)dt + \sigma dW(t), \quad (12)$$

where  $W(t)$  is a standard Wiener process.

On the basis of stochastic calculus principles and after some lines of algebra, it is possible to obtain the following covariance and variance functions of the process  $\int_0^{t+h} X(s)ds$ :

$$\begin{aligned} \text{cov} \left[ \int_0^{t+h} X(s)ds, \int_0^{t+k} X(s)ds \right] &= \frac{\sigma^2}{\beta^2} \min(h, k) + \\ &+ \frac{\sigma^2}{2\beta^3} e^{-2\beta t} \left[ (1 - e^{-2\beta t}) (-1 + e^{-\beta h} + e^{-\beta k}) - e^{-\beta(h+k)} + e^{2\beta t} e^{-\beta|h-k|} \right] \\ \text{var} \left[ \int_0^{t+h} X(s)ds \right] &= \frac{\sigma^2}{\beta^2} h + \frac{\sigma^2}{2\beta^3} e^{-2\beta t} \left[ (1 - e^{-2\beta t}) (-1 + 2e^{-\beta h}) - e^{-2\beta h} + e^{2\beta t} \right] \end{aligned}$$

For a simpler representation of the formulas for the variance and the covariance functions of the evaluation stochastic factor in formula (9), we pose:

$$\text{cov} \left[ \int_0^{t+h} X(s)ds, \int_0^{t+k} X(s)ds \right] = \Phi(h, k)$$

and

$$\text{var} \left[ \int_0^{t+h} X(s)ds \right] = \nu(h).$$

Considering, in our hypotheses, that the evaluation factor is log-normally distributed, we can write:

$$\text{cov} \left[ e^{-\int_0^{t+h} X(s)ds}, e^{-\int_0^{t+k} X(s)ds} \right] = e^{\frac{1}{2}(\nu(h)+\nu(k))} \left[ e^{\Phi(h,k)} - 1 \right]$$

Now we are able to compute the impact of the risk sources on a life annuity portfolio. As an exemplification we implement our model in the case of a portfolio consisting of  $c=1000$  policies at issue, each one related to a life aged  $x$ , with duration  $n=20$ . With regard to the instantaneous interest rate, we fix a constant deterministic component  $r=0.06$ , and the parameters  $\beta=0.11$ ,  $\sigma=0.005$  for the stochastic one. In figures 4 and 5 we can observe the behaviour of the two risk components as functions of the age  $x$  and the reserve valuation time  $t$  (and, implicitly, of the policy residual duration).

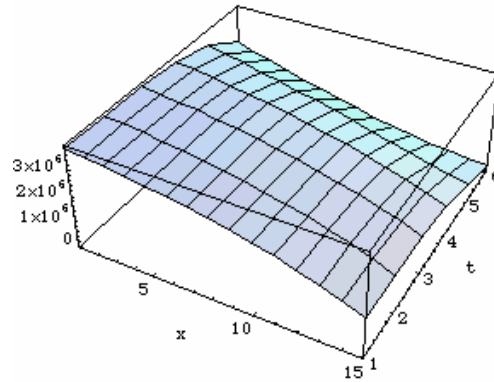


Fig. 4. The Financial Risk ( $x=20, \dots, 70$ ;  $t=10, \dots, 15$ )

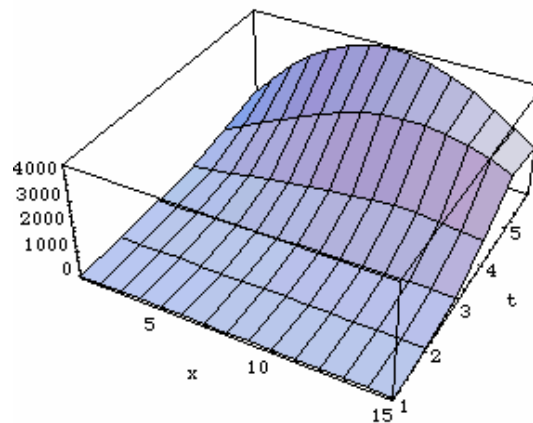


Fig. 5. The Insurance Risk ( $x=20, \dots, 70$ ;  $t=10, \dots, 15$ )

The financial risk, as function of  $x$ , decreases when  $t$  is fixed; the same behaviour appears when the roles of  $x$  and  $t$  are interchanged.

The insurance risk, for every fixed value of  $x$ , increases with  $t$  until a certain valuation time; the younger the insured is, the longer the period of the increasing behaviour is. This phenomenon depends on the reserve amount exposed to the demographic risk, taking into account the influence of the insureds' age and the residual duration of the portfolio

When  $t$  is fixed, the demographic risk generally increases with  $x$ , but for great values of  $t$  it decreases when the age becomes great, as it is clear reasoning as in the previous case.

#### 4. Conclusions

The application of risk factor analysis has given the opportunity to build up a methodology set able to evaluate the impact of different risk drivers, both in a deterministic and in a stochastic contexts. Therefore, it could be of valuable application for both external and internal controls.

It has been shown that, in both the contexts, there is a significant and consistent evolution along time of main risk indicators, thus suggesting the opportunity to scale the capital requirement according to the size and time evolution of the business.

Finally, it is necessary to underline that the extension of the analysis to other kind of policies could give rise to very different result as far the relevance and the time evolution of the risk indicators are concerned.

#### References

1. Babbel D.F., F.J. Fabozzi. Investment management for insurers. - New York: Wiley, 1999.
2. Babbel, D.F., J. Gold, C.B. Merrill. Fair value of liabilities: the financial economics perspective//North American Actuarial Journal 2002. N<sup>o</sup> 6 (1) - pp.12-27.
3. Coccozza, R. La gestione del rischio di tasso di interesse nelle assicurazioni del ramo vita. - Padova: CEDAM, 2000.
4. Coccozza, R., E. Di Lorenzo. Solvency of life insurance companies: methodological issues//Working paper 2003a.
5. Coccozza, R., E. Di Lorenzo. Risk profiles of life insurance business: a combined approach//VI Italian-Spanish Conference on Financial Mathematics (Trieste), 2003b.
6. Di Lorenzo, E., M. Sibillo, G. Tessitore. A Stochastic Model for Financial Evaluation: Applications to Actuarial Contracts//Applied Stochastic Models in Business and Industry 1999. N<sup>o</sup> 15 - pp. 269-275.
7. Frees, E. Relative Importance of Risk Sources in Insurance Systems//North American Actuarial Journal 1998. N<sup>o</sup> 2 (2) pp. 34-48.
8. IAIS Solvency & Actuarial Issues Subcommittee. On Solvency, Solvency Assessments and Actuarial Issues. 2000. IAIS, March.
9. IAIS Solvency & Actuarial Issues Subcommittee. Principles on capital adequacy and solvency. 2002. IAIS, January.
10. IASB Steering Committee. Insurance Issue paper. 1999. IASB, December
11. Knight, K. Mathematical Statistics - Boca Raton, Florida: Chapman and Hall/CRC, 2000.
12. Parker, G. Stochastic analysis of the interaction between investment and insurance risks//North American actuarial journal 1997. N<sup>o</sup> 1 (2) – pp. 55-84.
13. Vanderhoof, I.T., E.I. Altman (eds). The fair value of insurance liabilities - Boston. Dordrecht, London: Kluwer Academic Publishers. 1998.