A DYNAMIC APPROACH FOR THE EVALUATION OF PORTFOLIO PERFORMANCE UNDER RISK CONDITIONS

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Abstract

In this paper, we examine the behavior of a stock portfolio in reaction to possible fluctuations in the stock market. Our analysis is based on stress test and simulation techniques. Using historical data, we construct a portfolio (profile portfolio) with specific characteristics for volatility, return and beta coefficient. Our simulations suggest that adjustments must be made in a portfolio's composition in order to obtain maximum return or minimum risk, with respect to investors' preferences. We conclude that an asset manager can achieve the highest return by keeping his original position in the medium risk shares and adjusting his original position for the low and high risk shares. Finally, we compare different portfolios based on market capitalization and we obtain an optimum composition.

Key words: beta coefficient, portfolio management, stress test, simulation, dynamic system.
JEL Classification: C61, G11, G29.

1. Introduction

There are many recent quantitative studies that examine portfolio behavior, especially the years when financial markets experienced large fluctuations. The quantitative methods are usually based on "stress testing" and simulation analysis. Moreover, many mathematical models have been developed using tools from dynamic programming and optimization techniques. This paper is based on Markowitz's portfolio theory and on the assumption that portfolio optimizers respond to the risk of an investment by selecting portfolios that maximize return subject to a specified level of calculated risk, or equivalently minimize risk subject to achieving a specified level in expected return (Markowitz, 1952, 1959, 1991; Merton, 1972; Kroll, Levy and Markowitz, 1984). The difference of this paper from previous research is that we develop a system equation method, representing a "profile" portfolio in order to examine the behavior of a portfolio. The selection of the portfolio's shares is based on investors' profile which is determined by the beta coefficient as a proxy variable for risk.

Portfolio optimization problems were first introduced by Merton (1969) who considered a case where transactions can be carried out cost free with the aim to maximize discounted consumption utility. In this setting, the optimal policy, according to Merton, is to continuously transact in order to hold fixed fractions of total wealth in various shares and even if very small transaction costs were present, transactions would be necessary only if the fraction of stock holding is "sufficiently" far away from Merton's optimal fraction to warrant the transaction. Magill and Constantinides (1976) first considered a portfolio optimization problem with one risky asset, proportional transaction costs and an interval in the target risky asset proportion. They show that if the proportion varies randomly within this interval, no trading is needed. When the proportion lies outside the interval, the optimal policy would be to buy or sell just enough to bring the proportion back into the interval.

In a number of studies, dynamic programming is used in the formulation of financial problems (Consigli and Dempster, 1998; Walter et al., 2004; Tenney, 2005; Muthuraman and Zha, 2005). The methodologies in these studies are based on dynamic stochastic programming, Monte Carlo simulation, free-boundary Hamilton Jacobi Bellman (HJB) equations (Bellman, 1957) etc. These studies seek for the best allocation of wealth, given a set of assets and targeting the mean variance portfolio optimization. According to a recent study (Bai et al., 2006), it has been demonstrated that

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the estimated return for the Markowitz mean-variance optimization (1952, 1959) is significantly different from the value which the theory predicts. In the above mentioned study, it is proved that the estimated optimal return is always larger than the relevant theoretical parameter. In addition, developing new bootstrap estimators for the optimal return and its asset allocation, they prove that bootstrap estimates are consistent with their counterpart parameters. Nevertheless, this result is not new. Several studies (Sharpe, 1967, 1971; Stone, 1973; Elton, Gruber, and Padberg, 1976, 1978; Markowitz and Perold, 1981; and Perold, 1984) computing the corresponding estimates, indicate that there have been persistent doubts, held by both academics and practitioners, who hesitate to apply the mean variance paradigm, since they are cautious about the performance of the estimates.

Michaud (1989) reports that a relativly low level of analytical sophistication in the culture of institutional equity management is one often-cited reason for the lack of mean variance optimization. He considers mean-variance optimization to be one of the outstanding puzzles in modern finance that it has yet to meet with widespread acceptance by the investment community, particularly as a practical tool for active equity investment management. He names this puzzle the "Markowitz optimization enigma".

As we mentioned before, in this paper we examine the behavior of a profile portfolio in which investors' preferences are determined by the beta coefficient, the proxy variable for risk in our model. Simulating our model, we observe differences in the portfolio's composition in order to obtain the optimal portfolio, with respect to investors' preferences.

The rest of the paper is organized as follows: in section 2 we discuss the methodology and the data sets, in section 3 we present our results, and finally, in section 4 we report the conclusions of our paper.

2. Methodology and Data

The purpose of this study is to examine the behavior of a shares portfolio, and to calculate some basic factors of the portfolio that reflect its future behavior. In order to achieve this, we follow a stress test method by simulating a dynamic system that represents the portfolio's "profile". The dynamic system includes the equations describing the portfolio composition as well as the portfolio characteristics.

Stress testing is a simple form of scenario analysis. In our analysis we assume that the portfolio managers may specify certain fixed assumptions (defined in terms of percent changes in applicable risk factors) and based on these assumptions, we perform periodic stress testing. Rather than considering the evolution of risk factors over several time intervals, stress testing examines changes in risk factors over a single time period. In our simulation analysis, we consider as a risk factor the fact that the shares portfolio might experience possible changes in terms of shares risk exposure. We use stress testing in order to assess market risk, since we have assumed that the shares' beta coefficient might change over time.

We perform a single scenario that consists of projected values for the probabilities of migration at the end of the horizon. The result is compared with the portfolio's current market value, and the portfolio loss is calculated as the difference between the two. In this respect, the portfolio manager might present stress test results in his daily risk report. Such stress scenarios may be hypothetical, reflecting possible large fluctuations in the stock market. Nevertheless, they can also be historically based. In this case, stress scenarios may reflect the percentage changes in risk factors experienced during historical periods selected, such as stock market crashes, currency devaluations, etc.

We have developed a method based on the optimization techniques, in order to obtain the maximum return regarding to specific portfolio characteristics. The general optimization problem can be set as follows:

$$\max f(x_1,...,x_n), \tag{1}$$

subject to the constraints:

$$g(x_1,...,x_n) \le b_i,$$
$$\varphi(x_1,...,x_n) \le c_i, \text{ with } i = 1,...,m, \tag{2}$$

By solving this problem, we try to find the values of variables $x_1,...,x_n$ that give us the maximum value of the function f . However, the values of $x_1,...,x_n$ must satisfy the constraints. We find among the values of variables $x_1,...,x_n$, satisfying the above constraint, the one giving the largest value to the objective function f . In the case of only two variables, say x, y, one might solve the $g(x, y) \le b$ for y in terms of x , and then substitute into f to get an unconstrained problem in the single variable x . However, in a more general case of an optimization problem, the solution should be found with a more complex method.

In order to find the values of $x_1,...,x_n$, and then the maximum value of function f , we should calculate the Lagrange multipliers for the given optimization problem. The Lagrangian multiplier is the following :

$$L(x_1,...,x_n,\lambda_i) = f(x_1,...,x_n) + \sum_{i=1}^{m} \lambda_i \cdot [b_i - g(x_1,...x_n)] + \sum_{i=1}^{m} \lambda_i' \cdot [c_i - \varphi(x_1,...x_n)] \quad (3)$$

then we set the partial derivatives of the above relation equal to zero, as shown:

$$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i (b_i - g_i), \quad (4)$$

$$\frac{\partial L}{\partial x_j} = \frac{\partial \varphi}{\partial x_j} - \sum_{i=1}^{m} \lambda_i' (c_i - \varphi_i) \text{ with } j = 1,...,n,$$

$$\frac{\partial L}{\partial \lambda_i} = b_i - g_i, \; \frac{\partial L}{\partial \lambda_i'} = c_i - \varphi_i, \quad \lambda \ge 0, \; \lambda(b - g) = 0 \text{ with } i = 1,...,m, \quad (5)$$

where $\frac{\partial L}{\partial x_j}$ is the partial derivative of f with respect to x_j . Similarly, g_i and φ_i are the partial derivatives of g and φ with respect to x_i . The above relations are the necessary conditions for the solution of the optimization problem. The first part of Lagrangian relations restates the constraint, while the second part is the new conditions of the problem. Nonnegativity of λ reflects the fact that increasing b enlarges the feasible region for values of $x_1,...,x_n$ and therefore cannot reduce the maximum value attainable. The last equation says that either the multiplier is zero or else the constraint is tight.

Based on the above optimization theory, we construct a dynamic system with equations that describe the portfolio behavior. We develop an equation system using historical data in order to estimate important factors such as portfolio return, beta coefficient and the proportion of the stocks for each beta category. For our analysis, we have categorized the stocks into 3 categories based on risk exposure. Initially, we have defined the number of stocks, actually the percentage per beta category that already belongs to the portfolio, as well as the return and volatility of each beta category. Table 1 shows the variables in our model that reflect the characteristics of the stocks belonging to portfolio.

Table 1

Portfolio characteristics

Rating	Risk category	Beta	Return	Volatility	Portfolio weight
1	Low risk stocks	b_1	r_1	v_1	w_1
2	Medium risk stocks	b_2	r_2	v_2	w_2
3	High risk stocks	b_3	r_3	v_3	w_3

Implementing the model, we can measure basic factors, such as the portfolio return, volatility, beta and finally, the portfolio composition. This implies that by changing the initial conditions of the system, the characteristics of the stocks, e.g. the return or the volatility and the portfolio weight, we can calculate the maximum return that this specific portfolio can achieve. Simulating the model, we can find the portfolio composition and moreover the characteristics of it, that provide us with the maximum return. The purpose of the equation system is to maximize the portfolio return with specific initial conditions and subject to the constraints.

The equation system has the following form:

$$\max r_p \tag{6}$$

subject to the following constraints:

$$\begin{gathered} 1)\ b_l < b_p < b_h, \\ 2)\ 0 < w_1 < 1,\ 0 < w_2 < 1,\ 0 < w_3 < 1, \\ 3)\ 0 < w_1 + w_2 + w_3 < 1, \end{gathered} \tag{7}$$

where w_1 is the factor denoting the percentage of stocks with beta 1 in the portfolio, w_2 is the factor denoting the percentage of stocks with beta 2 in the portfolio, and w_3 is the factor denoting the percentage of stocks with beta 3 in the portfolio, r_1 is the factor that reflects the return of stocks with beta 1, r_2 is the factor that reflects the return of stocks with beta 2, and r_3 is the factor that reflects the return of stocks with beta 3, and finally the portfolio return equals $r_p = w_1 r_1 + w_2 r_2 + w_3 r_3$ and b_l, b_h are the low and high beta coefficients of portfolio, respectively.

The variables that can be measured and describe the portfolio behavior, have the following form:

$$b_p = w_1 b_1 + w_2 b_2 + w_3 b_3,$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + 2w_1w_3\rho_{13}\sigma_1\sigma_3 + 2w_2w_3\rho_{13}\sigma_2\sigma_3 \text{ ,(8)}$$

where σ_p^2 represents the volatility of portfolio, σ_i^2 is the factor that represents the volatility of stocks that belong to i (i = 1,2,3) beta category and ρ_{12} is the correlation coefficient between stocks that belong to 1st and 2nd beta categories, ρ_{13} is the correlation coefficient between stocks that belong to 1st and 3rd beta categories, and similarly, ρ_{23} is the correlation coefficient between stocks that belong to 2nd and 3rd beta categories.

An important feature of our work is that in the simulation of our dynamic system, we have considered that stocks in a specific beta category might migrate to another beta category through time. We calculate the basic factors of the portfolio, and we maximize the portfolio return by taking into account possible changes in beta categories. This implies that the possibilities of beta changes may reflect the trend of the market. Implementing the model, we can predict the future behavior and compute the characteristics of the portfolio only by changing the possibilities of the system. Thus, one can find the maximum portfolio return and predict the future behavior of it just by setting the possibilities of changes in beta categories or/and by setting a range of portfolio beta, which he or she wishes for the portfolio. Thus, the simulation of the system provides us with the maximum portfolio return regarding significant factors, such as volatility, portfolio composition and portfolio beta.

The data used in this analysis have been collected from two sources. The statistical analysis concerning portfolio's stocks, weights and possibilities of changing risk category due to beta change is based on the performance of one of the biggest mutual funds in Greece, the Delos Mutual Fund of the National Bank of Greece. From the whole set of data, which is available on a three month data period basis, we used only the information connected with stocks, as we assume that the portfolio consists of stocks at a 100% weight.

The beta coefficients on a three month basis and the fluctuations over time have been collected from the "Beta Book" of the Center of Financial Studies of the National and Kapodistrian University of Athens. The Beta Book is published every three months, the same time interval that mutual funds are obligated to publish their tables of investments. The whole period for our statistical analysis is October 2004 - March 2007.

3. Results

We perform a stress test method by simulating a dynamic system that represents the stock portfolio's "profile". The dynamic system includes the equations that describe the portfolio composition as well as the portfolio characteristics. Implementing stress testing, as a simple form of scenario analysis, we assume that the portfolio management may specify certain fixed scenarios. In our analysis, the fixed scenarios are presented by the possibilities that a stock of a specific risk category might migrate to another beta category through time because of market fluctuations.

Simulating the model, we can compute portfolio's volatility, return, beta and composition given a range of portfolio beta (according to the desired profile). Given the portfolio beta ranges, we observe the differences in the portfolio composition and we extract conclusions on how an asset manager has to operate in order to achieve the maximum return given the portfolio minimum risk. The way that an asset manager has to react because of the market fluctuations, as he has to restructure the portfolio, is very important as it determines portfolio's characteristics and especially the return and the volatility. In addition, the way that he will achieve the optimum portfolio composition affects the total cost of transactions, consequently portfolio's return and maybe volatility (because of the lesser transactions).

The portfolio that we use in our analysis has the following initial characteristics:

Table 2

The initial characteristics of the portfolio

Risk category	Return	Volatility	Beta	Portfolio weight
Low risk stocks	3.40%	0.64%	0.75	33%
Medium risk stocks	5.10%	0.98%	1.00	34%
High risk stocks	6.90%	1.38%	1.25	33%

In order to simulate the behavior of the stock portfolio, we have considered that the stocks of specific risk category might migrate to other risk category through time. Using historical data for our analysis, we estimate the following probabilities. The probabilities calculated from historical data denote the possibility that a stock that belongs to a specific risk category may move to other risk category.

Table 3

Possibilities of beta category migration

Risk category		Initial risk category		
		Low	Medium	High
Probabilities of changing at t+1	Low	71.00%	10.00%	6.00%
	Medium	21.00%	75.00%	26.00%
	High	8.00%	15.00%	68.00%

The above matrix reflects the probabilities that the stocks may change risk category, which implies that a stock of low risk may migrate to the medium risk category, or to the high risk, or remain in the same risk category in the next time period, etc.

Next, in our model we use two different sets of correlation coefficients denoting the different characteristics of the portfolio. We use the dynamic system to maximize the portfolio return with constraint that the value of beta coefficient may vary in a range determined by the portfolio manager. Simulating the model, we get the maximum portfolio return; and, moreover, we can calculate the optimal portfolio composition, as well as the volatility and the portfolio beta.

Performing the stress test method using the dynamic model with $\rho_{12} = 0.32$, $\rho_{13} = 0.20$ and $\rho_{23} = 0.40$ (case 1), we obtain the results of Table 4.

Table 4

Portfolio characteristics of case 1

Range of portfolio beta	Max return	Volatility	Optimum portfolio beta	Optimum portfolio composition		
				w_1	w_2	w_3
0.95<b<1.05	5.49%	0.85%	1.05	27.84%	24.32%	47.84%
0.90<b<1.10	5.84%	0.95%	1.10	17.44%	25.12%	57.44%
0.85<b<1.15	6.19%	1.07%	1.15	7.04%	25.92%	67.04%
0.80<b<1.20	6.22%	1.08%	1.15	6.00%	26.00%	68.00%
0.75<b<1.25	6.22%	1.08%	1.15	6.00%	26.00%	68.00%
0.70<b<1.30	6.22%	1.08%	1.15	6.00%	26.00%	68.00%

Table 5 shows the results of portfolio simulations concerning that the correlation coefficients have higher values. This is case 2 where $\rho_{12} = 0.50$, $\rho_{13} = 0.10$ and $\rho_{23} = 0.50$.

Table 5

Portfolio characteristics of case 2

Range of portfolio beta	Max return	Volatility	Optimum portfolio beta	Optimum portfolio composition		
				w_1	w_2	w_3
0.95<b<1.05	5.49%	0.92%	1.05	27.84%	24.32%	47.84%
0.90<b<1.10	5.84%	1.03%	1.10	17.44%	25.12%	57.44%
0.85<b<1.15	6.19%	1.15%	1.15	7.04%	25.92%	67.04%
0.80<b<1.20	6.22%	1.16%	1.15	6.00%	26.00%	68.00%
0.75<b<1.25	6.22%	1.16%	1.15	6.00%	26.00%	68.00%
0.70<b<1.30	6.22%	1.16%	1.15	6.00%	26.00%	68.00%

Concerning the results, we observe that the maximized portfolio return reaches a stable "point" for each case of correlation coefficients. This implies that the value of maximized portfolio return cannot be above a specific level, i.e. 6.22%, and similarly the volatility can be above 1.08% and 1.16% for the cases 1 and 2, respectively. So, even if we widen the range of the portfolio beta in the model in order to have the chance for a higher portfolio return, the maximum portfolio return that we can obtain is specific. At this point, we can say that the equation model has reached a steady state, or, differently, to an equilibrium point. Any further change to the constraint of range in portfolio beta doesn't influence the result of the system.

The most interesting result is connected with the portfolio composition for every beta range. The percentage of the medium risk stocks in the portfolio remains stable and around 25%. As the asset manager restructures the portfolio, he achieves the highest return – minimum risk point by substituting low risk stocks with high risk stocks. The final portfolio composition is characterized by high risk stocks in a proportion 2/3 for risky stocks. This result leads to an asset management strategy as a manager can achieve the highest return by retaining stable his position in the medium risk stocks and restructuring the proportions of the low – high risk stocks for the riskier stocks.

Next, we simulate the dynamic model and calculate the maximum portfolio return by changing the return of stocks per beta category. In so doing we want to examine the influence of stocks per beta category in the portfolio return. Figure 1 shows the change of maximized portfolio return as the return of stocks per beta category changes. The black line indicates the relationship between the maximized portfolio return and return of stock per beta category without taking into account the possibilities that the stocks may migrate to other beta category. On the other hand, the grey line describes the same relationship, however, in the case that there are possibilities of change, as we have mentioned in the previous part of the work.

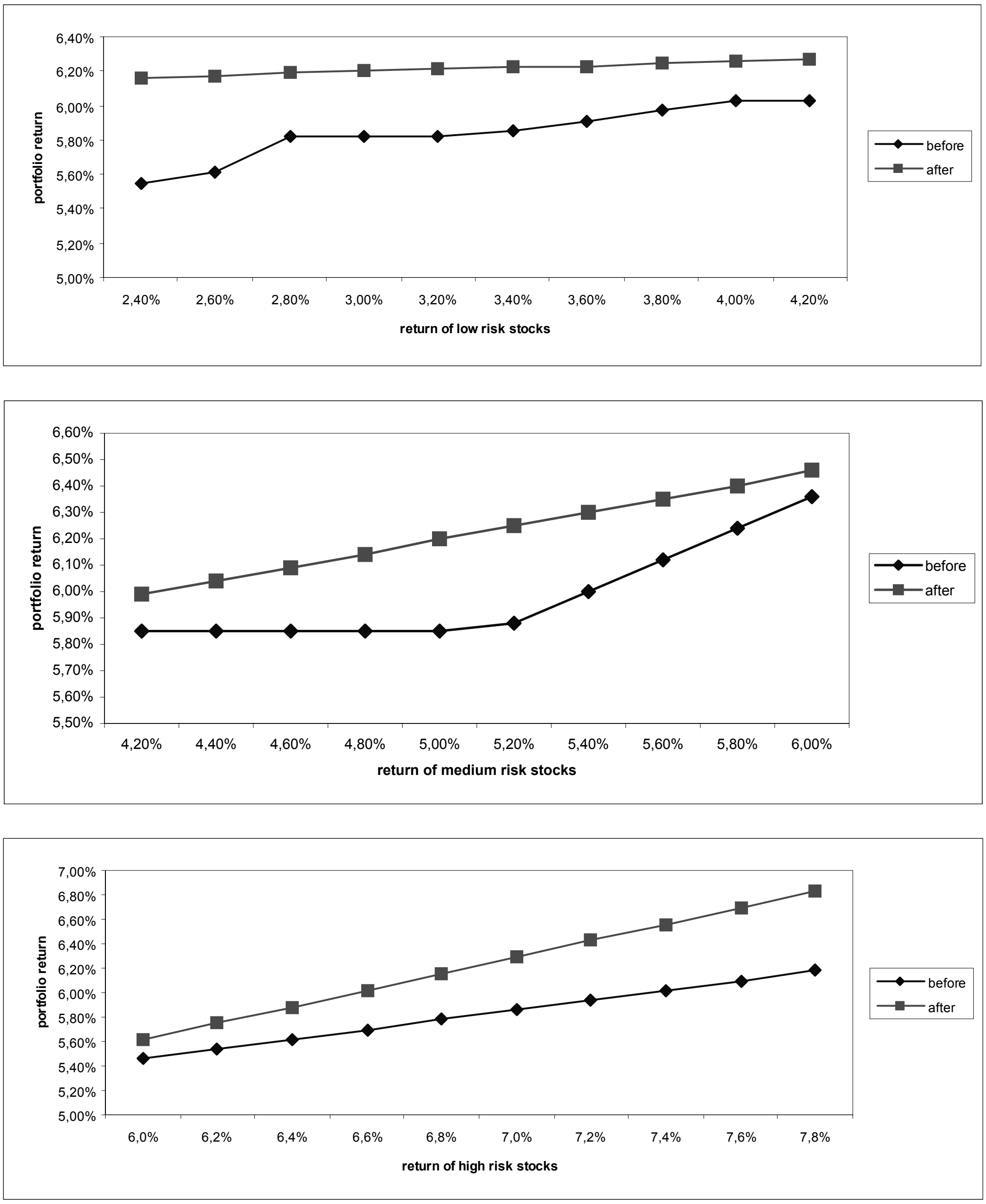

Fig. 1. Return of low, medium and high risk stocks

It is obvious that the increase in portfolio return is dependent on the possibilities of migration. The possibilities of migration indicate that the stocks of a specific beta category will migrate to other beta category. This implies that the amount of stocks in a low/medium/high risk category, at the next time period, will not be the same, since they change beta category. This has happened because stocks with low return migrate to other beta categories with higher return, like the high beta category. The same result holds in the case of stocks with medium risk. A percentage of "medium return" stock moves to high return stocks and a low percentage moves to low return category. So, the location of lines in the graphs depends on the possibilities of migration. This is illustrated in the 3 diagrams, that have been created regarding to the possibilities that reflect the percentage of stocks that change beta category (see Table 3).

4. Conclusions

We used a stress test method in order to study the behavior of a stock portfolio concerning specific characteristics, such as its composition and the portfolio's return and volatility. We constructed a dynamic system with equations that describe the behavior of a stock portfolio. The system parameters are estimated from quarterly data, using statistical and econometric techniques. By implementation of dynamical system equations, we chose the suitable portfolio composition among shares with different betas in order to obtain the maximum return with regard to the characteristics of the portfolio. Simulating the model, we achieved the maximum portfolio return related to the specific portfolio characteristics that might be given by the portfolio manager. We concluded that the best asset management strategy, given that the manager targets to achieve the highest possible return, is maintaining a stable portfolio position in the medium risk shares and restructuring the portfolio asset proportions of the low and high risk ones.

Moreover, we enhanced the above model by introducing the possibility that shares with a specific beta characterization may migrate to another risk category. Similarly, we computed portfolio's characteristics (volatility, return, beta) and its composition given a range of portfolio beta and the possible stocks' migrations from a risk category to another. Finally, this model may be used to compare shares portfolios, and calculate basic financial indices.

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