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Multiscale hedge ratio between Taiwan stock and futures index: an application of wavelet analysis

Abstract

The objective of this study is to examine the relationship between the stock and futures markets of Taiwan in terms of lead-lag relationship, correlation, and hedge ratio using wavelet analysis. We sampled 1,510 observations, from January 2002 to December 2007, of TAIEX and TAIFEX for analysis. Empirical results show that (1) there is a feedback relationship between the stock and futures markets regardless of time scales; (2) wavelet correlation between two markets varies over investment horizons but remains at high level; and (3) hedge ratio and the effectiveness of hedging strategies increase as the wavelet time scale increases.

Keywords: hedge ratio, wavelet analysis, lead-lag relationship.
JEL Classification: C10, G13.

Introduction

With the diversification of financial instruments and rapid economic growth, it is more convenient and necessary for financial managers, as well as investors, to hedge their investments using the popular derivatives, futures. If the movement of futures markets could be anticipated precisely, then we could take some measures to reduce risk in advance. However, to hedge effectively in futures markets, it depends not only on an accurate hedging model but also a relative insight of it. Thus, precision in adoption of hedging strategy had become a heated research issue.

As we know, there exists a certain relationship between stock and futures market. In order to estimate hedge ratio, many researchers usually employed Ordinary Least Squares (OLS) regression. Early work simply used the slope of an OLS regression of stock on futures prices. However, there is wide evidence that simple regression model is inappropriate to estimate hedge ratios as it suffers from the problem of serial correlation in the OLS residuals and the problem of heteroskedasticity often encountered in cash and futures price series. An improvement has been made on past literature to adopt a univariate GARCH, bivariate GARCH, or E-GARCH, as well as other stochastic volatility models. Although these studies are successful in capturing the time-varying covariance/correlation features, many of them focus mainly on the myopic hedging problem. Another new analysis method of this topic is Wavelet analysis, the analysis of change. A wavelet coefficient measures the amount of information that is gained by increasing the frequency at which the data is sampled, or what needs to be added to the data in order for it to look like it had been measured more frequently. Owing to such advantages, we employ this method for the research.

The main purpose of this study is to introduce a new approach for investigation of the relationship between the stock and futures markets of Taiwan in terms of the lead-lag relationship, covariance (correlation), and the hedge ratio in various time scales using wavelet analysis. In this study, we employ a different testing methodology as compared with previous studies.

Applying wavelet analysis to examine these factors has at least three salient features. First, the main advantage of using wavelet analysis is the ability to decompose the data into several time scales (investment horizons). Consider the large number of regulators and speculative investors who trade in the stock and futures markets and make decisions over different time scales. In fact, owing to the different decision-making time scales among traders, the true dynamic structure of the relationship between the stock and futures markets itself will vary over different time scales associated with those different horizons. Economists and financial analysts have long recognized that there are several time periods in decision making, whereas economic and financial analyses have been restricted to at most two time scales (the short run and the long run) because of the lack of analytical tools to decompose data into more than two time scales (In and Kim, 2006). Unlike previous studies, in this paper we use wavelets to produce an orthogonal decomposition of correlation and the hedge ratio between the stock and futures indices over several different time scales. In particular, this feature of time scale decomposition enables us to examine the lead-lag relationship between the stock and futures markets at different investment horizons.

Secondly, the wavelet co-variance decomposes the covariance between two stochastic processes over different time scales. A wavelet covariance in a particular time scale indicates the contribution to the covariance between two stochastic variables (Lindsay et al., 1996). This feature of wavelet analysis allows us to examine the covariance/correlation over different time scales.

The final feature of wavelet analysis is related to the calculation of the hedge ratio. We note that the conventional estimation of the hedge ratio has three problems: First, it is an unreliable estimator because of a handful of independent observations generated from long-horizon return series (see Geppert, 1995). Second, computation is burdensome and difficult to calculate over longer investment horizons. Third, it requires an assumption for the error term for GARCH/SV model estimation (see Lien and Luo, 1993), which can cause inaccurate results. Thereby, applying wavelet analysis enables us to overcome these obstacles through the time scale decomposition and provision of a non-parametric method.

The remainder of the paper is organized as follows: section 1 reviews related literature. Section 2 discusses minimum variance hedging and the degree of hedging effectiveness that we derive and describes the fundamental methods of wavelet analysis that we employ. Section 3 presents the data and basic statistics and a discussion of the empirical results. The last section presents some concluding remarks.

1. Literature review

Majority of the studies investigating the hedging on stock index futures relates to the U.S. In the first analysis of hedging effectiveness of stock index futures, Figlewski (1984) calculated the risk and returns combination of different capitalization portfolios underlying five major stock indices that could have been achieved by using the S&P 500 stock index futures as a hedging instrument. The risk minimizing hedge ratios were estimated by OLS on historical spot and futures returns. He discovered that for all indices represented diversified portfolios, minimum variance hedge ratios (MVHRs) were better than the beta hedge ratios. With large capitalization portfolios, risk was considerably reduced in contrast to smaller stocks portfolios. Moreover, Figlewski (1984) pointed out that dividend risk was not an important factor, whereas time to maturity and hedge duration were. Junkus and Lee (1985) investigated the hedging effectiveness of three U.S stock index futures under alternative hedging strategies. The optimal hedge ratios were calculated using the OLS conventional regression model. Their results indicated the superiority of MVHR. Moreover, there was little evidence about the impact of contract expiration and hedging effectiveness. Ghosh (1993) extended studies of lead and lag relationships between stock index and stock index futures prices by using an ECM, arguing that the standard OLS approach is not well specified in estimating hedge ratios because it ignores lagged values. Holmes (1996) tried to assess the appropriate econometric technique when estimating optimal hedge ratios of the FTSE-100 stock index by applying a GARCH (1, 1) as well. He showed that in terms of risk reduction a hedge strategy based on MVHRs estimated using OLS outperforms optimal hedge ratios that are estimated using more advanced econometric techniques such as an error correction model or a GARCH (1, 1) approach.

Wavelet analysis is employed extensively in science or engineering filed but far less is applied in economics and finance in the past. Many of wavelet analysis properties are suitable for description of time series data, which drive wavelet analysis flourishing development within this decade. Recent applications of wavelets in economics and finance include Ramsey and Lampart (1998), study the permanent income hypothesis, and conclude that the time-scale decomposition is very important for analyzing economic relationships. Norsworthy, Li and Gorener (2000) apply wavelet analysis to estimate the systematic risk of an asset (the beta of an asset). The main conclusion of Norsworthy et al. is that the major part of the market's influence on an individual asset return is at higher frequencies. In other words, the beta coefficient will generally decrease when regressing an individual asset return on the smoother components of the market portfolio. Fernandez (2006) extended the literature in this area to analyze the impact of time scaling on the computation of value at risk and conclude that risk is concentrated at the higher frequencies of the data.

2. Data and methodology

We will conduct our study adopting the procedure as in Figure 1. First, we transform our data from time domain into frequency domain using wavelet transform. The next step we take is to reconstruct the data into different scales under the condition that all of original message is complete reserved. Finally, we will use these data sets to compute statistic estimates like variance, covariance, and correlation for the purpose of estimating hedge ratio.

2.1. Data description. The data set used in the analysis is obtained from TAIEX and TAIFEX, covering the period from January 4, 2002 through December 30, 2007. After matching the daily observations, we have 1,510 observations for TAIEX. These indices are transformed to daily rates of return by calculating continuously compounded index returns, $R_t = 100 \times \ln(S_t/S_{t-1})$. Note that these 'daily' rates of return on a given calendar day may represent the returns realized over different time interval depending on trading day schedules.

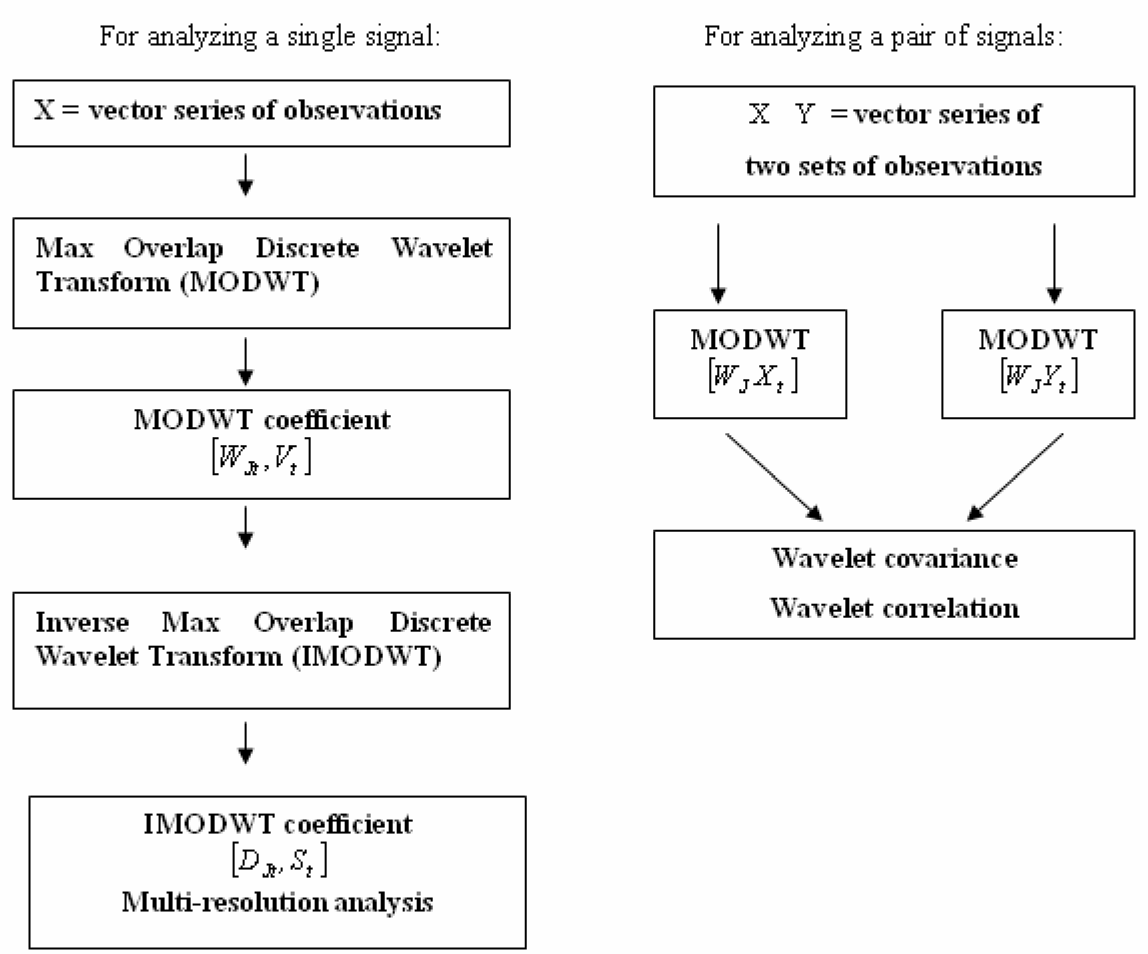

Fig. 1. MODWT Workflow

2.2. Mean variance hedge. Assume an individual has taken a fixed position in some assets and that this person is long one unit of the asset without loss of generality. Let h_t represent the short position taken in the futures market at time *t* under the adopted hedging strategy. Besides, assuming the investor has the mean-variance expected utility function.

$$EU(\Delta HP) = E(\Delta HP) - \gamma Var(\Delta HP), \tag{1}$$

where γ is the degree of risk aversion ($\gamma > 0$), the hedger's objective within this framework is to maximize the utility subject to Equation (1):

$$\underset{h}{\text{Max}}\ EU(\Delta HP_t) = \max_h \left[E(\Delta S) - hE(\Delta F) - \gamma(\sigma_s^2 + h^2\sigma_f^2 - 2h\sigma_{sf}) \right], \tag{2}$$

where ΔHP – change in the value of the hedge portfolio; ΔS – change in the log of the stock prices; ΔF – change in the log of the futures prices; h_t – optimal hedge ratio; σ_s^2, σ_f^2 – variances of the change in the stock and futures prices; σ_{sf} – covariance between the stock and futures prices.

Suppose the hedger decides to pursue a dynamic hedging strategy. The optimal hedge ratio is determined by solving Equation (2).

$$\frac{\partial Var(\Delta HP_t)}{\partial h_t} = \frac{-E(\Delta F) + 2\gamma\sigma_{sf}}{2\gamma\sigma_f^2}. \tag{3}$$

If the futures rate follows a martingale (i.e., $E(F_1) = F_0$), Equation (3) becomes

$$h_t^* = \frac{\sigma_{sf}}{\sigma_f^2}. \tag{4}$$

This corresponds to the conventional hedge ratio when changes in both stock and futures prices are homoskedastic. Therefore, the optimal hedge ratio is the conditional covariance between price changes in proportion to the conditional variance of change in futures prices. It is known that the optimal hedge ratio h_t^* is the one-period variance minimization solution and termed the myopic optimal hedge ratio. In the absence of conditional heteroskedasticity, both $Cov(\Delta S_t, \Delta F_t)$ and $Var(\Delta F_t)$ are independent of the information set. As a result, h_t^* is a constant term regardless of whatever information is available.

Duffie (1989) shows that the optimal hedge ratio for a person with mean-variance utility can be decomposed into two portions: one reflecting speculative demand (which varies across individuals according to their risk aversion) and another reflecting a pure hedge (which is the same for all mean-variance utility hedgers). Because the pure hedge term is common to all hedgers and the speculative demand term is both difficult to estimate and often close to zero, it is reasonable to focus attention on the pure hedge.

The degree of hedging effectiveness is measured by the percentage reduction in the variance of the naked stock prices changes (Geppert, 1995). Therefore, the degree of hedging effectiveness can be expressed as follows:

$$EH = \frac{Var(\Delta S_t) - Var(\Delta HP_t)}{Var(\Delta S_t)} = 1 - \frac{Var(\Delta HP_t)}{Var(\Delta S_t)} = \rho^2_{sf,t}, \quad (5)$$

where $\rho^2_{sf,t}$ is the square of the correlation between the change in the stock and futures prices. As seen in Equations (2) and (3), the variances, covariance, and correlation coefficients need to be calculated for the hedge ratio and the hedging effectiveness. In the next section, we briefly present the wavelet analysis and describe how to derive the wavelet variance, covariance, and correlation coefficients in the wavelet analysis.

2.3. Wavelet analysis. Wavelet analysis is a new development in the area of applied mathematics. A wavelet coefficient measures the amount of information that is gained by increasing the frequency at which the data are sampled, or what needs to be added to the data in order for it to look like it had been measured more frequently.

We offer a brief exposition of wavelet analysis by focusing on the basic framework of discrete wavelet transform. Wavelet is similar to sine and cosine functions in that they also oscillate about zero. However, oscillations of a wavelet fade away around zero, and the function is localized in time or space. In wavelet analysis, a signal (i.e., a sequence of numerical measurements) is represented as a linear combination of wavelet functions (Schleicher, 2002).

There are father wavelets φ and mother wavelets ψ such that:

$$\int \phi(t)dt = 1, \qquad \int \psi(t)dt = 0. \quad (6)$$

Father wavelets are good at representing the smooth and low frequency parts of a signal, whereas mother wavelets are good at representing the detailed and high frequency parts of a signal. The most commonly used wavelets are the orthogonal ones. In particular, the orthogonal wavelet series approximation to a continuous signal f(t) is given by:

$$f(t) \approx \sum_k S_{J,K}\phi_{J,K}(t) + \sum_k d_{J,K}\psi_{J,K}(t) + \sum_k d_{J-1,K}\psi_{J-1,K}(t) + ... + \sum_k d_{1,K}\psi_{1,K}(t), \quad (7)$$

where J is the number of multi-resolution components or scales, and K ranges from 1 to the number of coefficients in the corresponding component, $s_{J,K}$, $d_{J,K}$,..., $d_{1,K}$ are the wavelet transform coefficients, $\phi_{\mathrm{j,k}}(t)$ and $\psi_{\mathrm{j,k}}(t)$ are the approximating wavelet functions.

These functions are generated from ϕ and ψ as follows:

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(\tfrac{t-2^j k}{2^j}\right),$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\tfrac{t-2^j k}{2^j}\right). \quad (8)$$

These coefficients are a measure of the contribution of the corresponding wavelet function to the total signal.

Applications of wavelet analysis commonly make use of a discrete wavelet transform (DWT). The DWT calculates the coefficients of the approximation in Equation (7) for a discrete signal of final extent, $f_1, f_2, \ldots, f_n$. That is, it maps the vector $f = (f_1, f_2, \ldots, f_n)'$ to a vector ω of n wavelet coefficients that contains $s_{J,k}$ and $d_{J,k}$, j = 1, 2,..., J. The $s_{J,k}$ are called the smooth coefficients and the $d_{J,k}$ are called the detail coefficients. Intuitively, the smooth coefficients represent the underlying smooth behavior of the data at the coarse scale 2^J, whereas the detail coefficients provide the coarse scale deviations from it.

When the length of the data n is divisible by 2^J, there are $n/2$ coefficients $d_{1,k}$ at the finest scale $2^1=2$. At the next finest scale, there are $n/2$ coefficients $d_{2,k}$. Similarly, at the coarsest scale, there are $n/2^J d_{J,k}$ coefficients and $n/2^J s_{J,k}$ coefficients. Altogether, there are $n\left(\sum_{i=1}^{J}\frac{1}{2^i} + \frac{1}{2^J}\right) = n$ coefficients.

The number of coefficients at a given scale is related to the width of the wavelet function. For instance, at the finest scale, it takes n/2 terms for the functions $\omega_{1,k(t)}$ to cover the interval $1 \le t \le n$.

The wavelet coefficients are ordered from coarse scales to fine scales in the vector ω. If n is divisible by 2^J, ω will be given by:

$$\varpi = \begin{pmatrix} s_J \\ d_J \\ d_{J-1} \\ \vdots \\ d_1 \end{pmatrix},$$

where

$$s_J = (s_{J,1}, s_{J,2}, \cdots, s_{J,n/2^J})',$$
$$d_J = (d_{J,1}, d_{J,2}, \cdots, d_{J,n/2^J})',$$
$$d_{J-1} = (d_{J-1,1}, d_{J-1,2}, \cdots, d_{J-1,n/2^{J-1}})',$$
$$\vdots$$
$$d_1 = (d_{1,1}, d_{1,2}, \cdots, d_{1,n/2^J})'.$$

Each of the sets of coefficients $s_J, d_J, ..., d_1$ is called a crystal.

Equation (7) can be rewritten as

$f(t) \approx S_{J,K} + D_{J,K} + D_{J-1,K} + ... + D_{1,K}$, where

$$S_{J,K} = \sum_k d_{J,K} \varphi_{J,K}(t),$$

$$D_{J,K} = \sum_k d_{J,K} \psi_{J,K}(t),$$

$$D_{1,K} = \sum_k d_{1,K} \psi_{1,K}(t),$$

are denominated by the smooth signal and the detail signals, respectively.

The terms in Equation (7) represent a decomposition of the signal into orthogonal signal components $S_J(t)$, $D_J(t)$, $D_J - 1(t)$, ...,D_1(t) at different scales. These terms are components of the signal at different resolutions. That is why the approximation in (7) is called a multi-resolution decomposition (MRD).

2.3.1. Estimation of wavelet variance. Wavelet variance analysis consists in partitioning the variance of a time series into pieces that are associated to different time scales. It tells us what scales are important contributors to the overall variability of a series (Percival and Walden, 2000). In particular, let $x_1, x_2, ..., x_n$ be a time series of interest, which is assumed a realization of a stationary process with variance σ_X^2. Let $n_j^{'} = n/2^j$ be the number of discrete wavelet transform (DWT) coefficients at level *j* where *n* is the sample size, and $L_j^{'} = (L-2)(1-2^{-j})$ be the number of DWT boundary coefficients at level *j* (provided that $n_j^{'} > L_j^{'}$), where *L* is the width of the wavelet filter. An unbiased estimator of the wavelet variance is defined as:

$$v_X^2(\lambda_j) \equiv \frac{1}{\hat{N}} \sum_{t=l_j^{'}}^{N} \left[d_{j,t}^{(X)} \right]. \qquad (9)$$

It has been shown that the wavelet variance $v_X^2(\lambda_j)$ can decompose the variance of a time series on a scale-by-scale basis, instead of the frequency-by-frequency basis used the spectrum (Percival and Walden, 2000). The wavelet variance is defined to be the variance of the wavelet coefficients associated with scale $\lambda_j \equiv 2^{j-1}$. This is equivalent to the expected value of the squared wavelet coefficients. Constructing an estimator of the wavelet variance using a variation of the DWT, the maximal overlap DWT (MODWT), has been shown to be superior to that of the DWT-based estimator (Percival and Walden, 2000). The MODWT coefficients of X_1,…,X_N are denoted by $W_{j,t}^{(X)}$ for *j* = 1,…,J and *t* = 1,…,N. The wavelet variance estimated by the MODWT coefficients for scale λ_j is given by:

$$\tilde{v}_X^2(\lambda_j) \equiv \frac{1}{\tilde{N}_j} \sum_{t=L_j}^{N} \left[\tilde{d}_{j,t}^{(x)} \right]^2, \qquad (10)$$

where $\tilde{N}_j = N_j - L_j$ and $L_j = (L-2)(1-2^{-j})$.

2.3.2. Estimation of wavelet covariance. Let X_t, Y_t $t = 1,...,N$ be a realization of a portion of a zero mean stationary process $\{X_t, Y_t\}$ with cross spectrum $S_{XY}(.)$ and auto spectra $S_X(.)$ and $S_Y(.)$, respectively. Just as the periodogram was used in the univariate case, the cross periodogram:

$$\tilde{S}_{XY}^{(P)}(f) = \sum_{\tau=-(N-1)}^{N-1} \hat{C}_{\tau,XY} e^{-i2\pi f\tau} \qquad (11)$$

is utilized here to estimate the cross spectrum. The sample cross covariance sequence is defined to be $\hat{C}_{\tau,XY} \equiv \sum_t X_t Y_{t+\tau}$, where the summation goes from t = 1 to N-τ for $\tau \geq 0$ and from t -1 to N for $\tau \leq 0$. The multi-taper estimator of the cross spectrum is given by:

$$\hat{S}_{XY}^{(mt)}(f) = \frac{1}{K}(\sum_{l=1}^{N} h_{k,l} X_l e^{-i2\pi fl})^* (\sum_{l=1}^{N} h_{k,l} X_l e^{-i2\pi fl}), \quad (12)$$

where $\{h_{k,t}\}$ is the k_{th}-order data taper for a sequence of length N normalized such that $\sum_t h_{k,t}^2 = 1 \quad k = 1,...,K$. Let X_t and $Y_t, \quad t = 1,...,N$ be defined as before. For $N \geq L_j$, we can define an unbiased estimator $Cov_{XY}(\lambda_j)$ of the wavelet covariance based upon the MODWT via

$$Cov_{XY}(\lambda_j) \equiv \frac{1}{\tilde{N}_j} \sum_{t=L_j}^{N} \tilde{d}_{j,t}^{(X)} \tilde{d}_{j,t}^{(Y)}. \quad (13)$$

The estimator does not include any coefficients that make explicit use of the periodic boundary conditions. We can construct a biased estimator of the wavelet covariance by simply including the MODWT wavelet coefficients affected by the boundary and renormalizing.

2.3.3. Estimation of wavelet correlation. Given the covariance does not take into account the variation of the univariate time series, a natural next-step is to introduce the concept of wavelet correlation. As with the usual estimator for correlation in time series, the wavelet correlation is simply made up of the wavelet covariance for $\{X_t, Y_t\}$ and wavelet variances for $\{X_t\}$ and $\{Y_t\}$. The DODWT estimator of the wavelet cross-correlation is simply

$$\tilde{\rho}_{\tau,XY}(\lambda_j) \equiv \frac{\tilde{r}_{\tau,XY}(\lambda_j)}{\tilde{v}_X(\lambda_j)\tilde{v}_Y(\lambda_j)}, \quad (14)$$

where $\tilde{r}_{\tau,XY}(\lambda_j)$ is the wavelet covariance, $\tilde{v}_X(\lambda_j)$ and $\tilde{v}_Y(\lambda_j)$ are the wavelet variances. When $\tau = 0$, we obtain the MODWT estimator of the wavelet correlation between $\{X_t, Y_t\}$.

3. Empirical results

Table 1 summarizes selected basic statistics. All sample means are positive in the sample period. Variances are 2.736 for the futures index and 3.591 for the stock index, showing that the futures market has higher volatility than the stock market. The signs of skew are all negative. The values of Ljung-Box up to 14 lags (LB(14)) for the return series are significant at the 1% level. The LB(14)'s for squared return series are highly significant for both markets, suggesting the possibility of the presence of autoregressive conditional heteroskedasticity.

Table 1. Basic statistics

	STOCK	FUTURE
Mean	-0.019270	-0.019451
Variance	2.736416	3.5911387
Skewness	-0.003464	-0.098053
Kurtosis	4.606049	5.492309
Q value LB(14) for Rt	35.082 (0.001)	32.317 (0.004)

Notes: Sample period from January 4, 2002 to December 30, 2007. Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags.

Table 2. Granger causality test in wavelet domain

	Original	d1	d2	d3	d4
Future → Stock	2.083* (0.0004)	2.011* (0.0008)	2.118* (0.0003)	2.383* (0.0001)	2.400* (0.000)
Stock → Future	2.699* (0.000)	1.541* (0.030)	1.790*(0.005)	2.185* (0.000)	2.962* (0.000)
	d5	d6	d7	d8	
Future → Stock	2.678* (0.000)	3.476* (0.000)	3.669* (0.000)	4.470* (0.000)	
Stock → Future	3.126* (0.000)	6.553* (0.000)	3.533* (0.000)	5.127* (0.000)	

Notes: The original data have been transformed by the wavelet filter (LA(8)) up to time scale 8. The significance levels are in parentheses. The first detail (wavelet coefficient) d1 captures oscillations with a period length two to four days. The last detail d8 captures oscillations with a period length of 128-256 days. * Significant at 1% level.

The main purpose of this paper is to examine the lead-lag relationship, correlation, and the hedge ratio between the stock and futures markets over the various time scales using wavelet analysis. To examine the lead-lag relationship in wavelet analysis, we first test for Granger causality up to level 8. The results of the Granger causality tests are stated in Table 2. As seen in Table 2, the stock and futures markets show a feedback relationship contemporaneously as well as in various time scales. This result is consistent with the assumption of the cost-of-carry (COC) model and indicates that the two markets are efficient and frictionless. The COC model states that as new information arrives simultaneously to the stock and futures markets and is reflected immediately in both stock and futures prices, profitable arbitrage should therefore not exist, under the assumption that the two markets are perfectly efficient and frictionless and act as perfect substitutes. In other words, if both markets are efficient, the COC model indicates that the two markets have a feedback relationship in terms of Granger causality.

Turning to the second purpose of our paper (correlation in the various time scales), we first examine the variances of the futures and the stock markets' returns in various time scales. An important characteristic of the wavelet transform is its ability to decompose the variance of the stochastic process. Figure 2 illustrates the MODWT-based wavelet variance of two series against the wavelet scales. The straight lines indicate the variance and the box bar indicates the 95% confidence interval.

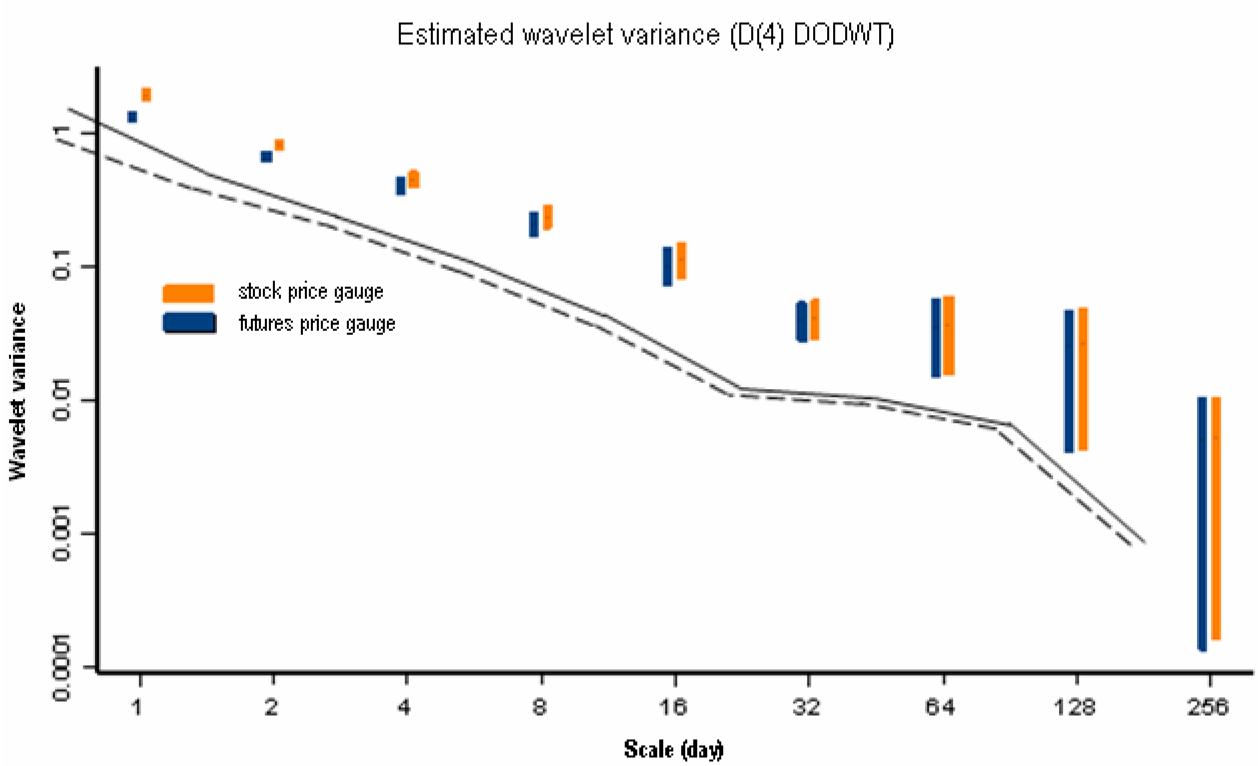

Fig. 2. Estimated wavelet variance of stock and futures returns

There is an approximate linear relationship between the wavelet variance and the wavelet scale. The variances of both the stock and futures markets decrease as the wavelet scale increases. Note that the variances versus wavelet scale curves show a broad peak at the lowest scale (d1) in both markets, which is consistent with the result of In and Kim (2006). More specifically, a wavelet variance in a particular time scale indicates the contribution to sample variance. The sample variances of the stock and futures markets are 2.736 and 3.591, respectively. Notice that the wavelet variances show that the futures market is more volatile than the stock market regardless of the time scale. This is consistent with the results of Lee (2001), who found that the futures market has higher volatility than the stock market using a GARCH model.

In addition to the examination of variances of the two time series, a natural question is to consider how the two series are associated with one another. Note that a wavelet covariance in a particular time scale indicates the contribution to the covariance between two series. Figure 3 shows the MODWT based wavelet covariance of the returns of the stock and futures markets using the LA(8) wavelet filter. Approximate confidence intervals are also presented. Overall, the movements of covariance are decreasing as the time scale increases. The sample covariance between the stock and futures markets is 2.943, and presents a decreasing trend. Although there is a decreasing association between the stock and futures markets, it is difficult to compare the wavelet scales because of the different variability exhibited by them. In this case, dividing by the variance of each series is a natural way to standardize the covariance, thereby overcoming this influence and making it possible to compare the magnitude of the association across scales. Therefore, the wavelet correlation should be constructed to examine the magnitude of the association of each series.

Figure 4 illustrates the estimated wavelet correlation between the stock and futures returns. The high and significant positive relationship can be observed in all time scales. The correlation between the two markets varies over time scales but remains very high.

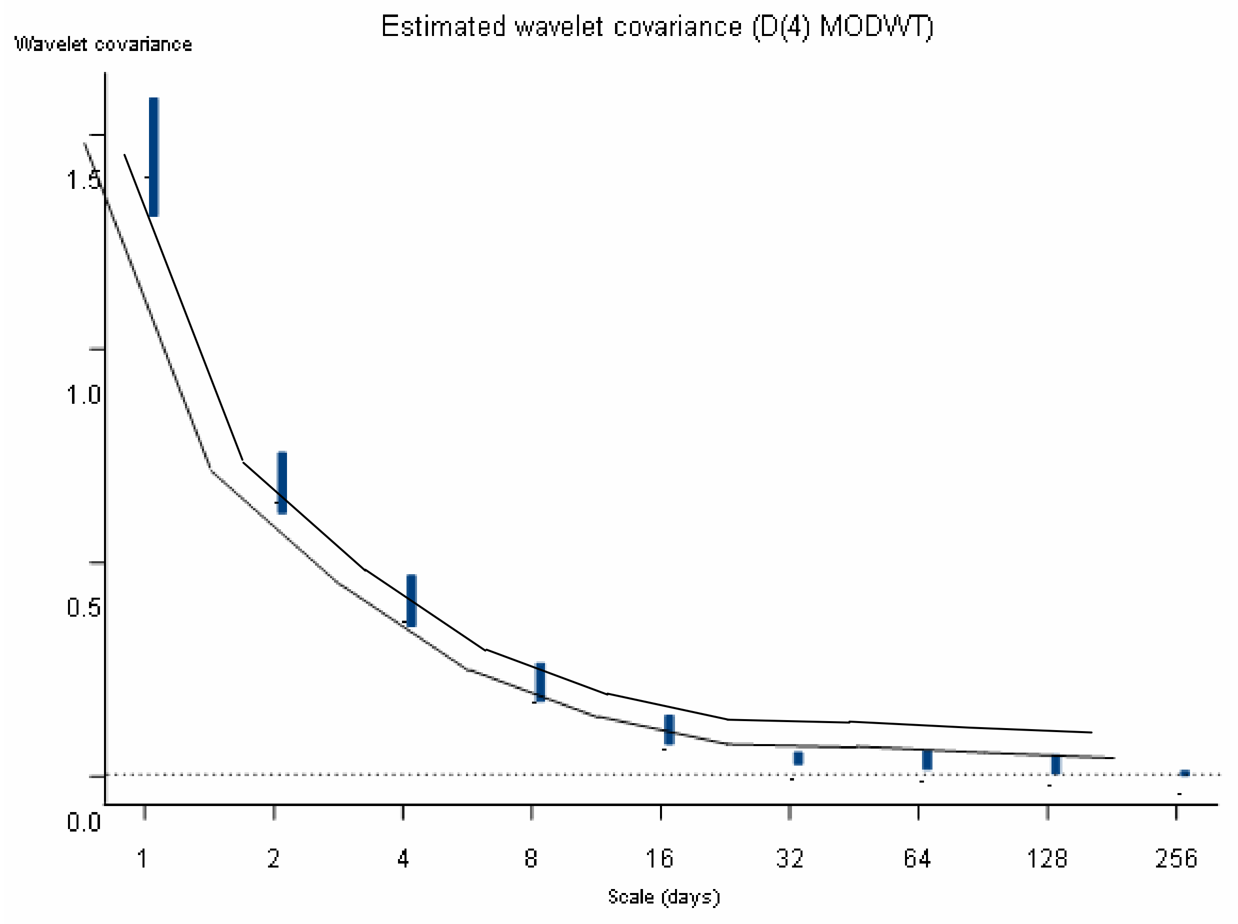

Fig. 3. Estimated wavelet covariance between the stock and futures returns

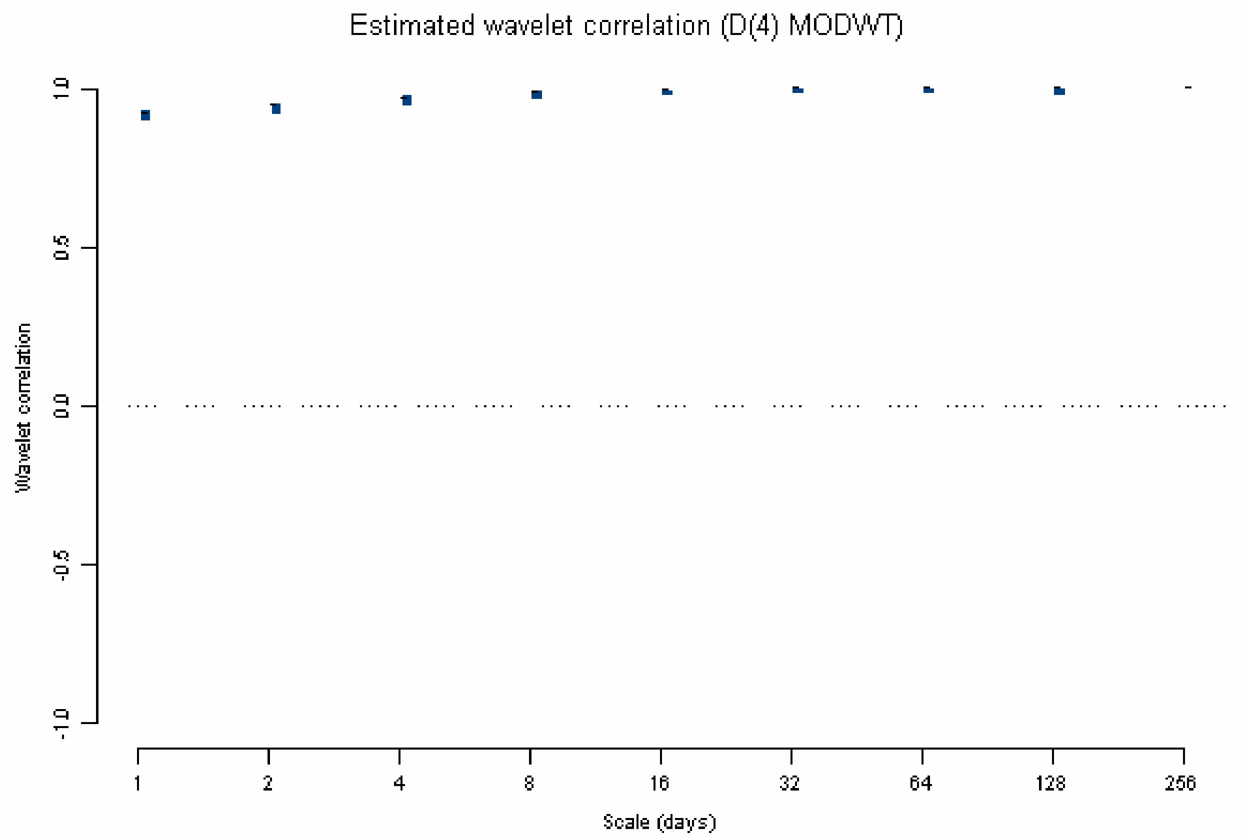

Fig. 4. Estimated wavelet correlation between the stock and futures returns

The final purpose of this paper is to examine the multi-period hedge ratio, based on the results of variance and co-variance obtained from wavelet analysis. As indicated in Lien and Luo (1993), realism suggests that the hedger's planning horizon usually covers multiple periods. Therefore, examining the multi-period hedge ratio is more appropriate than examining the one-period hedge ratio. Figure 6 shows the hedge ratio and hedging effectiveness using the various time scales up to level 8. Note that these wavelet hedge ratios are estimated by a nonparametric method. Therefore, it is not necessary to assume a particular distribution of the error term as in the GARCH/SV model. As seen in Figure 5, the decomposition hedge ratio increases monotonically at a decreasing rate, converging toward the long-horizon hedge ratio of one. This result is consistent with the findings in In and Kim (2006). As indicated in Figure 6, the degree of hedging effectiveness approaches one as the wavelet time scale increases. Intuitively, hedging effectiveness approaches one because, over long horizons, the shared permanent component ties the stock and futures series together and the effect of the transitory components becomes negligible. In the long run, the stock and futures prices are perfectly correlated. This result is consistent with the results of Geppert (1995) and Low et al. (2002), who compare the hedge ratios and hedging effectiveness obtained from various models.

Figures 6 and 7 show the MODWT MRA of the stock and futures returns using various time scales. There is a fierce fluctuation in the original series in November 2002, which is captured in the d1-d3 component. Interestingly, the shock of November 2002 is getting smaller as the time scale increases, implying that the short-term shock does not affect the long-run movement of the stock and futures markets. At the highest time scale, d8, representing the deviation from the long-term trend, there is a smooth and similar movement for both markets.

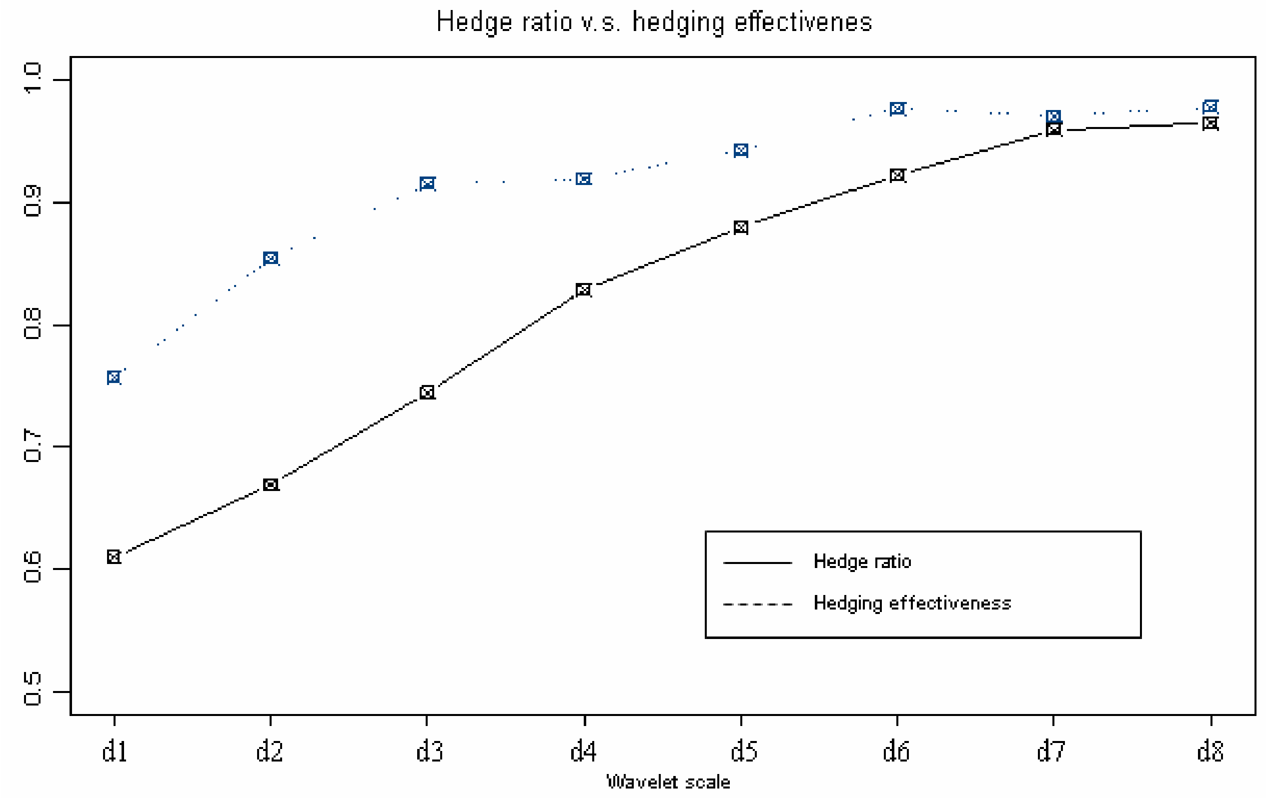

Fig. 5. Hedge ratio and hedging effectiveness with different wavelet domains

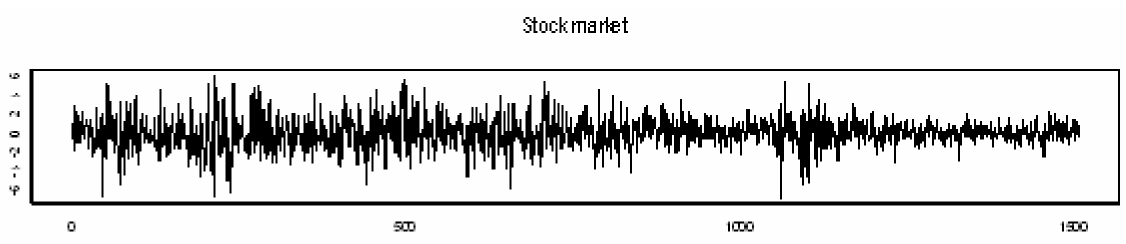

Fig. 6. MODWT multi-resolution analysis – stock market

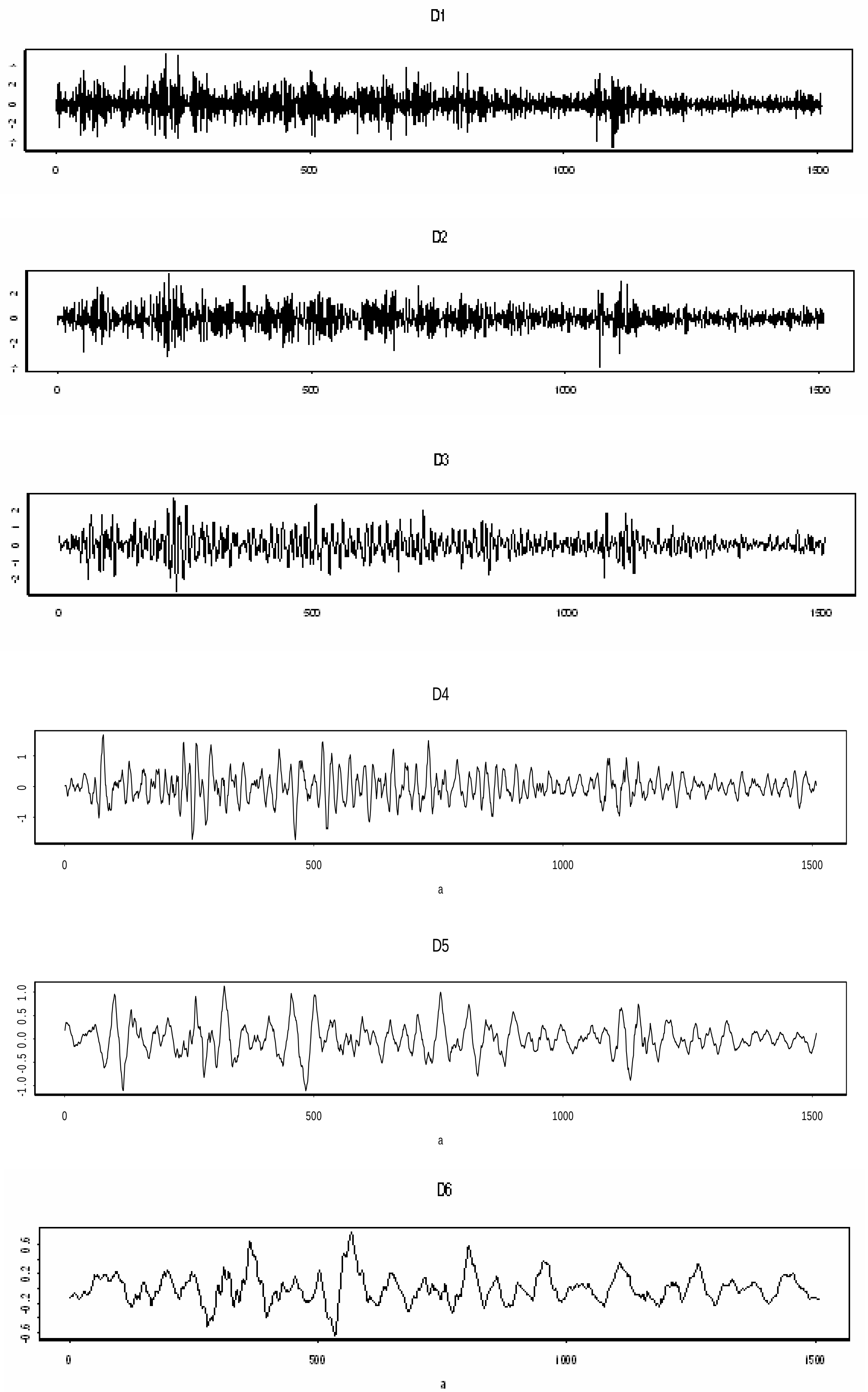

Fig. 6 (cont.). MODWT multi-resolution analysis – stock market

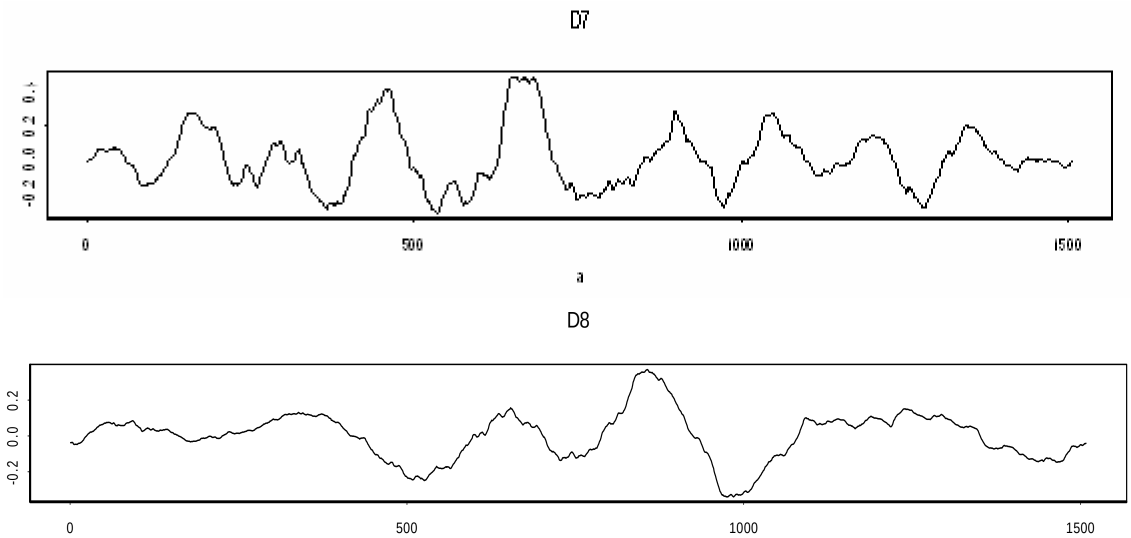

Fig. 6 (cont.). MODWT multi-resolution analysis – stock market

Futures market

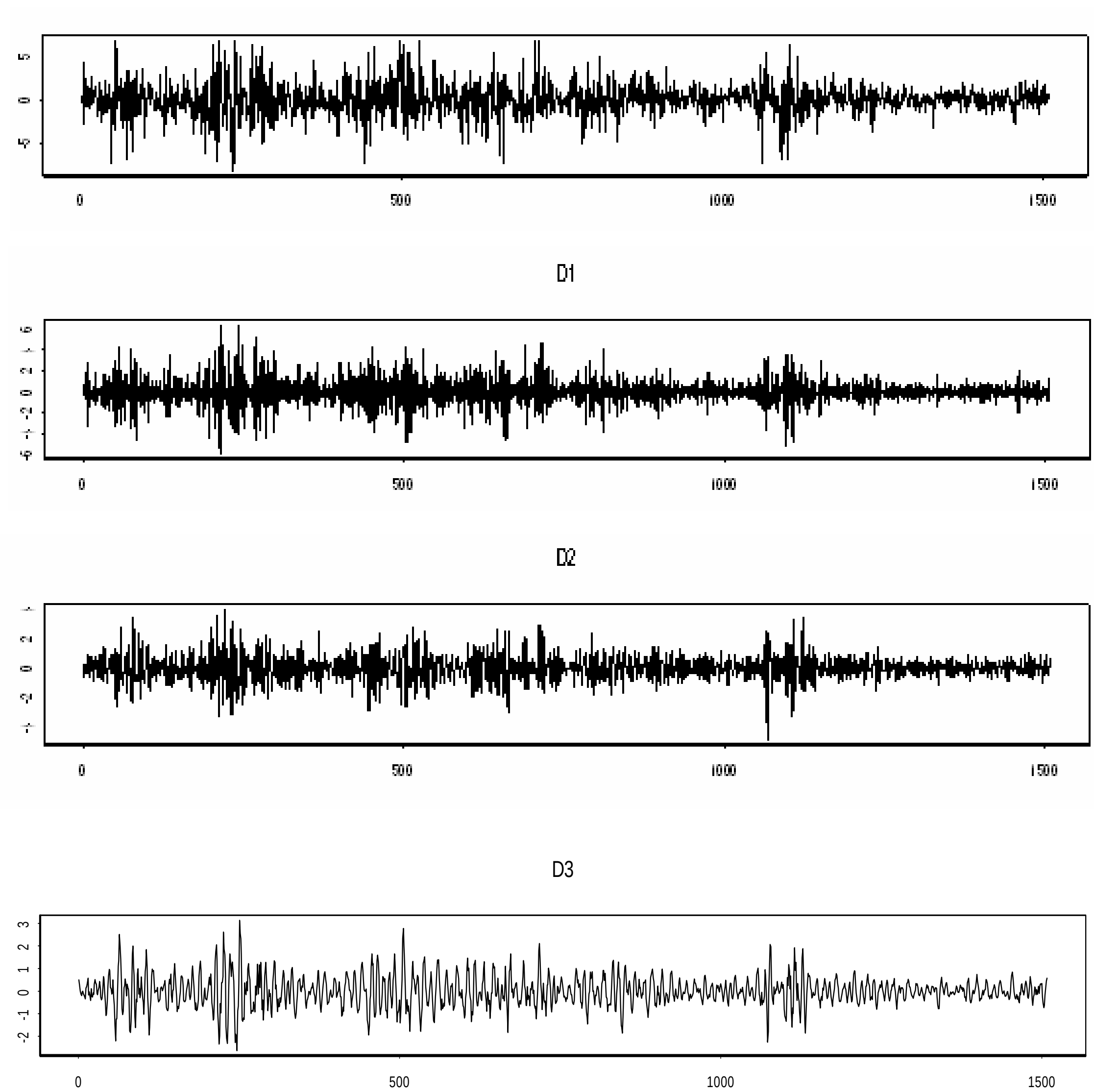

Fig. 7. MODWT multi-resolution analysis – futures market

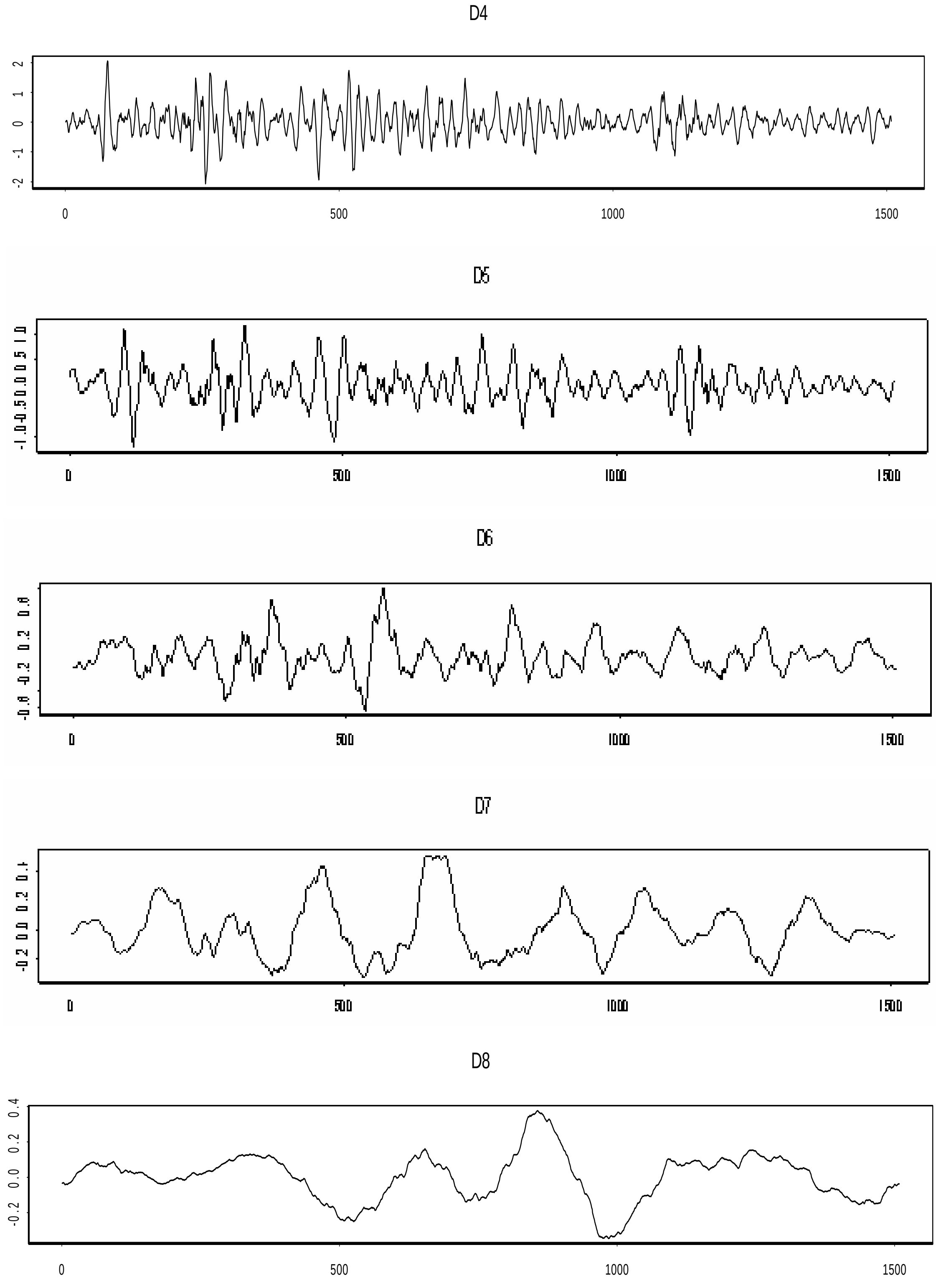

Fig. 7 (cont.). MODWT multi-resolution analysis – futures market

Another finding, in average, there are about two recycles within one year in Taiwan, especially observed in the higher time scale. However, in the finer scale (d1-d4), it is difficult to find the trend of stock market. Overall, it is interesting to observe that as the time scale increases from the finer time scale (d1) to the highest time scale (d8), the wavelet coefficients show a smooth movement, implying that short-term noise in the market is cancelled out as the wavelet time scale increases, and consequently the "true" underlying economic relationship between stock and futures prices will prevail in the long run. These wavelet MRA figures indicate that the wavelet stock decomposition gathers information that cannot be captured by conventional analysis. In other words, the decomposition of data into several time scales is important in economics and finance since it detects the frequency burst in various time scales. In the co-integration literature on hedge ratios, the presence of both long-run and short-run components in the stock and futures markets causes the hedge ratio and the degree of hedging effectiveness to depend on the time horizon. As the wavelet time scale increases, the decomposed data are close to the long-run trend. Therefore, over long horizons, the shared long-run component ties the stock and futures series together, and the two prices will be perfectly correlated.

Conclusions

In this paper, we use wavelet analysis to propose a new approach to investigate the relationship between the stock and futures markets over different time scales. The paper examines this relationship in three ways: (1) the lead-lag relationship, (2) covariance/correlations, and (3) the hedge ratio. To examine the lead-lag relationship between the two markets, we employ the Granger causality test for various time scales. The wavelet correlation is estimated by testing the correlation between the two markets in the various time scales from the wavelet coefficients. The hedge ratio, defined by the co-variance between the stock return and the futures return divided by the volatility of futures return, is calculated from the wavelet covariance and variance. The main advantage of using wavelet analysis is the ability to decompose the data into the various time scales. This advantage allows researchers to investigate the relationship between two variables in various time scales, whereas the traditional methodology only allows examination of only two time scales: short- and long-run scales.

The wavelet analysis is undertaken using the LA(8) wavelet filter and supports our main conclusions. First, it is found that the stock and futures markets show a feedback relationship regardless of the time scale. According to the assumption of the cost-of-carry model, this could imply that the two markets are perfectly efficient and frictionless and act as perfect substitutes. This result also implies that a profitable arbitrage does not exist between the two markets, regardless of the time scale.

Second, it is found that there is an approximate linear relationship between the wavelet variance and the wavelet scale. Both wavelet variances and covariance of the stock and futures returns decrease as the wavelet time scale increases. Overall, the wavelet variances show that the futures market is more volatile than the stock market regardless of the time scale, which is consistent with the results of Lee (2001). It is also interesting to observe that the wavelet correlation between the two markets varies over time but remains very high, over 0.92 on average.

Third, we examine the multi-period hedge ratio, based on the variances and covariance obtained from wavelet analysis. We find that (1) each hedging horizon has a unique hedge ratio, (2) the long-horizon hedge ratio converges to one, and (3) hedging effectiveness converges to one as the hedging horizon (wavelet time scale) increases. This is consistent with the results of Low et al. (2002) on Nikkei index futures. As indicated in Baillie and Myers (1991), there are two reasons for this. The economic rationale is that the arrival of information in the market resolves price uncertainty. More uncertainty is resolved, and over a longer amount of time, the basis risk is reduced. The statistical rationale is that the noise in the market tends to be canceled over time; the true underlying relationship between the stock and futures prices emerges in long investment horizons, evidenced by the wavelet MRA. Our result indicates that as the wavelet time scale increases, the decomposed data are close to the long-run trend. Therefore, over long horizons, the shared long-run component ties the stock and futures series together, and the two prices will be perfectly correlated.

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