Hassan Shirvani (USA), Barry Wilbratte (USA), Natalya Delcoure (USA)

Testing for periodic integration and cointegration of the stock prices of the G7 countries

Abstract

This paper examines the seasonal properties of the stock prices of the G7 countries. Using a number of empirical tests, the paper finds both deterministic seasonal dummies and seasonal unit roots to be inadequate to explain the seasonal behavior of these prices. However, drawing on more recent advances in the time series of periodic autoregressive processes, the paper finds evidence of periodic integration, but not periodic cointegration, in the underlying data. The paper also explores the implications of these findings for efficient international portfolio management.

Keywords: seasonal integration, periodic integration and cointegration, international portfolio diversification.
JEL Classification: C22, G11, G15.

Introduction

Seasonality is a critically important component of many macroeconomic and financial time series, tending in many cases to dominate other non-trend components (Barsky and Miron, 1989; Miron, 1994). Despite this fact, and until recently, most econometricians tended to either completely ignore the issue of seasonality in their applied work, or to filter it away through a host of adjustment techniques, such as the inclusion of deterministic seasonal dummies in their estimated equations or the use of the well-known Census Bureau X-11 and ARIMA X-11 methods. More recent research, however, has thrown considerable doubt on the validity of these approaches. On the one hand, it has been shown that seasonality is an independent aspect of economic and financial behavior, thus deserving explanation in its own right (Miron, 1986; Miron and Zeldes, 1988; Osborn, 1988; Birchenhall et al., 1989). On the other hand, evidence has accumulated that questions the treatment of seasonality as a constant feature of the data, to be adjusted away through deterministic dummies (Ghysels, 1994; Franses, 1996).

Mirroring the parallel empirical debate about whether trend or difference stationarity best characterizes the trend component of most time series data, econometricians have increasingly tended to model the seasonality component as a stochastic process, to be subjected to stationarity tests. Indeed, it has been shown that if seasonality is ignored, the unit root and cointegration test results are spurious (Franses, 1994; Ghysels, 1994). Consequently, extending the econometrics of unit roots and cointegration to the study of seasonality, Hylleberg et al. (1990), Engle et al. (1993), and Ghysels and Perron (1993), among others, have developed similar seasonal integration and cointegration tests. Since inappropriate seasonal

adjustment methods, such as using seasonal dummies to purge stochastic seasonality, can complicate standard unit root and cointegration results, pretesting the data for seasonal unit roots has now become standard practice among many researchers. At the same time, more recent work has come to consider the seasonal unit roots approach, with its assumption of constant autoregressive coefficients for all seasons, too restrictive. By providing evidence to the contrary, this work advocates a more general approach, the so-called periodic integration approach, in which the autoregressive coefficients are allowed to vary across seasons, while at the same time satisfying a newly defined condition for unit root behavior (Osborn, 1991; Franses, 1996; Ghysels and Osborn, 2004). Clearly, for variables that display periodic as opposed to seasonal unit roots, one can simply replace the concept of seasonal with periodic cointegration.

The purpose of this paper is to employ some of the above methodological advances to examine the seasonal behavior of the stock prices of the G7 countries. We present evidence that the seasonal components of the stock prices are neither deterministic nor characterized by seasonal unit roots. Rather, they seem to be periodically integrated, but not cointegrated. In addressing the issue of seasonality, the paper employs as a test case the topic of international investment in equities, the implications of which are discussed briefly at the end of the paper.

The remainder of the paper is organized as follows. Section 1 summarizes the methodology used in the paper. Section 2 presents the empirical results. The last section concludes the paper and discusses the implications of our findings for international portfolio management.

1. Methodology

The primary goal of this paper is to examine the seasonal properties of the stock prices of the G7 countries. As our data are quarterly, we use four

seasons per year. As a first step, we must ascertain that there is indeed a seasonal pattern in our underlying data. This is accomplished through a standard Wald test of joint significance of the seasonally dummied variables in the following autoregression equation:

$$y_t = a_0 + b_0 T + c_0 y_{t-1} + \sum_{i=1}^{3} a_i D_i +$$

$$+ \sum_{i=1}^{3} b_i \ D_i T + \sum_{i=1}^{3} c_i \ D_i y_{t-1} + \varepsilon_t, \qquad (1)$$

where y_t is the stock price in quarter t, T is a quarterly time trend, D_i is a seasonal dummy with one for the i-th quarter and zeroes elsewhere, and ε_t is a white noise error term.

Since the results presented later in the paper do indicate the presence of a seasonal pattern in our data, we need to find an appropriate approach for modeling such seasonal behavior. There is substantial evidence of seasonal variation in many economic and financial time series (Hylleberg, 1994; Canova and Hansen, 1995). Thus, it is of considerable interest to determine whether the seasonal pattern in our data follows a stationary stochastic process, and if not, how it can be rendered stationary. At the same time, it is well known that any quarterly time series can be purged of its seasonal effects through the use of the fourth-differencing filter, Δ_4. If this filter renders a seasonal series stationary, the series are said to be seasonally integrated, i.e., to have seasonal unit roots. Hylleberg et al. (1990; henceforth HEGY) provide a methodology to test for the presence of seasonal unit roots in quarterly time series data. As HEGY show, underlying the fourth-differencing filter is the assumption that there are four non-seasonal and seasonal unit roots in the underlying data. Specifically, with B denoting the lag operator, we can solve the fourth differencing equation to obtain $\Delta_4 = 0 = (1 - B^4) = (1 - B)(1 + B)(1 + B^2)$, with four unit roots of 1, -1, i, and -i, with i representing the imaginary number. (In frequency domain, the unit root 1 corresponds to the zero frequency, the root -1 corresponds to frequency π, and the roots i and -i correspond to frequency $\pi/2$). The unit root 1 is the non-seasonal unit root, while the other three are seasonal unit roots. Clearly, should any of these unit roots be absent, the fourth-differencing filter may result in over-differencing of the data.

To determine the number of unit roots, we perform the HEGY test, which is based on the following auxiliary regression:

$$\Phi(B) y_{4,t} = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} +$$

$$+ \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \varepsilon_t, \qquad (2)$$

where $\Phi(B)$ is an autoregressive polynomial in B with order r chosen to render the error term in the above equation white noise, μ_t is a combination of seasonal dummies and a time trend, and the y variables are defined as follows:

$$y_{4,t} = (1 - B^4) y_t,$$

$$y_{1,t} = (1 + B + B^2 + B^3) y_t,$$

$$y_{2,t} = - (1 - B)(1 + B^2) y_t,$$

$$y_{3,t} = - (1 - B^2) y_t.$$

HEGY show that the test for the presence of non-seasonal and seasonal unit roots amounts to testing for the significance of the π terms in the above auxiliary equation. If π_1 equals zero, then the null hypothesis of a non-seasonal unit root 1 cannot be rejected. If π_2 equals 0, the null of a seasonal unit root -1 cannot be rejected. Finally, if $\pi_3 = \pi_4 = 0$, then the null of two seasonal unit roots of +i and -i cannot be rejected. To test the above hypotheses, HEGY show that we can use t-tests for π_1 and π_2 and a joint F-test for π_3 and π_4, using the non-standard critical values they provide.

A drawback of the HEGY test is the restriction that the coefficients of the auxiliary regression (2) are constant over the seasons. Franses (1993) proposes an alternative periodic approach that relaxes this restriction by allowing the coefficients to vary across seasons. In addition, by stacking quarterly data into a 4x1 vector of annual series, Franses (1994) offers an alternative multivariate approach to testing for non-seasonal and seasonal unit roots, using the Johansen (1988) cointegration method. Under this alternative approach, the hypotheses concerning the presence of non-seasonal and seasonal unit roots can be replaced by testable hypotheses on the coefficient values of the Johansen cointegration vectors. Indeed, as we show later in the paper, application of the HEGY approach to our data finds only non-seasonal unit roots, with no evidence of seasonal unit roots. Given these results, we proceed to also perform the multivariate Franses test and, based on this test, we reject even the non-seasonal unit roots. In light of these negative findings, we then test for periodic unit roots and find them to characterize the stock prices of each country. As part of our periodic unit root tests, we find a first-order periodic autoregression to be appropriate for our data. Hence, in the remainder of this section, we briefly describe the periodic integration approach only in the context of a simple first order periodic process, given by following expression:

$$y_t = \varphi_{1s} y_{t-1} + \varepsilon_t, \qquad (3)$$

where $\varphi_{1s} \neq \varphi_1$ for s = 1 to 4. In addition, we can stack the quarterly series y_t into a 4x1 vector Y_T of

annual series, where $Y_T = (Y_{1T}, Y_{2T}, Y_{3T}, Y_{4T})'$, and Y_{sT} is the season s observation in the year T. Thus, equation 3 can be expressed in vector notation as follows:

$$A_0 Y_T = A_1 Y_{T-1} + \varepsilon_T, \qquad (5)$$

with

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \varphi_{12} & 1 & 0 & 0 \\ 0 & -\varphi_{13} & 1 & 0 \\ 0 & 0 & \varphi_{14} & 1 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \varphi_{11} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

We now define the parameter vector $\varphi = (\varphi_{11}, \varphi_{12}, \varphi_{13}, \varphi_{14})$. The vector process Y_T is stationary provided so that the root of the characteristic equation

$$|A_0 - A_1 z| = (1 - (\varphi_{11}\varphi_{12}\varphi_{13}\varphi_{14})z) = 0 \qquad (5)$$

lies outside the unit circle, i.e. if $\varphi_{11}\varphi_{12}\varphi_{13}\varphi_{14} < 1$. On the other hand, the process Y_T can be said to be integrated if (5) has a unit root, i.e., if it is the case that the product

$$\varphi_{11}\varphi_{12}\varphi_{13}\varphi_{14} = 1. \qquad (6)$$

To test the above restriction, we first estimate the following unrestricted equation

$$y_t = \sum_{s=1}^{4} \varphi_{1s} D_{st} y_{t-1} + \varepsilon_t. \qquad (7)$$

Next, by imposing the above restriction, we have the following restricted equation:

$$y_t = \varphi_{11} D_{1t} y_{t-1} + \varphi_{12} D_{2t} y_{t-1} + \varphi_{13} D_{3t} y_{t-1} +$$
$$+ (\varphi_{11}\varphi_{12}\varphi_{13})^{-1} D_{4t} y_{t-1} + \varepsilon_t, \qquad (8)$$

which can be estimated by non-linear least squares (NLS). A likelihood ratio test can then be expressed in the following form:

$$LR = n \cdot ln(RSS_0 / RSS_1), \qquad (9)$$

where n is the number of quarterly observations, and RSS_0 and RSS_1 denote the residual sums of squares from (7) and (8), respectively. As shown by Boswijk and Franses (1995), the above likelihood ratio can then be used to construct the following one-sided test, which follows a Dickey-Fuller (1979) t distribution:

$$LR_\tau = \left[sign(\varphi_{11}\varphi_{12}\varphi_{13}\varphi_{14} - 1) \cdot LR^{1/2} \right] \qquad (10)$$

Having established periodic integration for our stock series of the G7 countries, it is of interest to also test for the presence of periodic cointegration among them. Boswijk and Franses (1995) extend the standard cointegration definition to periodically integrated series and formulate the periodic cointegration test as a Wald test of joint significance of the 4-quarter lagged variables in the following vector autoregression:

$$\Delta_4 y_t = \sum_{s=1}^{4} (\mu_{0s} D_{st} + \tau_s D_{st} t) +$$
$$+ \sum_{s=1}^{4} \delta_s' D_{st} x_{t-4} + \pi' \omega_t + \varepsilon_t, \qquad (11)$$

where y_t is the variable chosen to serve as the dependent variable, x_t represents all the other variables, and ω_t stands for the lagged values of the fourth-differenced variables in the system, needed to render the error term in a white noise process. Under the Boswijk-Franses cointegration test, the null of no cointegration can thus be rejected if the δ_s' coefficients in the above regression are found to be jointly and significantly different from zero. In light of the fact that the above test, an extension of the Engle-Granger (1987) two-stage cointegration test, fails to detect the number of cointegrating vectors and may yield results dependent on the choice of variable for the left-hand-side of the equation, we substantiate the Boswijk-Franses test results using the multivariate Johansen cointegration approach.

2. Empirical results

Before offering our quantitative results concerning the seasonal characteristics of the G7 stock prices, we provide a visual presentation of these characteristics in Figures 1 to 7. Each chart contains four curves corresponding to the seasonally unadjusted stock prices for the four quarters of the year, where all stock prices are real (deflated by the consumer price index), logarithmic, cover the 1960:1 to 2007:1 period, and are OECD data taken from the RATS database.

It is clear from the above figures that there is considerable seasonal variation in the real stock prices of each of the countries in the sample. This is apparent in the frequent crossings of the quarterly curves in the figures. To substantiate this visual impression, we conduct the Wald test of joint significance of the dummied seasonal variables in equation 1 presented earlier. The F values of the Wald test, which range from 2.12 to 18.34, all are significant at the 5 percent level, indicating the presence of seasonality for all the sample countries.

Given the presence of seasonal variations in the data, we proceed to model them. To this end, we employ the HEGY seasonal unit root tests discussed earlier, where a lag of eight quarters is chosen for each country through a process of testing down for significant lags. Table 1 presents the results of these tests. As the table shows, the stock prices of the G7 countries all are characterized by only one non-seasonal unit root and no seasonal unit roots at all. This means that a simple first differencing filter is all that is needed to render these prices stationary. In addition, given that our stock prices contain only non-seasonal unit roots, we can test for the presence of common stochastic trends among them through the standard Johansen cointegration procedures.

Before proceeding further, however, it is important to obtain independent verification of the HEGY test results through the multivariate Franses (1994) test of seasonal unit roots. As mentioned earlier, this test consists of stacking quarterly data into four separate vectors of quarters (VQs), with each vector corresponding to a particular season of the year. Under these conditions, the number of cointegrating vectors for these VQs provides information concerning the stochastic seasonal properties of the underlying data. In particular, to provide independent support for the above HEGY test results of only one non-seasonal unit root for our data, it is necessary (although not sufficient) to find three cointegrating vectors for our VQs. To render the finding of three cointegrating vectors also sufficient for a non-seasonal unit root, we test the restriction that first differencing the data renders them stationary. This means that the three cointegrating vectors, after appropriate normalizations, must be presentable as (-1,1,0,0), (0,-1,1,0), and (0,0,-1,1). Clearly, these vector forms can be tested as imposed restrictions on the cointegrating vectors using the Johansen method. At the same time, a finding of three cointegrating vectors indicates the presence of only one seasonal unit root, specifically the -1 root. Since under this seasonal unit root, the data can be rendered stationary by adding successive observations, it is clear that we can also test for the -1 root by testing whether the cointegrating vectors can be presented as (1,1,0,0), (0,1,1,0), and (0,0,1,1).

The results of the Franses test are presented in Tables 2 and 3. Table 2 reports tests for the number of cointegrating vectors (using the Johansen cointegration test with one lag, as selected by the Hannan-Quinn (1979) method for all sample countries). As shown by Table 2, for each country in the sample, there are three cointegrating vectors among the VQs, indicating that the stock prices in each of these countries are non-stationary and driven by a common stochastic seasonal factor. In addition, this

common factor can be either the non-seasonal unit root 1, the seasonal unit root -1, or neither. Thus, as mentioned earlier, we must perform an additional likelihood ratio test on the values of the cointegrating coefficients to determine whether the root is 1 or -1. This likelihood ratio test has a chi-squared distribution with $k - r(r - 1)$ degrees of freedom, with k denoting the number of restrictions imposed on cointegrating vectors and r the number of cointegrating vectors. The results of these tests appear in Table 3. As the table indicates, based on the Franses test results, we can reject both the non-seasonal and seasonal unit roots hypotheses for our sample data.

Having established the absence of standard nonseasonal and seasonal unit roots in the stock prices of the G7 countries, we turn to the issue of whether the non-stationarity of these prices can be captured by periodic unit roots. To this end, we simply employ the methodology outlined earlier in the paper. In particular, and as part of our periodic unit roots test, we use a first-order periodic autoregression, the lag length of one having been selected by the Hannan-Quinn method as indicated earlier. Next, we estimate equation 7 as our unrestricted version of the periodic model for each country in the sample (a Wald test of the constancy of the autoregressive coefficients in equation 7 is rejected for all countries, with the F ratios ranging from 2.50 to 21.53, all significant at the 5 percent level). Finally, under the null hypothesis that the underlying data are characterized by periodic unit roots, we also employ the non-linear least squares method to estimate equation 8 for our sample countries. Based on our estimation results for equations 7 and 8, we then form the likelihood ratio statistic defined by equation 10, which, as stated before, has a Dickey-Fuller non-standard t-distribution. A non-significant value for this statistic is indicative that the null hypothesis of the periodic unit root cannot be rejected. The test results are presented in Table 4. It is clear from the table that none of the likelihood ratio statistics is significant at the standard levels. Thus, we cannot reject the null hypothesis of a periodic unit root for any of the countries in the sample.

Given our finding that the stock prices of the G7 countries are periodically integrated, it is of considerable interest to determine whether these prices are periodically cointegrated, that is, whether there are linear combinations of these prices which lack periodic unit roots. For the reasons mentioned above, our cointegration tests are based on the Boswijk-Franses test (equation 11), substantiated by the Johansen method, where in the latter method we test for the presence of cointegrating relationships among the VQs of the stock prices of the G7 countries. Since the test results associated with both

methods are essentially the same, we report only the Boswijk-Franses results in Table 5. As the table shows, none of the F-statistics is significant at the standard levels, indicating no periodic cointegration for the stock prices of the sample countries. As stated earlier, we obtain the same results (not reported here but available from the authors) using the Johansen method.

Our conclusion that seasonality differs among the G7 counties is consistent with previous empirical findings and, upon reflection, is quite reasonable in light of institutional and behavioral differences among the sample countries. For instance, Corhay et al. (1987) find seasonal variations in stock returns in a subset of the G7 countries. In a closely related paper, Smith (2002) obtains similar results in the context of the bond markets for six of the G7 countries. The findings of seasonality in our and other research can be attributed to a number of factors arising from international differences in timing among events influencing investor behavior, despite the presence of common shocks to international firms whose stocks dominate the world market indices (Brooks and Del Negro, 2006). We would expect international differences to arise from country-specific factors with uneven effects in different markets. Such factors would include differences in timing of the tax years, unequal capital gains tax rates, variations in the timing and importance of executive bonuses, and non-uniformity in the lengths and dates of national holidays among the sample countries.

In touching upon the issue of stock investment, our findings bear interesting tactical implications for investors who practice frequent reallocations of their funds rather than for international portfolio optimization. It is well established that with increasing integration of capital markets, the potential long-term gains from efforts at portfolio optimization through international diversification have diminished over time (Longin and Solnik, 1995). In contrast, our results suggest that international diversification offers potential benefits to investors who make a practice of moving their funds frequently in order to exploit seasonal variations among foreign markets, even confining themselves to the rather similar G7 countries. Recognizing seasonal patterns, investors may minimize risk associated with such well-known seasonal events as the September US market declines through periodic movements of funds overseas. Following a strategy of moving their investments among countries to exploit differences in seasonal patterns, investors may thereby reduce variation in their portfolio values while keeping the equity allocation unchanged rather than attempting to avoid stock price fluctuations through portfolio reallocation between equities and fixed income securities. The key to following such an approach would be willingness to reallocate equity investments internationally on a seasonal basis.

Conclusion

This paper begins its investigation by identifying observable and statistically significant seasonal variations in the stock prices within the G7 countries. Application of various statistical tests reveals that deterministic seasonal dummies fail to adequately model the seasonal patterns in the data. Using more current advances in the analysis of stochastic autoregressive processes, this study finds no evidence of seasonal unit roots, but does find evidence of periodic unit roots. In addition, our tests fail to detect periodic cointegration, indicating that different seasonal patterns drive the stock prices of the sample countries.

The above findings bear interesting implications regarding the possible benefits of portfolio diversification among major industrial countries. It is well known that in recent years, as a result of the growing economic and financial integration, the stock markets of the industrial countries have displayed similar trend and cyclical movements. As a result, the scope for risk reduction through diversification across these countries has become increasingly limited. However, the stock prices of the industrial countries also fluctuate in response to seasonal factors, which, as documented in this paper, can diverge across industrial countries. The fascinating implication of this finding is that although international diversification fails to protect investors from fundamental economic and financial shocks affecting all countries concurrently, such diversification can still prove useful by protecting investors from exposure to volatilities associated with unfavorable seasonal patterns within individual countries.

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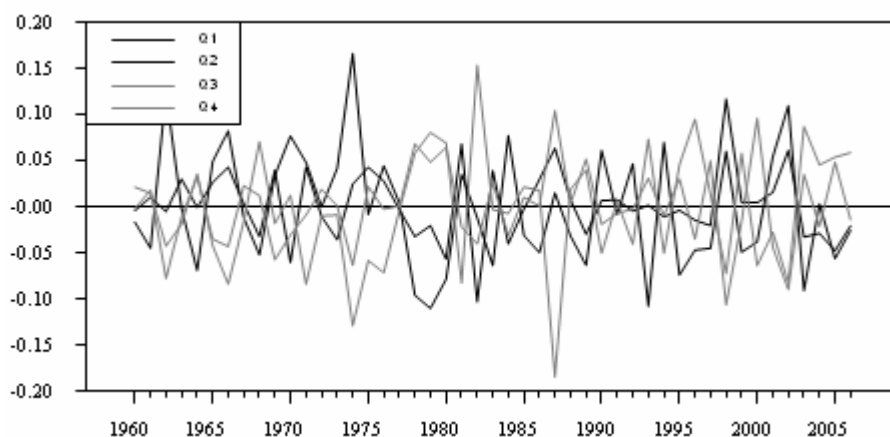
Appendix

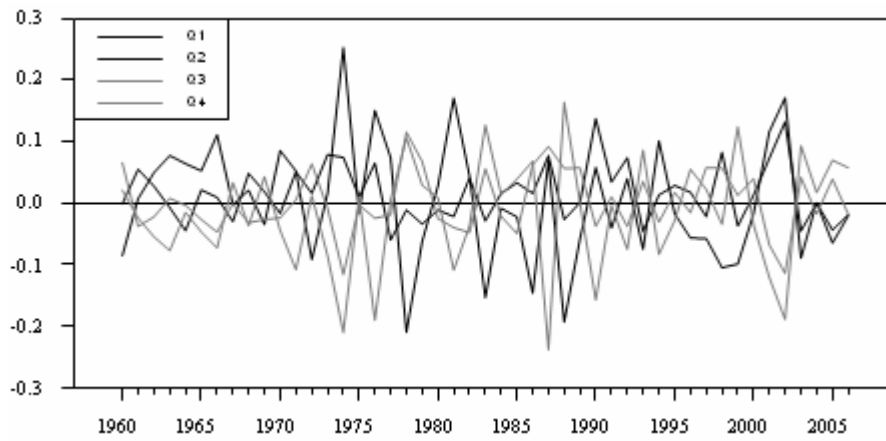**Fig. 1. Real stock prices by quarter of the year: Canada**

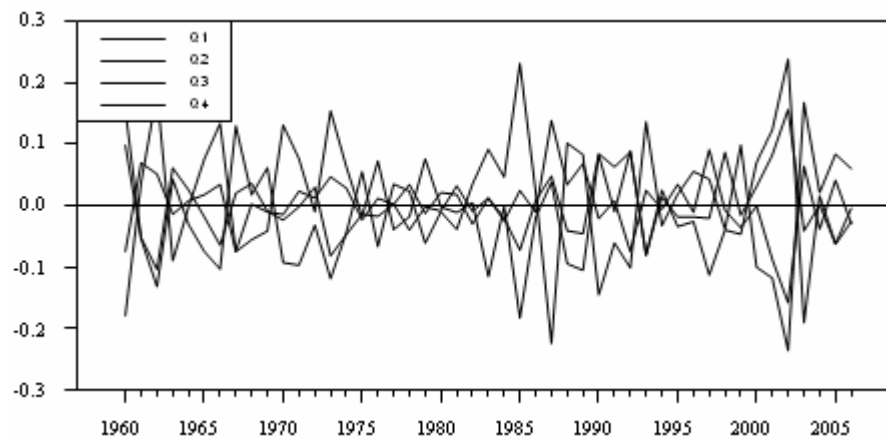**Fig. 2. Real stock prices by quarter of the year: France**

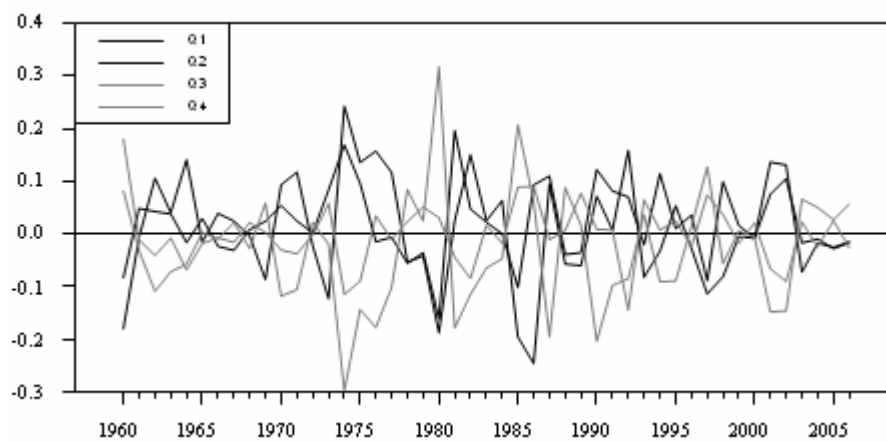**Fig. 3. Real stock prices by quarter of the year: Germany**

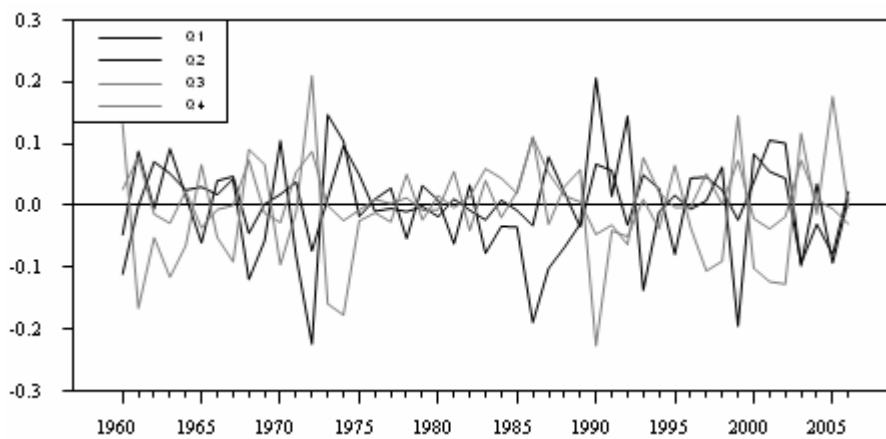**Fig. 4. Real stock prices by quarter of the year: Italy**

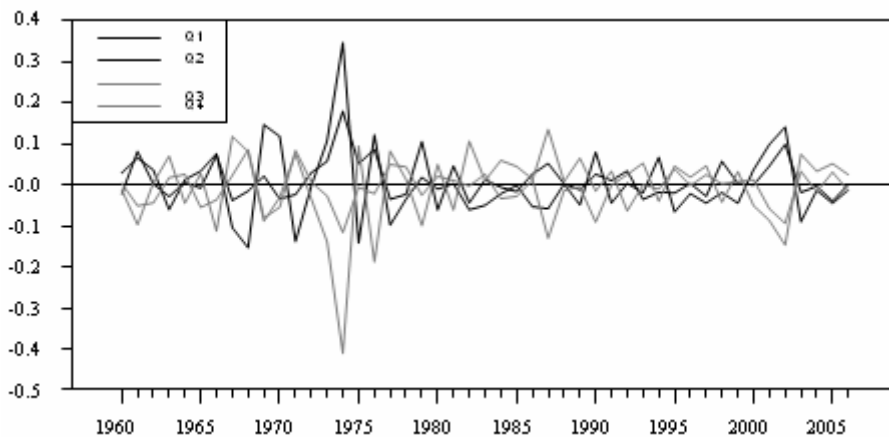**Fig. 5. Real stock prices by quarter of the year: Japan**
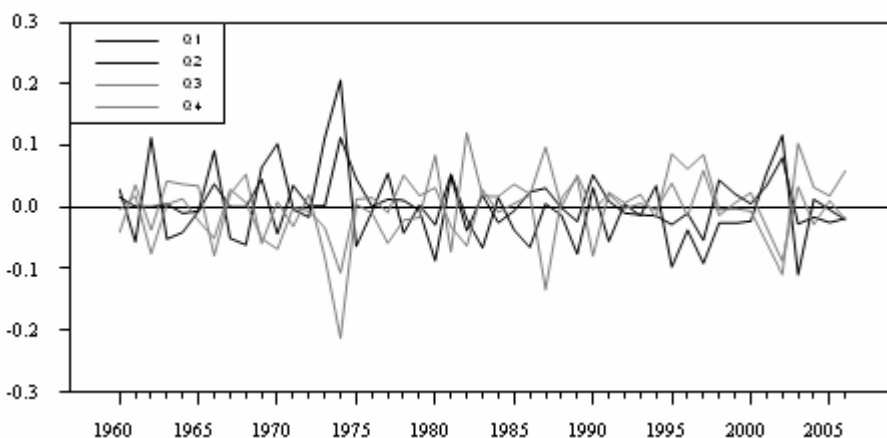
Fig. 6. Real stock prices by quarter of the year: UK

Fig. 7. Real stock prices by quarter of the year: US

Table 1. HEGY seasonal unit roots test

Country	Unit root: +1	Unit root: -1	Unit root: $\pm i$
Statistic	t-value for $\pi1 = 0$	t-value for $\pi2 = 0$	F-value for $\pi1 = \pi2 = 0$
Canada	-1.96	-4.06*	23.02*
France	-2.19	-4.37*	14.79*
Germany	-2.21	-4.21*	15.95*
Italy	-1.99	-3.79*	13.65*
Japan	-2.36	-5.35*	20.86*
UK	-1.81	-3.73*	16.07*
US	-1.06	-4.62*	16.58*

Note: * indicates significant at the 5 percent level.

Table 2. Franses seasonal unit roots test (number of cointegrating vectors)

Country	Canada	France	Germany	Italy	Japan	UK	US
Null hypothesis	Trace test						
r = 0	195.69*	203.25*	193.11*	224.29*	242.26*	199.85*	190.52*
r ≤ 1	95.81*	65.64*	73.98*	100.42*	90.09*	76.21*	73.98*
r ≤ 2	44.34*	29.88*	32.22*	48.52*	37.70*	36.34*	32.63*
r ≤ 3	3.11	1.04	1.81	3.75	2.34	1.38	1.97

Note: * indicates significant at the 5 percent level.

Table 3. Franses seasonal unit roots test (restrictions on cointegrating vectors)

Country	Canada	France	Germany	Italy	Japan	UK	US
Unit root	Likelihood ratio test (Chi-squared with 6 degrees of freedom)						
1	33.94*	21.31*	45.95*	22.37*	22.42*	12.14*	12.13*
-1	59.23*	74.56*	10.82**	92.27*	68.21*	64.71*	50.51*

Note: * and ** indicate significant at the 5 and 10 percent levels, respectively.

Table 4. Periodic unit roots test (likelihood ratios)

Country	Canada	France	Germany	Italy	Japan	UK	US
Statistic	- 1.93	-1.19	-1.77	-1.13	-1.48	-1.67	-0.76

Table 5. Periodic cointegration test (Boswijk-Franses F-tests)

Country used as dependent variable	Canada	France	Germany	Italy	Japan	UK	US
Statistic	1.34	2.38	1.49	3.34	2.57	1.29	0.96