Michael Dempsey (Australia)

The Fama and French three-factor model and leverage: compatibility with the Modigliani and Miller propositions

Abstract

The issue of whether the Fama and French (FF) three-factor model is consistent with the propositions of Modigliani and Miller (MM) (1958, 1963) has received surprisingly little attention. Yet, unless it is so, the model is at variance with the foundations of finance. Fama and French (FF) (1993, 1995, 1996, 1997) argue that their three-factor asset pricing model is representative of equilibrium pricing models in the spirit of Merton's (1973) inter-temporal capital asset pricing model (ICAPM) or Ross's (1976) arbitrage pricing theory (APT) (FF, 1993, 1994, 1995, 1996). Such claims, however, are compromised by the observations of Lally (2004) that the FF (1997) loadings on the risk factors lead to outcomes that are contradictory with rational asset pricing. In response, we outline an approach to adjustment for leverage that leads by construction to compatibility of the FF three-factor model with the Modigliani and Miller propositions of rational pricing.

Keywords: leverage, asset pricing, CAPM, factor models, MM propositions.
JEL Classification: G10, G12.

Introduction

The issue of whether the Fama and French (FF) three-factor model is consistent with the propositions of Modigliani and Miller (MM) (1958, 1963) has received surprisingly little attention (with the exception of Lally, 2004). The MM propositions hold that equations of risk and return that purport to reflect rationality in perfect capital markets (assuming no taxes) should comply with a risk-reward structure for the combination of the firm's debt and equity holders that does not change with variations of leverage. The assumption is that the firm's cash return to investors derives from its output goods and services, which need not be affected by financial leverage.

If the FF model is to be viewed as the outcome of systematic mispricing – the outcome of behavioral or psychological propensities (see, for example, Lakonishok, Shleifer and Vishny, 1994; Daniel, Hirshleifer and Subrahmanyam, 2001) – we have little basis for expecting that the model's outcome implications will adhere to a systematic rational framework as in the Modigliani and Miller propositions. We would, in effect, be obliged to shift to an empirically driven edifice, whereby one empirical observation has equal standing with another. A theoretical or rational development of a model that itself flies in the face of theoretical rationality is always likely to falter against the next empirical observation. As Lally (2004) points out, the "price" of diverging from arbitrage models, such as those of Merton (1973) and Ross (1976), is that the "logical" developments that are the outcome of the Modigliani and Miller propositions (the equations for the firm's cost of equity, its weighted average cost of capital (*WACC*) and beta as a function of leverage, for example) are made redundant.

Fama and French (1993, 1995, 1996, 1997) have aggressively interpreted their model as consistent with Merton's (1973) intertemporal capital asset pricing model (ICAPM) and the arbitrage pricing theory (APT) of Ross (1976). The big-minus-small (*BMS*) "market size" portfolio and high-minus-low (*HML*) "book-to-market equity" portfolio factors in their model are viewed as mimicking the underlying risk factors that can be hedged by investors. Fama and French note that the co-variabilities of return for the stocks of such firms do not appear to be captured by the ongoing movements of the market's returns, but nevertheless appear to be compensated for in average returns (FF, 1995; also Huberman and Kandel, 1987; Chan and Chen, 1991). Thus the *BMS* and *HML* portfolios are interpreted as capturing premiums for risk as it affects the cash flows of firms, the outcome of both "more distressed" firms (with higher book-to-market equity ratio) and "less-responsive-to-economic-cycles" (smaller) firms having less reliable earnings. The loadings on risk factors, in addition to the market return, reflect these aspects of risk exposure.

If the Fama and French three-factor model represents a version of Merton's ICAPM or Ross's APT, it clearly must conform to the fundamental principles of arbitrage, as captured by the Modigliani and Miller (MM) propositions. On the face of it, however, the FF model is algebraically inconsistent with the MM propositions, and hence with the no-arbitrage conditions of leverage. Lally (2004) demonstrates how, by applying increased leverage to a firm, the FF three-factor model can quite easily predict that leverage itself can cause the cost of equity *and* the *WACC* to simultaneously fall. This directly contradicts mean-variance theory, which holds that higher equity risk is commensurate with a higher cost of equity capital. In one industry, Lally actually

finds that the *WACC* declines with leverage to the point of being negative. In this manner, Lally observes that substantial violations of rationality are inherent in the Fama and French (1997) expressions for the loadings on the risk factors (*BMS* and *HML*). To attain the required consistency with the MM propositions, Lally applies a "leverage patch" to the FF (1997) loadings on the market size and book-to-market equity factors. However, Lally concedes that there is little empirical justification for his reconstructions.

This paper argues that Lally's "leverage patch" is inadequate on account of being applicable only under a restricted set of assumptions in regard to leverage, including the availability of risk-free debt for firms under financial stress. We respond by presenting our preferred approach to leverage, which maintains consistency between the FF three-factor model and the MM propositions.

The rest of the paper is organized as follows. In the next section, we outline the framework of a general multi-factor asset pricing model that is compatible with the MM propositions. In Section 2, we address the issue that Fama and French's (1997) empirically-derived expressions for loading on the portfolio factors are not algebraically consistent with leverage. Section 3 presents our recommended approach to leverage of the FF loadings, which allows for compatibility between the FF three-factor model with the MM propositions. The final section concludes the paper.

1. A general asset pricing model consistent with the MM propositions

In this section, we develop a general asset pricing model (of which the CAPM is a special case) consistent with the Modigliani and Miller propositions.

In the absence of market frictions, such as corporate taxes, Modigliani and Miller demonstrate that the overall return on the levered firm (R_L) (debt plus equity) is determined as:

$$R_L \equiv \frac{B}{B+E} R_B + \frac{E}{B+E} R_E = R_U,\qquad(1)$$

which is the statement that the weighted average of the returns on the firm's equity (R_E) and its bonds (R_B) must be equal to the return on the otherwise equivalent unlevered firm (equity only) (R_U). That is, $R_L = R_U$. The intuition is that the firm's cash returns to its debt and equity holders derive from the firm's operations, which are assumed to be independent of the firm's financing arrangements. It follows that the market value of the levered firm (V_L) is determined as:

$$V_L = V_E + V_B = V_U,\qquad(2)$$

which is the statement that the total value of the firm's equity (V_E) and bonds (V_B) is the same as that of the unlevered counterpart firm (V_U). This is Modigliani and Miller's proposition I. A violation of the above implies an opportunity for arbitrage. If corporate tax exists (at rate T_c) and debt is held in perpetuity, the additional after-tax cash flow to the firm created by the tax-deductibility of the interest payments may be expressed as $B\,R_B T_c$ in each period, where B is the market value of the firm's debt with market interest rate R_B. Assuming additionally that such cash flows have the same risk as the debt itself, and summing the flows as a perpetuity, their market value is $B\,T_c R_B / R_B = B\,T_c$. In which case, we have:

$$V_L = V_E + V_B = V_U + B.T_c.\qquad(3)$$

It follows that the additional value of the "tax shield" for the levered firm ($B.T_c$) has expected return equal to that of the underlying bonds (R_B). Hence with corporate taxes, we have:

$$R_L \equiv \frac{B}{B+E} R_B + \frac{E}{B+E} R_E =$$
$$= \frac{B+E-BT_c}{B+E} R_U + \frac{BT_c}{B+E} R_B$$

yielding:

$$R_E = \left[1 + \frac{B}{E}(1-T_c)\right] R_U - \frac{B}{E}(1-T_c) R_B,\qquad(4)$$

which is Modigliani and Miller's proposition II[1]. Defining the weighted average cost of capital (*WACC*) as:

$$WACC_E = \frac{E}{V} R_E + \frac{B}{V}(1-T_c) R_B,\qquad(5)$$

with *V* defined as the sum of *E* and *B*, and substituting in R_E from equation 4 gives:

$$WACC_E = R_U \left[1 - \frac{B}{V} T_c \right], \qquad (6)$$

which is Modigliani and Miller's proposition III.

Assuming no corporate taxes, the overall beta for the levered firm (β_L) (debt plus equity) – which, by the mathematics of covariance, is the weighted average of the betas of the firm's equity stock (β_E) and bonds (β_B) – is equal to the beta of the unlevered firm (equity only) (β_U). That is, $\beta_L = \beta_U$:

$$\beta_L = \frac{E}{B+E} \beta_E + \frac{B}{B+E} \beta_B = \beta_U . \qquad (7)$$

With corporate tax, the "additional" value for the levered firm ($B\, T_c$) has beta equal to that of the underlying bonds (β_B). In this case we have:

$$\beta_L = \frac{E}{B+E} \beta_E + \frac{B}{B+E} \beta_B =$$
$$= \frac{B+E-BT_c}{B+E} \beta_U + \frac{BT_c}{B+E} \beta_B ,$$

giving:

$$\beta_E = \left[1 + \frac{B}{E}(1-T_c) \right] \beta_U - \frac{B}{E}(1-T_c)\beta_B . \qquad (8)$$

Combining equations 4 and 8, we have:

$$\frac{R_E - R_U}{R_U - R_B} = \frac{\beta_E - \beta_U}{\beta_U - \beta_B} . \qquad (9)$$

If the debt has a risk-free return (R_f) ($\beta_B = 0$) and the unlevered firm has a return equal to the return on the market (R_m) ($\beta_m = 1$), equation 9 can be written as:

$$R_E - R_f = \beta_E.(R_m - R_f). \qquad (10)$$

Taking expectations of equation 10 gives:

$$E(R_E) - R_f = \beta_E.(E(R_m) - R_f), \qquad (11)$$

which is the CAPM. The CAPM is thereby demonstrated to be consistent with leverage equations 4 and 8, and hence with Modigliani and Miller's propositions I, II and III. More generally, we consider a model for the return R_j of firm j as:

$$R_j - R_f = b_j.(R_m - R_f) + s_j.R_1 + h_j.R_2 + ..., \qquad (12)$$

where $R_m, R_1, R_2 \ldots$ are risky variables and the $b_j, s_j, h_j \ldots$ are either held constant with leverage or with leverage act as:

$$b_j = \left\{ \left[1 + \frac{B}{E}(1-T_c) \right] b_{U,j} - \frac{B}{E}(1-T_c) b_{B,j} \right\}, \qquad (13a)$$

$$s_j = \left\{ \left[1 + \frac{B}{E}(1-T_c) \right] s_{U,j} - \frac{B}{E}(1-T_c) s_{B,j} \right\}, \qquad (13b)$$

$$h_j = \left\{ \left[1 + \frac{B}{E}(1-T_c) \right] h_{U,j} - \frac{B}{E}(1-T_c) h_{B,j} \right\}, \qquad (13c)$$

where $b_{U,j}$, $s_{U,j}$ and $h_{U,j}$ are the values of b_j, s_j, h_j for the unlevered firm, respectively, and $b_{B,j}$, $s_{B,j}$ and $h_{B,j}$ are the values of b_j, s_j, h_j for the firm's debt, respectively. In this case, equation 12 is equally consistent with the Modigliani and Miller propositions. To see this, observe that equation 12 applied to the firm's levered return (R_j), its unlevered return ($R_{U,j}$) and the return on its debt ($R_{B\,j}$), respectively, implies:

$$b_j = (R_j - R_f - s_j.R_1 - h_j.R_2 - ...)/(R_m - R_f), \qquad (14a)$$

$$b_{U,j} = (R_{U,j} - R_f - s_{U,j}.R_1 - h_{U,j}.R_2 - ...)/(R_m - R_f), \qquad (14b)$$

$$b_{B,j} = (R_{B,j} - R_f - s_{B,j}.R_1 - h_{B,j}.R_2 - ...)/(R_m - R_f). \qquad (14c)$$

Substituting equations 14 into equation 13a, and using equations 13b and 13c, give:

$$R_j = \left[1 + \frac{B}{E}(1-T_c) \right] R_{U,j} - \frac{B}{E}(1-T_c) R_{B,j}. \qquad (14)$$

In perfect markets with no corporate tax, equations 13 are the statements that the firm's total exposure to the risky returns $R_m, R_1, R_2. \ldots$ is neither created nor destroyed by variations in leverage. In effect, such models state that an asset's expected return (R_j) relates to fixed variables and exposure to risky variables ($R_m, R_1, R_2 \ldots$, as in equation 12) and that such exposures in perfect markets without corporate taxes are neither created nor destroyed for the firm as a whole due to the firm's choice of capital structure. By simply taking expectations of equation 12, we can express the expected return $E(R_j)$ for the general firm j as:

$$E(R_j) - R_f = b_j.[E(R_m) - R_f] + s_j. E(R_1) +$$
$$+ h_j. E(R_2) + ... \qquad (15)$$

This is consistent with Modigliani and Miller's propositions, where the $R_m, R_1, R_2 \ldots$ are risky variables and the $b_j, s_j, h_j \ldots$ are either held constant with leverage or with leverage act as equations 13.

2. The Fama and French three-factor model and leverage

The Fama and French expectations model for the expected return $E(R_j)$ on a portfolio is expressed:

$$E(R_j) - R_f = b_j.[E(R_m) - R_f] +$$
$$+ s_j.E(R_{SMB}) + h_j.E(R_{HML}). \qquad (16)$$

Its implied unlevered counterpart is:

$$E(R_{U,j}) - R_f = b_{U,j}.[E(R_m) - R_f] +$$
$$+ s_{U,j}.E(R_{SMB}) + h_{U,j}. E(R_{HML}), \qquad (17)$$

where R_{SMB} is the difference in the expected returns of a portfolio of stocks of small firms, and a portfolio of stocks of large firms; R_{HML} is the difference in the expected returns of a portfolio of stocks with high book-to-market equity, and a portfolio of stocks with low book-to-market equity; and the coefficients b, s, and h are the "covariabilities" of the firm's equity returns with R_m, R_{SMB}, and R_{HML}, respectively. Fama and French (1996, 1997) state that using loadings on the SMB factor to explain stock market returns is in line with the evidence of Huberman and Kandel (1987), who find that there is co-variation in the returns of stocks of small firms that is not captured by the ongoing or continuous market return, but is nevertheless compensated by average returns. They also find that using loadings on the HML factor to explain stock market returns is in line with the evidence of Chan and Chen (1991), who show that there is return co-variation related to relative distress that is missed by the market return, but is compensated in average returns. Thus Fama and French interpret their model as portraying a security's expectation of return dependent on the sensitivity of its returns to the market return, together with the sensitivity of its return to the SMB and HML portfolios, as mimicking additional risk factors.

Identifying equation 16 with equation 15, we obtain the result that the Fama and French model is consistent with the Modigliani and Miller propositions, provided that either:

- the coefficients b_j, s_j and h_j are held constant with leverage, or

- the coefficients b_j, s_j and h_j with leverage act as equations 13.

In the particular case that the firm's bonds can be assumed to be risk-free, the weightings $b_{B,j}$, $s_{B,j}$ and $h_{B,j}$ on the debt in equations 13 can be assumed equal to zero. Equations 13 then become:

$$b_i = \left[1 + \frac{B}{E}(1 - T_c) \right] b_{U,j} , \qquad (18a)$$

$$s_j = \left[1 + \frac{B}{E}(1 - T_c) \right] s_{U,j} , \qquad (18b)$$

$$h_j = \left[1 + \frac{B}{E}(1 - T_c) \right] h_{U,j} , \qquad (18c)$$

as Lally (2004). Fama and French (1997) model s_j as a function of the firm's market value of equity (ME) as:

$$s_j = s_{1,j} + s_{2,j} \ln (ME_j) \qquad (19a)$$

and h_j as a function of the firm's book equity to market equity as:

$$h_j = h_{1,j} + h_{2,j} \ln (ME_j / BE_j). \qquad (19b)$$

In this case, consistency with the Modigliani and Miller propositions requires either that s_j and h_j are held constant with leverage, or with leverage act as:

$$s_j = \left[1 + \frac{B}{E}(1 - T_c) \right] \cdot \left\{ s_{U1,j} + s_{U2,j} \, ln(ME_U)_j \right\} -$$
$$- \frac{B}{E}(1 - T_c) . s_{B,j} , \qquad (20a)$$

$$h_j = \left[1 + \frac{B}{E}(1 - T_c) \right] \cdot \left\{ h_{U1,j} + h_{U2,j} \, ln\left(\frac{BE_U}{ME_U} \right)_j \right\} -$$
$$- \frac{B}{E}(1 - T_c) . h_{B,j} . \qquad (20b)$$

Clearly neither s_j nor h_j, as defined in equations 19 are constant with leverage. Neither is there any reason to suppose that either s_j or h_j so defined are determined with leverage as equations 20. If the firm has risk-free debt ($s_{B,j} = h_{B,j} = 0$), equations 20 become:

$$s_j = \left[1 + \frac{B}{E}(1 - T_c) \right] \cdot \left\{ s_{U1,j} + s_{U2,j} \, ln(ME_U)_j \right\}, \quad (21a)$$

$$h_j = \left[1 + \frac{B}{E}(1 - T_c) \right] \cdot \left\{ h_{U1,j} + h_{U2,j} \, ln\left(\frac{BE_U}{ME_U} \right)_j \right\}, \quad (21b)$$

as Lally (2004). If the firm has risk-free debt, Lally demonstrates that the outcome from applying equations 19 is inconsistent with equations 21. It appears that we must conclude that the Fama and French three-factor model (equations 16 and 17) is generally incompatible with the Modigliani and Miller propositions.

3. Recommended approach to leverage in the Fama and French three-factor model

Lally's (2004) approach is to replace Fama and French's equations 19 (above) for the loadings on s_j and h_j with equations 21. However, the approach contradicts Fama and French's own recommended equations 19. Additionally, the approach presents several computational difficulties. To start, there is difficulty in computing the *unlevered* $s_{U1,j}$, $s_{U2,j}$, $h_{U1,j}$, $h_{U2,j}$ in the equations[1]. In addition, the assump-

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[1] In addition, Fama and French do not show how the b coefficients on the market premium R_m - R_f might be adjusted with leverage, which creates additional difficulties for our leverage calculations. Lally (2004) responds by assuming in his examples that b is invariant with leverage, which is clearly erroneous.

tion in equations 21 is that the firm enjoys risk-free debt, which is unreasonable in the context of distressed firms. So we are obliged to resort to equations 20 with the additional complication that we need to estimate $s_{B,j}$ and $h_{B,j}$ loadings for the firm's risky debt. Finally, neither of the above approaches has an empirical justification.

An alternative approach, and the one recommended here, is to interpret the Fama and French (1997) estimates for the $s_{1,j}$, $s_{2,j}$, $h_{1,j}$, $h_{2,j}$ across industries as representing an *averaging over leverage* for that particular industry. In other words, we assume that the outcome for any particular firm using their tables of recommended $s_{1,j}$, $s_{2,j}$, $h_{1,j}$, $h_{2,j}$ values relates to a firm of typical industry leverage. This assumption reflects the way the results are estimated by Fama and French. The Lally-type calculation in which the Fama and French equations 19 are applied directly to a firm with an arbitrary level of leverage is then inappropriate. Instead, the approach should be to apply equations 19 to the firm of interest, assuming that it has a typical industry leverage, and then re-lever with MM proposition II (equation 4) to compensate for the firm's particular leverage[1].

As a worked example, we borrow from Lally and consider a firm in the Beer industry (say, BeerZZZ) with market equity $4.6b, leverage debt/equity (B/S) equal to 0.1 (hence debt, B = $0.46b), and a book-to-equity ratio equal to 0.51 (hence book value of equity = $2.35 billion). Also following Lally, we assume for simplicity that there are no taxes and that the firm's total value is invariant to pure leverages. Here we also assume that the typical leverage for the Beer industry is 30%, which reflects the typical U.S. firm. For the Beer industry we take from Fama and French (1997) that $b = 0.90$, $s_1 = 0.1$, $s_2 = -0.15$, $h_1 = 0.27$, $h_2 = 0.73$, and the market risk premiums for $R_m - R_f$, SMB, and HML equal 0.052, 0.032, 0.054, respectively. Also for simplicity, we follow Lally in assuming that the firm is able to borrow at risk-free debt at 6%.

The process for applying leverage to the Fama and French three-factor model would then follow three stages:

Stage 1: Adjust the leverage for the firm being considered (BeerZZZ) to reflect the typical leverage for the industry. Thus, in this case, we require that BeerZZZ borrow $X billion such that:

$$\frac{0.46 + X}{4.6 - X} = 0.3 \ .$$

Hence $X = $0.71, and the new value of market equity falls to $(4.6 − 0.71) = $3.89 billion, and the book equity falls to $(2.35 − 0.71) = $1.64 billion, implying that *BE/ME* falls to 1.64/3.89 = 0.42. Substituting these values into equations 19 yields:

$$s = 0.10 - 0.15 \ln (3.89) = - 0.104;$$

and

$$h = 0.27 + 0.73 \ln(0.42) = -0.36$$

and substituting this into the Fama and French model equation 16:

$$R = 0.06 + 0.9(0.052) - 0.104 \, (0.032) -$$

$$- 0.36(0.054) = 0.84$$

gives the cost of equity for the re-financed (with typical industry leverage) BeerZZZ as 8.4%.

Stage 2: Apply Modigliani and Miller's proposition II (equation 4) to calculate the cost of equity (R_U) for the unlevered version of BeerZZZ. We can write equation 4 as:

$$R_E = \left[1 + \frac{B}{E}\right] R_U \ - \frac{B}{E} \, R_B$$

giving:

$$0.084 = \left[1 + 0.3 \right] R_U \ - 0.3(0.06) = 0.078.$$

Hence the cost of equity for unlevered BeerZZZ (R_U) is 7.8%.

Stage 3: Apply Modigliani and Miller's proposition II to calculate the cost of equity for the required levered versions of BeerZZZ. Hence with leverage 0.10, as in Lally's first example, we would calculate equation 4 as:

$$R_E = \left[1 + \frac{B}{E}\right] R_U \ - \frac{B}{E} \, R_B =$$

$$= [1 + 0.1] \, 0.078 - 0.1(0.06) = 0.08.$$

With leverage 0.7, as in Lally's second example, we would similarly find:

$$R_E = [1 + 0.7] \, 0.078 - 0.7(0.06) = 0.127.$$

Hence the cost of equity for the required levered versions of BeerZZZ is 8% (10% leverage) and 12.7% (70% leverage).

The above approach allows us to accept the industry estimates of Fama and French (1997) and simulta-

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[1] This is the approach we would recommend when manipulating industry betas with leverage: if the leverage for BeerZZZ is not typical of the industry, do *not* assume that the beta for BeerZZZ is the same as it is for the rest of the Beer industry. Rather, assume that the beta provided for the Beer industry is for a leverage that is typical of the industry. Then proceed to de-lever to find the beta for the unlevered firm in the Beer industry, before re-levering with the leverage of BeerZZZ to determine its beta.

neously, by construction, to have the outcome returns remained consistent with Modigliani and Miller's proposition II.

Conclusion

This paper clarifies the conditions of leverage that must apply to a factor model for it to be considered algebraically consistent with the rules of no arbitrage. The structure of the FF three-factor model itself does not reveal accordance with such conditions. It is possible, however, to interpret the FF model as a proxy for a true analytical model that complies with the conditions, but which is difficult to test. This paper shows how, under such conditions, leverage of the FF three-factor model can be applied.

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