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The role of SGT distribution in Value-at-Risk estimation: evidence from the WTI crude oil market

Abstract

This study assesses market risk in the international crude oil market from the perspective of VaR analysis. A GARCH-SGT approach is thus proposed capable of coping with fat-tails, leptokurtosis and skewness using SGT returns innovations and catering for volatility clustering with the GARCH(1,1) model in modeling one-day-ahead VaR. This technique is illustrated using daily returns of West Texas Intermediate crude oil spot prices from December 2003 to December 2007. Empirical results indicate that the VaR forecast obtained by the GARCH-SGT model is superior to that of the GARCH-T and GARCH-GED models through a series of rigorous model selection criteria. Overall, the sophisticated SGT distributional assumption significantly benefits VaR forecasting for WTI crude oil returns at low and high confidence levels, indicating a need for VaR models that consider fat-tails, leptokurtosis and skewness behaviors. The GARCH-SGT model thus is a robust forecasting approach that can practically be implemented for VaR measurement.

Keywords: risk management, crude oil, SGT distribution, conditional coverage.
JEL Classification: C52, C53, Q42.

Introduction

Assessing and managing market risk against financial uncertainty are of priority concern among academics, practitioners, regulators and common investors, particular since the financial world has witnessed the bankruptcy or near bankruptcy of various institutions that have incurred huge losses due to their exposure to unexpected market moves over the last 15 years. According to the "Amendment of the Basel Accord" in 1997, the Basel Committee allowed banks to use internal risk models to fulfill their capital adequacy requirements. Within this framework, the Value-at-Risk (VaR) methodology was developed as a practical tool in response to the financial catastrophe of the 1990s and became a key measure of market risk. With VaR, financial institutions can have a sense on the minimum amount that is expected to lose with a small probability c over a given time horizon k (usually 1- or 10-day). Taking $c = 5\%$ as an example, a 1-day VaR of $10 million indicates that on one out of 20 days, a loss of at least $10 million can be expected. Restated, VaR is defined as the maximum loss over a given time horizon at a given confidence level $1 - c$.

Since the industrial revolution at the end of the 18th Century, oil has become the primary source of energy upon which mankind relies, making it the lifeblood of modern economies. The world has witnessed oil price volatility for a long time; however, crude oil price volatility not only affects national economies as a whole, but also affects most sectors within those economies[1]. Over the past few decades, energy mar-

ket price fluctuations have been caused primarily by supply and demand imbalances originating from the business cycle, political upheavals, wars and extreme weather conditions. Recently, the worldwide deregulation[2] of oil markets has created increased opportunities and incentives for market participants to trade crude oil via the spot market and derivatives. This has resulted in increased price volatility owing to the trading behavior of market participants with either long or short positions. Owing to the intensely competitive nature of the deregulated energy market, oil prices have become highly volatile. Modeling risk for the crude oil market thus is a challenging task because oil prices exhibit more extreme price movements of magnitudes than traditional financial assets, and are inherently complex due to the strong interaction between the trading of products and economic supply and demand imbalances. Arguably, approaches to VaR measurement that are common in financial markets may not necessarily be appropriate in turbulent oil markets.

VaR methodology benefits from the quantile quality of an appropriate distribution for the innovation process. Various studies related to VaR applications have demonstrated an improvement in VaR estimations associated with GARCH models with returns innovations that allow fat-tailed distributions. Some studies have found evidence in favor of GARCH models using student-t distribution, for the case of stock and exchange rate returns predictions. Examples include Huang and Lin (2004), Bams et al. (2005), Ané (2006) and So and Yu (2006). Other studies have found evidence in favor of the generalized error distribution (GED), with examples including Angelidis et al. (2004), Marcucci (2005) and Su and Knowles (2006), all of whom examined global stock markets.

[1] Examples include the oil-related industries (oil exploration, production and refining), the highly oil sensitive transportation industries (airlines, trucking and railroads) and the highly oil intensive manufacturing industries (aluminum, polymer and steel).

[2] See Ghouri (2006) and Narayan and Narayan (2007) for more details.

The student-t and GED distributions are not without their faults. However, the aforesaid distributions impose the restriction of symmetry and thus are not always valid for financial data. Moreover, Brooks and Persand (2003) provided supportive evidence that VaR models which do not accommodate for asymmetries in the unconditional distribution of returns underestimate predicted VaR[1].

Since asymmetry is an important issue in the VaR framework, recent developments account for the asymmetrically distributional assumption in returns innovations. On the one hand, Giot and Laurent (2003a) estimated daily VaR for stock indices using the skewed student-t distribution, indicating that it outperforms the pure symmetric one. Giot and Laurent (2003b) and Wu and Shieh (2007) also calculated daily Value-at-Risk for commodities and T-bond interest rate futures returns, respectively, based on the same distribution. Both of them concluded that the predictive accuracy of GARCH-type models with skewed fat-tailed distribution is better than that of traditional ones. On the other hand, Lee et al. (2008) first incorporated the skewed generalized error distribution (SGED) into the GARCH model to analyze the predictive performance of VaR. Empirical results demonstrated that the use of the SGED distribution which explicitly accommodates both skewness and kurtosis, is essential for out-of-sample VaR forecasting when applied to U.S. stock markets. Furthermore, Mittnik et al. (2000) demonstrated that more general GARCH specifications with skewed fat-tailed distributions significantly improve the precision of out-of-sample VaR forecasts. Recently, Theodossiou (1998) developed the skewed generalized t (SGT) distribution, which provides a flexible tool for modeling the empirical distribution of financial data exhibiting fat-tails, leptokurtosis and skewness. Using stock market indices, exchange rates and the price of gold, Theodossiou showed that the SGT provides a good fit to the empirical distribution of the data. Bali and Theodossiou (2007) employed ten popular variations of the GARCH model for the estimation of VaR and expected shortfall measures based on the SGT distribution using daily returns of the S&P-500 composite index. Empirical results indicated that the TS-GARCH and EGARCH models have the best overall performance. Notably, because of the absence of SGT-GARCH computer programs, Bali and Theodossiou used an indirect approach to calculate model-based VaR[2]. Such an approach is prone to model risk. To sum up, these facts can be construed as strong empirical evidence in favor of the adoption of sophisticated distribution, which altogether embody considerable characteristics of returns innovations, and of GARCH models to allow for heteroskedasticity.

To the best of our knowledge, Cabedo and Moya (2003) were the first to evaluate Value-at-Risk for daily spot returns of Brent oil prices via historical simulation with the ARMA forecasts (HSAF) approach. Subsequently, a similar approach was presented by Sadeghi and Shavvalpour (2006). To take account of heteroskedasticity, Sadorsky (2006) and Hung et al. (2008) are two recent studies that model Value-at-Risk using GARCH models for crude oil returns. Sadorsky (2006) indicated that the GARCH(1,1) model fits well for daily volatility of crude oil returns. Extending the analysis of Sadorsky (2006) to various oil-related commodities, Hung et al. (2008) found evidence that the proposed GARCH-HT model-based VaR approach provides good accuracy and efficiency at both low and high confidence levels for alternative energy commodities when asset returns exhibit leptokurtic and fat-tailed features.

Although our analysis is somewhat similar to the previous papers, there are still significant differences. This study implements GARCH volatility models under three distributional assumptions to estimate the 95% and 99% one-day-ahead VaR[3] for WTI crude oil returns, whereas most previous research has focused on stock markets or foreign exchange markets. The different distributions (student-t, GED and SGT distribution) will allow the selection of a model for the return tails and infer whether a sophisticated distribution of returns innovations benefits VaR forecasting. A group of rigorous tests are then employed for comparative evaluation of the predictive performance of these VaR models in risk management. At the first stage, the models are back-tested to determine their predictive accuracy by simultaneously using the unconditional coverage test (Kupiec, 1995) and the conditional coverage test (Christoffersen, 1998). Note that the former approach can reject a model having either too high or too low failures, while the latter enables the rejection of models that generate either too many or too few clustered VaR violations. At the second stage, two utility-based loss functions (regulatory and firm loss functions) are defined to further evaluate models that have met the prerequisites of both

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[1] Bams et al. (2005) also indicated that if a particular distribution does not allow for an empirical phenomenon which is present in the data, then the accuracy of VaR predictions will suffer accordingly.
[2] See page 252 of Bali and Theodossiou (2007) for more details.

[3] VaR forecasts are considered over a daily horizon because this horizon is considered relevant for trading purposes, and is therefore believed to be interesting to academics, regulatory bodies and practitioners who engaged in risk management.

back-testing criteria (Sarma et al., 2003). Under this framework, a model that minimizes the total loss is preferred to other models. Moreover, this study employs the same one-sided sign tests as Diebold and Mariano (1995) and Sarma et al. (2003) by further examining the competing models in terms of loss functions which can reveal the superiority of one model over another. Risk managers thus can select a unique model among the various candidates.

The remainder of this study is organized as follows. Section 1 describes the econometric methodology under consideration, including the various VaR models and the criteria for evaluating VaR estimates. Section 2 presents the data description and model estimates, while Section 3 details the comparative analysis of VaR performance of competing models. Finally, the last Section concludes.

1. Econometric methodology

1.1. Conditional volatility model. The existing literature has long been recognized that the time series data of financial assets appear to exhibit autocorrelation and clustering effects in volatility. Many studies support the GARCH genre of volatility models as providing a good description of the stylized facts in volatility. In fact, a great deal of financial literature finds evidence in favor of parsimonious models, such as GARCH(1,1), for the case of stock and crude oil returns predictions. Examples include Bollerslev

et al. (1992), Sadorsky (2006) and Hung et al. (2008). This present study thus relies on the simplest GARCH(1,1) specification for modeling conditional volatility of daily crude oil prices returns. Let $r_t = (\ln P_t - \ln P_{t-1}) \cdot 100$ denote the daily return series, where P_t is the crude oil price at time t, and Ω_{t-1} denotes the information set of all observed returns up to time t-1. The GARCH(1,1) model with a basic mean can be formulated as follows:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \big|_{\Omega_{t-1}} \sim F(0,1), \tag{1}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2}$$

where μ and σ_t^2 denote the conditional mean and variance of returns, respectively. ε_t is the innovation process, while $F(0,1)$ is a density function with a mean of zero and a unit variance. Furthermore, ω, α and β are nonnegative parameters with the restriction of $\alpha + \beta < 1$ to ensure the positiveness of conditional variance and stationarity as well.

1.2. Alternative distributions. Theodossiou (1998) developed the skewed generalized t-distribution which provides a flexible tool for modeling the empirical distribution of financial data exhibiting fat-tails, leptokurtosis and skewness. The skewed generalized t (SGT) probability density function for the standardized residuals (z_t) of returns can be defined as follows:

$$F(z_t; N, \kappa, \lambda) = C \left(1 + \frac{|z_t + \delta|^\kappa}{((N+1)/\kappa)(1 + sign(z_t + \delta)\lambda)^\kappa \theta^\kappa} \right)^{-(N+1)/\kappa}, \tag{3}$$

where

$$C = 0.5\kappa \cdot \left(\frac{N+1}{\kappa} \right)^{-1/\kappa} \cdot B\left(\frac{N}{\kappa}, \frac{1}{\kappa} \right)^{-1} \cdot \theta^{-1}, \tag{4}$$

$$\theta = \left(g - \rho^2 \right)^{-1/2}, \tag{5}$$

$$\rho = 2\lambda \cdot B\left(\frac{N}{\kappa}, \frac{1}{\kappa} \right)^{-1} \cdot \left(\frac{N+1}{\kappa} \right)^{1/\kappa} \cdot B\left(\frac{N-1}{\kappa}, \frac{2}{\kappa} \right), \tag{6}$$

$$g = (1 + 3\lambda^2) \cdot B\left(\frac{N}{\kappa}, \frac{1}{\kappa} \right)^{-1} \cdot \left(\frac{N+1}{\kappa} \right)^{2/\kappa} \cdot B\left(\frac{N-2}{\kappa}, \frac{3}{\kappa} \right), \tag{7}$$

$$\delta = \rho \cdot \theta, \tag{8}$$

where z_t is the standardized residual with zero mean and unit variance; N is a tail-thickness parameter with constraint $N > 2$; κ is a leptokurtosis parameter with $\kappa > 0$; λ governs the skewness of the density obeying the constraint $|\lambda| < 1$; sign is the sign function. Moreover, B(•) denotes the beta

function. Notably, the skewed generalized t distribution nests several well-known distributions. Specifically, it gives for $\kappa = 2$ and $\lambda = 0$ the student-t distribution and for $N = \infty$ and $\lambda = 0$ the generalized error distribution.

In the empirical investigation, three conditional distributions for the standardized residuals of returns innovations were considered: (I) a standardized student-t distribution, (II) a generalized error distribution, and (III) a skewed generalized t-distribution, since the first two distributional assumptions are commonly used with previous studies and the last one reveals its predominance to cater for the empirical distribution of financial data. Accordingly, we construct three competing model specifications in modeling volatility of the WTI crude oil returns in our comparative analysis: GARCH-T, GARCH-GED and GARCH-SGT models. The parameter vector $\Psi = [\mu, \omega, \alpha, \beta, \cdots]$ is obtained from the maximization of the sample log-likelihood function, using QMLE (Quasi

maximum likelihood estimation, QMLE) as follows:

$$LL(\Psi) = \sum_{t=1}^{T} ln\big(F(z_t \mid N, \kappa, \lambda)\big), \qquad (9)$$

where $F(\bullet)$ is the likelihood function of the GARCH model with various distributional assumptions.

1.3. Calculating Value-at-Risk. Under the framework of the parametric techniques (Jorion, 2000), the conditional VaR estimate for a one-day holding period is obtained as follows:

$$VaR_{t+1} = z_c \cdot \hat{\sigma}_t + \mu \text{ with } F(z_c) = c, \qquad (10)$$

where z_c denotes the corresponding quantile of the z_t distribution at a given confidence level 1-c, while $\hat{\sigma}_t$ is the volatility forecast generated from Eq (2).

1.4. Evaluating VaR performance of competing models. To compare the forecasting ability of different models in terms of VaR, this study employs a group of rigorous tests to further examine the predictive performance of VaR models in risk management.

1.4.1. Unconditional coverage test (LR_{uc}). To backtest the VaR results, this study first employs a likelihood-ratio test by Kupiec (1995) to test whether the true failure rate is statistically consistent with the VaR model's theoretical failure rate (referred to as the unconditional coverage in Christoffersen, 1998). The null hypothesis of the failure rate p is tested against the alternative hypothesis that the failure rate is different from p, in which statistics is given by:

$$LR_{uc} = 2\ln\left[\frac{\hat{\pi}^{n_1}(1-\pi)^{n_0}}{P^{n_1}(1-P)^{n_0}}\right] \sim \chi^2(1), \qquad (11)$$

where $\hat{\pi} = \dfrac{n_1}{n_0 + n_1}$ is the maximum likelihood estimate of p, and n_1 denotes a Bernoulli random variable representing the total number of VaR violations[1,2].

1.4.2. Conditional coverage test (LR_{cc}). In a risk management framework, it is of paramount importance that VaR exceptions be uncorrelated over time, which prompts independence and conditional coverage tests based on the evaluation of interval forecasts. Christoffersen (1998) devel-

oped a conditional coverage test (LR_{cc}) that jointly investigates whether the total number of failures is equal to the expected one, and the VaR exceptions are independently distributed. Given the realizations of the return series r_t and the set of VaR estimates, the indicator variable I_t can be defined as follows:

$$I_t = \begin{cases} 1 & if \ r_{t+1} < VaR_t \\ 0 & if \ r_{t+1} \geq VaR_t \end{cases}. \qquad (12)$$

Since accurate VaR estimates exhibit the property of correct conditional coverage, the I_t series must exhibit both correct unconditional coverage and serial independence. The LR_{cc} test is a joint test of these two properties, and the corresponding test statistics is LR_{cc} = LR_{uc} + LR_{ind} as we condition on the first observation. Consequently, under the null hypothesis that the failure process is independent and the expected proportion of exceptions equals p, the appropriate likelihood ratio is represented as follows:

$$LR_{cc} = -2\ln\frac{(1-P)^{n_0} P^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}} \hat{\pi}_{01}^{n_{01}} (1-\hat{\pi}_{11})^{n_{10}} \hat{\pi}_{11}^{n_{11}}} \sim \chi^2(2), \quad (13)$$

where $n_{i,j}$ = the number of observations with value i followed by value j (i,j=0, 1), $\pi_{ij} = P\{I_t = j \mid I_{t-1} = i\}$ (i,j=0, 1), $\hat{\pi}_{01} = n_{01}/(n_{00} + n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$[3].

1.4.3. Risk management loss functions. In most cases, there is more than one VaR model that can pass these coverage tests, and therefore a risk manager cannot select a unique volatility forecasting technique. Consequently, this study follows the two-step model selection criterion of Sarma et al. (2003) by further selecting one model among the various candidates through utility-based loss functions which are closer to the real risk manager's utilities. In this section, two utility-based loss functions are introduced as follows:

♦ Regulatory loss function (RLF). The regulatory loss function (RLF) (Sarma et al., 2003; Marcucci, 2005) reflecting the regulator's utility function is given by:

$$L_{t+1}^R = (r_{t+1} - VaR_t)^2 \cdot I_{\{r_{t+1} < VaR_t\}}, \qquad (14)$$

where I denotes the usual indicator function. Moreover, the RLF penalizes large violations more severe than small violations.

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[1] A violation occurs if the predicted VaR cannot cover the realized dollar loss.
[2] The Kupiec's (1995) LR-test can reject a model having either too high or too low failures, but has been criticized for its inability in response to volatility clustering.

[3] In comparison with Kupiec's (1995) LR-test, the advantage of Christoffersen's procedure is that it can reject a model that generates either too many or too few clustered violations.

♦ Firm loss function (FLF). Sarma et al. (2003) argued that firms use VaR as a pragmatic tool in international risk management; nevertheless, there is a conflict between the goal of safety and the goal of profit maximization. A VaR estimator which reported too high values of VaR would force the firm to hold too much capital, imposing the opportunity cost of capital upon the firm. On the contrary, the firm's capital set aside may not be enough to cover it from market risks. Thus, the firm loss function (FLF) reflecting the utility of a firm is given by:

$$L_{t+1}^F = \left(r_{t+1} - VaR_t\right)^2 \cdot I_{\{r_{t+1} < VaR_t\}} - \delta \cdot VaR_t \cdot I_{\{r_{t+1} > VaR_t\}}, \quad (15)$$

where δ measures the opportunity cost of capital, and can be linked to the risk-free interest rate. Thus, we set $\delta = 1.5\%$ in our empirical illustrations. While both loss functions take into account the magnitude of the VaR violations into account, the FLF which considers the opportunity cost of capital is closer to the real risk managers' utilities (Marcucci, 2005).

1.4.4. Superiority tests in terms of utility-based loss functions. For those models which can meet the prerequisite of both correct unconditional and conditional coverage, this study employs the same one-sided sign tests as Diebold and Mariano (1995) and Sarma et al. (2003) by further examining the competing models in terms of utility-based loss functions which can assess the superiority of one model over another. To consider two VaR models, model i and model j, defining the loss differential between model i and j as $e_t = L_{i,t} - L_{j,t}$[1], the null hypothesis of a zero-median loss differential is tested against the alternative hypothesis of a negative median, with a studentized version of the sign test given by:

$$\hat{S}_{ij} = \left(S_{ij} - 0,5T\right)\left(0.25T\right)^{-0,5}, \quad (16)$$

where $\hat{S}_{ij} = \sum_{t=1}^{T} I_{\{e_t > 0\}}$, $I_{\{\}}$ is the indicator function, and T denotes the evaluation period. Under the null,

$\hat{S}_{ij}$ is asymptotically distributed as a standard normal. The null hypothesis is rejected at the 5% significance level if $\hat{S}_{ij} < -1.645$. Rejections of $\hat{S}_{ij}$ ($\hat{S}_{ji}$) would imply that model i (j) is significantly superior to model j (i).

2. Data and estimation results

2.1. Data description. The data examined in this paper are daily West Texas Intermediate (WTI) crude oil spot prices retrieved from Energy Information Administration (EIA) website. The data set for WTI crude oil covers the period from December 30, 2003 to December 28, 2007 for a total of 1001 observations. The daily price returns are calculated as, $r_t = \left(\ln P_t / P_{t-1}\right) \cdot 100$ where r_t and P_t denote the rate of return and the closing spot price on day t, respectively[2].

Summary statistics of crude oil returns for the whole sample period are provided in Table 1. From panel A, the average daily returns are positive and very small compared with the variable standard deviation. The oil price returns display significant evidence of skewness and kurtosis. That is, the returns series is skewed towards the left, indicating that there are more negative than positive outlying returns in crude oil market and the returns series is characterized by a distribution with tails that are significantly thicker than for a normal distribution. J-B test statistic further confirms that the spot returns of WTI crude oil price is non-normal distributed. Moreover, the Q^2 and LM-test statistics display linear dependence of squared returns and strong ARCH effects. Consequently, the preliminary analysis of the data indicates the use of a conditional model for capturing the fat-tails and time variation of volatility. To avoid the spurious results in time series analysis, Panel B reports the Augmented Dickey and Fuller (1979) (ADF) and Phillips and Perron (1988) (PP) unit root tests. The test results indicate no evidence of non-stationarity in the spot prices returns.

Table 1. Summary statistics of crude oil returns

Panel A. Basically statistical characteristics						
Mean %	Std. dev.	Skewness	Kurtosis	J-B	Q²(12)	LM(12)
0.1067	2.0650	-0.2030*	1.5706*	109.6551*	36.182*	39.086*

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[1] Negative values of e_t indicate a superiority of model i over j.
[2] To save space, we do not report the descriptive graphs of WTI crude oil returns. Note that the QQ-plot against the normal distribution shows that returns distribution exhibits fat-tails. Moreover, QQ-plot also indicates that fat-tails are not symmetric, providing evidence in favor of SGT distribution with flexible treatment of fat-tails, leptokurtosis and skewness in the conditional distribution of crude oil returns.

Table 1 (cont.). Summary statistics of crude oil returns

Panel B. Unit root tests for stationarity				
ADF	Lags		PP	Bandwidth
-33.5201*	0		-33.6067*	12

Notes: 1. * denotes significance at the 1% level. 2. J-B represents the statistics of Jarque and Bera (1987)'s normal distribution test. 3. $Q^2(12)$ denotes the Ljung-Box Q test for 12th order serial correlation of the squared returns. 4. LM test also examines for autocorrelation of the squared returns. 5. Each of the unit root test statistics is calculated with an intercept in the test regression. 6. The lag length for the ADF test regression is set using the Schwarz Information Criteria (SIC). 7. The bandwidth for the PP test regressions is set using a Bartlett Kernel.

2.2. Estimation results and diagnostic tests. In this study, the parameters are estimated by quasi maximum likelihood estimation (QMLE) in terms of the BFGS optimization algorithm using the econometric package of WinRATS 7.0. Model estimates and diagnostic tests for WTI crude oil returns during the in-sample period are provided in Table 2.

Panel A of Table 2 reports the estimates of models considered. First, the GARCH parameters (ω, α and β) are all positive and found to be highly significant (at least at the 5% level). Second, the GARCH component in each model exhibits persistence in the volatility parameter, as the $a + \beta \approx 1$. Third, the coefficients N, κ and λ of VaR models all meet the parameters' constraints. On the one hand, the tail-thickness parameters (N) of GARCH-SGT and GARCH-T models which range from 6.8979 to 12.3083, are statistically significant at the 1% level from two, indicating that the distribution of returns series is fat-tailed. On the other hand, the leptokurtosis parameters (κ) of GARCH-GED and GARCH-SGT models which range from 1.6849 to 2.5947, are statistically significant at the 1% level from zero, showing that the empirical returns distribution exhibits leptokurtic. Furthermore, the skewness parameter (λ) of the GARCH-SGT model is negative (-0.0739) and found to be significant at the 5% level, implying that the returns of WTI crude oil prices display evidence of a leftwards skew.

Table 2. Estimation results for alternatively competing VaR models

Panel A. Model estimates			
Parameter	GARCH-T	GARCH-SGT	GARCH-GED
μ	0.1528[a] (0.0616)	0.1463[a] (0.0575)	0.1464[a] (0.0607)
ω	0.6385[a] (0.2572)	0.5639[a] (0.2504)	0.7823[a] (0.3263)
α	0.0669[b] (0.0230)	0.0686[b] (0.0224)	0.0606[b] (0.0227)
β	0.7840[b] (0.0673)	0.8002[b] (0.0663)	0.7562[b] (0.0826)
N	12.3083[b] (3.6537)	6.8979[b] (2.2409)	∞
κ	2	2.5947[b] (0.4152)	1.6849[b] (0.1002)
λ	0	-0.0739[a] (0.0320)	0
Panel B. Diagnostic tests			
$Q^2(12)$	5.895	5.632	6.907
LL	-2127.2812	-2125.3182	-2132.1543
LR-test	3.9260		13.6722[c]

Notes: 1. In this table, N, κ and λ are specific parameters of the SGT distribution, where N and κ are positive kurtosis parameters controlling the tails and height of the density with $N > 2$ and $\kappa > 0$, respectively; λ denotes the skewness parameter obeying the constraint $|\lambda| < 1$. 2. a and b denote significance at the 5% and 1% levels, respectively. 3. Standard errors for the estimators are included in parentheses. 4. $Q^2(12)$ is the Ljung-Box Q test for serial correlation in the squared standardized residuals with 12 lags. 5. LL indicates the log-likelihood value. 6. The LR-test statistic is asymptotically distributed as a chi-square with two degrees of freedom. Its critical value at the one-percent level of significance is 9.21.

Panel B of Table 2 presents the diagnostic tests of competing models. The Ljung-Box Q statistic indicates that the GARCH(1,1) specification in these models is sufficient to correct the serial correlation of the returns series in the conditional variance equation. Moreover, the LR-test results indicate rejection of the GARCH-T and GARCH-GED models, providing evidence that the GARCH-SGT model achieves superior in-sample goodness of fit to alternatives for the data employed. While obtain-

ing the estimates of these VaR models, the model-based VaR can be computed using Eq (10) according to the corresponding quantile of alternative distributions[1]. A comprehensive evaluation of the predictive performance of the competing VaR models is then carried out in the next section.

3. Comparative analysis of VaR performance

3.1. Unconditional and conditional coverage tests results. Table 3 presents the forecasting performance summary for various VaR models using 95% and 99% confidence levels. From panel A of Table 3, the GARCH-GED model yields the highest average value of VaR estimates and the lowest unexpected loss at the 95% confidence level. Generally speaking, the empirical failure rates of these three

models are very close to the prescribed one. Therefore, all models can pass the Kupiec's (1995) unconditional coverage test, indicating that the sample point estimate is statistically consistent with the prescribed confidence level of these VaR models. Turning into the column of LR_{cc} statistics, we find that all models also pass the conditional coverage test, indicating that these three models do not incur too many or too few clustered VaR violations. In the case of 95% VaR confidence level, a risk manager cannot select a unique VaR technique when there is more than one model that has passed these coverage tests. Consequently, a two-step model selection procedure is recommended for further selecting one model among the various candidates through utility-based loss functions.

Table 3. Forecasting performance summary of Value-at-Risk statistics

Panel A. VaR results at the 95% confidence level						
	Mean VaR	Violations	Failure rate	LR_{uc}	LR_{cc}	UL
GARCH-T	-3.1893	54	5.4 %	0.3286	1.8153	-1.0170
GARCH-GED	-3.2468	55	5.5 %	0.5104	1.8214	-0.9462
GARCH-SGT	-3.1730	55	5.5 %	0.5104	1.8214	-1.0124
Panel B. VaR results at the 99% confidence level						
	Mean VaR	Violations	Failure rate	LR_{uc}	LR_{cc}	UL
GARCH-T	-4.8661	6	0.6 %	1.8862	**6.2899**	-2.4566
GARCH-GED	-4.8389	6	0.6 %	1.8862	**6.2899**	-2.4407
GARCH-SGT	-4.8824	7	0.7 %	1.0156	4.8722	-2.1098

Notes: 1. UL denotes the unexpected loss, which refers to the average dollar loss caused by the failures of VaR model. 2. The critical values of the LR_{uc} and LR_{cc} statistics at the 5% significance level are 3.84 and 5.99, respectively. 3. Boldface indicates significance at the 5% level.

For the case of 99% VaR confidence level, we find that the GARCH-SGT model generates the highest average value of VaR estimates which is accompanied with the lowest unexpected loss. Although these three models have very similar empirical failure rates, the LR_{cc} tests of Christoffersen (1998) suggest that the GARCH-T and GARCH-GED do indeed fail to provide adequate conditional coverage. In contrast, only the GARCH-SGT model can provide appropriate unconditional and conditional coverage. So, we could conclude that the proposed GARCH-SGT model generates the most accurate VaR forecasts for WTI crude oil returns with the approval of both coverage tests at the 99% confidence level. Such evidence indicates that the SGT distribution with flexible fat-tails, leptokurtosis and skewness parameters plays a key role in VaR estimates at high confidence level.

3.2. Model selection based on utility-based loss functions. For those models which can meet the prerequisite of the correct coverage tests, we employ the one-sided sign tests by further evaluating the remaining candidates through utility-based loss functions which can assess the superiority of one model over another[2, 3]. Table 4 reports the summary results of the standardized sign tests at the 95% confidence level. Panel A of Table 4 lists the average values of the two loss functions obtained by the various VaR models which survived at the first stage of the model selection procedure. These values indicate that GARCH-GED model produces the lowest economic losses, both for a regulator and for a firm. However, a lower average value of RLF (or FLF) does not reflect the superiority of that model among its competitors. Consequently, we use Diebold and Mariano (1995)'s sign

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[1] In this study, the quantiles of the SGT distribution are calculated using Mathematica 5.0. The subroutine is available upon request.

[2] The two back-testing measures discussed in the previous section can not compare different VaR models directly, as a lower test statistic of a model does not indicate the superiority of that model among its competitors (Angelidis et al., 2004).
[3] The superiority test of VaR models will not be performed in case of rejections of either LR_{uc} or LR_{cc} test.

test to check whether these losses are statistically significantly different.

Panel B of Table 4 lists the standardized sign test statistic. The test statistics in terms of RLF imply that none of the models is significantly better than the others. For example, the GARCH-GED model produces the smallest average RLF values, while its forecasting accuracy is not statistically superior to that of alternatives. On the contrary, the sign test

applied to these models with respect to the FLF shows that GARCH-SGT is significantly superior to GARCH-T and GARCH-GED models, while the GARCH-T is significantly better than GARCH-GED model. Consequently, the GARCH-SGT is considered the best model for a firm having this specific loss function at the 95% confidence level, indicating that it is important to take the opportunity cost of capital into account.

Table 4. Superiority tests in terms of utility-based loss functions at the 95% confidence level

Panel A. Average values of loss functions					
	RLF				FLF
GARCH-T	0.1696			GARCH-T	0.2148
GARCH-GED	0.1624			GARCH-GED	0.2084
GARCH-SGT	0.1720			GARCH-SGT	0.2169
Panel B. Sign tests					
Sign test of regulatory loss function				Sign test of firm loss function	
(i, j)	$\hat{S}_{ij}$	$\hat{S}_{ji}$	(i, j)	$\hat{S}_{ij}$	$\hat{S}_{ji}$
(1, 3)	-30.7373*	-29.0297*	(1, 3)	17.3292	-17.3292*
(2, 3)	-30.8638*	-28.8399*	(2, 3)	20.6180	-20.6180*
(1, 2)	-28.8399*	-30.9270*	(1, 2)	-21.7564*	21.7564

Notes: 1. The GARCH-T, GARCH-GED and GARCH-SGT VaR models are numbered by 1, 2 and 3, respectively. 2. The critical value of the $\hat{S}_{ij}$ ($\hat{S}_{ji}$) statistics at the 5% significance level is -1.645. 3. * denotes significance at the 5% level. 4. Rejections of $\hat{S}_{ij}$ ($\hat{S}_{ji}$) would imply that model i (j) is significantly superior to model j (i).

Taking all results of low and high confidence levels together, several conclusions have emerged from this study. First, a risk manager would remain indifferent between these models according to the regulatory loss function when these VaR models have met the prerequisite of correct coverage at lower confidence level, which is consistent with Hung et al. (2008). In contrast, the proposed GARCH-SGT is considered the best model for him/her having the firm loss function. Second, the firm loss function, taking into account the opportunity cost of capital along with the magnitude of the VaR violations, is essential to select the best model, and is consistent with Sarma et al. (2003) and Marcucci (2005). Finally, the proposed GARCH-SGT model provides more accurate VaR forecasts than alternatives for WTI crude oil returns at low and high confidence levels, regarding predictive accuracy.

Conclusion

The recent deregulation of international oil markets has created increased opportunities and incentives for market participants to trade oil via the spot market and derivatives. Owing to the increasingly volatile oil market environment, managing market risk is an important issue facing financial firms

investing in this area. Notably, in the wake of recent several financial disasters, the development of adequate measures of market risk has become a key issue for risk managers and policy makers seeking to develop adequate measures for use by financial institutions, investment banks, and even firms involved in oil-related industries. Currently, the most popular and practical tool in risk management is Value-at-Risk. This study contributes to the literature by assessing market risk in the international crude oil market using VaR analysis. Accordingly, this study proposes a GARCH-SGT approach for dealing with fat-tails, leptokurtosis and skewness using SGT returns innovations and catering for volatility clustering with the GARCH(1,1) model in modeling VaR. Illustration of this technique is presented for daily returns of West Texas Intermediate crude oil spot prices for the period from December 2003 to December 2007.

Using a GARCH-SGT approach, a series of solid evidence has emerged from the present study. First, VaR models that consider fat-tails, leptokurtosis and skewness behaviors are required since crude oil returns are influenced by such behaviors. Second, a risk manager would remain indifferent between

these models according to the regulatory loss function, whereas such a manager would consider the proposed GARCH-SGT the best model owing to having a firm loss function at a low confidence level. Meanwhile, the GARCH-SGT model provides more accurate VaR forecasts than alternatives for WTI crude oil returns at high confidence level, regarding predictive accuracy. Taking all the results together, empirical studies suggest that a utility-based loss function, taking into account the opportunity cost of capital along with the magnitude of the VaR violations, is essential to select the best model. Eventually, the sophisticated SGT distributional assumption significantly benefits VaR forecasting for WTI crude oil returns at low and high confidence levels. This makes the GARCH-SGT model be a robust forecasting approach which is practical to implement and regulate for VaR measurement. In addition, further research can be conducted to present an alternative VaR approach using expected shortfall to alleviate the problem of Value-at-Risk for not being sub-additive.

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