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Evans and Archer – forty years later

Abstract

The question of how large a diversified portfolio needs to be to significantly reduce risk is an important question in the academic field of finance. The first paper devoted to this question was Evans and Archer (1968). Later research has criticized the outcome and methodology of this paper, claiming that alternative weighting schemes or other measures of risk should have been considered. I argue that Evans and Archer's paper is still relevant. I use two weighting schemes, two measures of risk and different time periods to find that 40-50 stocks is all that is needed to achieve diversification in the US stock market.

Keywords: portfolio size, diversification, stock market, Evans and Archer.
JEL Classification: G11.

Introduction

In 1968, Evans and Archer published in the *Journal of Finance* what turned out to be one of the most consistently cited papers in Finance textbooks. Their paper was a logical continuation of the Capital Asset Pricing Model (CAPM), which states that total risk as measured by time series risk (or standard deviation) of a portfolio can be separated into two components: systematic and unsystematic risk. The reduction of time series risk due to increased diversification is entirely due to reduction of unsystematic, or unique risk. Evans and Archer (1968) evaluate how large a portfolio should be to become free of unique risk. They came to the conclusion that surprisingly few stocks are needed to eliminate unique risk and therefore achieve diversification. Since then, a large number of related papers have agreed with the conclusions of Evans and Archer, but none have been cited as much in Finance textbooks. In this paper I revisit the topic of diversification and comment on the continuous impact of Evans and Archer's paper on Finance textbooks.

According to the existing literature, the number of stocks required for a portfolio to be well-diversified has increased from 10-15 back in the 1950s to several hundred at the present time. Various papers have explored the issue of diversification using various measures of risk, different weighting schemes and different time periods. In this paper I use two risk measures and two weighting schemes to evaluate how large a well-diversified portfolio needed to be during the last twenty years of the 20th century.

The two risk measures and two weighting schemes are borrowed from the diversification literature. The first measure of risk is the time series standard deviation, simply known as standard deviation. It measures the dispersion of times series returns around the average return. The second measure is the standard deviation of terminal wealth, it captures the dispersion of portfolios terminal wealth around the market portfolio's terminal wealth. This measure is cross sectional. The two weighting schemes are equally weighted and market weighted portfolios. Later in this paper I formally introduce these measures.

The main conclusion of this paper is that regardless of how risk is measured or how a portfolio is constructed, a randomly-chosen portfolio can be considered well-diversified with about 40 to 50 stocks. This number is larger than suggested by Evans and Archer (1968), but similar to Campbell et al. (2001) finding. This should be regarded as a natural evolution of the risk structure of the American financial market. The claim, however, that a diversified portfolio requires hundreds of stocks, as argued by a large portion of the diversification literature, is not supported in this paper.

In this study I examine the reduction in time series risk as measured by time series standard deviation and cross sectional risk as measured by terminal wealth standard deviation. I also use two weighting schemes and different time periods. I also provide a new approach to estimating the diversification level (asymptote).

The paper is organized as follows. The first section surveys some of the existing literature on diversification. The second section defines the different measures of risk used in this paper. The third section states the hypotheses to be tested. The fourth section describes the data, the simulation procedure, the regression analysis, and highlights the results. Conclusions are contained in the last section.

1. Literature review

Ever since the influential work of Markowitz (1952) and Sharpe (1964), the number of common stocks required to achieve an adequate level of diversification has received considerable attention in the finance literature. Evans and Archer (1968) examine how portfolio size affects portfolio risk. They find that portfolio risk, as measured by time series standard deviation, decreases asymptotically. In other words, risk ceases to decrease beyond 15 stocks.

They conclude that no more than 15 stocks are needed to achieve diversification. Solnik (1974) uses the Evans and Archer's approach to examine the benefits of international diversification. He finds that the US market, because of its larger size and correlation structure, offers more opportunities for diversification than most of the European markets. Further, when the US and European markets are combined, there is an even greater reduction in risk. Fisher and Lorie (1970) address the consistency of stock returns issue by examining the frequency distributions and dispersion of wealth ratios as portfolio size increases. They find that portfolios containing eight stocks have frequency distributions similar to those of portfolios containing larger numbers of stocks and thus conclude that eight stocks is all that is needed to achieve diversification.

Upson, Jessup and Matsumoto (1975) confirm previous findings but note that while time series standard deviation falls quickly, the dispersion of portfolio returns around the market's return does not fall as quickly. In other words, once risk is measured cross sectionally the conclusions change dramatically. Thus, the way risk is measured is of paramount importance. When the traditional measure of risk, times series standard deviation, is used the conclusion is that a small number of stocks is needed to achieve diversification, but when a cross sectional measure of risk is used the conclusion is that many more stocks are needed to achieve diversification.

Elton and Gruber (1977) provide an analytical solution to the approach of Upson, Jessup and Matsumoto. Unlike earlier papers Elton and Gruber (1977) and Upson, Jessup and Matsumoto (1975) use a cross sectional measure of risk but do not measure the exact rate of risk reduction as diversification increases and do not provide specific recommendations for portfolio risk management. This paper fills that gap.

Using mutual funds quarterly returns O'Neil (1997) investigates portfolio diversification. He finds that when time series standard deviation is used as a measure of risk, one fund is all that is needed to achieve diversification. However, when O'Neil uses standard deviation of terminal wealth, a measure of cross sectional risk, he finds that a large number of funds are needed to achieve diversification. In this study I adopt O'Neil's measure as a proxy for cross sectional risk.

Newbould and Poon (1993) suggest that the minimum number of stocks needed to achieve diversification is much higher than suggested in earlier papers when market weighted portfolios are used instead of equally weighted portfolios. Newbould and Poon do not provide an exact estimate of when the asymptote is reached with market weights. In this paper I examine the size of a well diversified portfolio when both equal and market weights are used.

Campbell, Lettau, Malkiel and Xu (2001) trace changes in the volatility of individual stocks, industries and the overall market from 1962 to 1997. Campbell et al. come to the following conclusions: the volatility of individual stocks has risen over time, the correlation among stocks returns has fallen over time, the volatility of the market and most of the industries has not changed, and the number of stocks necessary to achieve diversification has increased. In other words, Campbell et al. acknowledge the existence of an asymptote and implicitly estimate it to be around 50.

More recent papers such as Statman (2004), Benjelloun and Siddiqi (2006) use a different approach to estimate the size of a well diversified portfolio. They contrast the benefits of diversification with its costs, as portfolio size increases risk decreases but management costs increase. Both papers' argument is built around the idea that it is worth increasing portfolio size as long as marginal benefit of increased diversification exceeds its marginal cost. They both reach the conclusion that a diversified portfolio contains hundreds of stocks.

2. Measures of portfolio risk

It is common wisdom that portfolio risk decreases as portfolio size increases. However, previous research uses more than one measure of portfolio risk and more than one weighting scheme. In this section I review two measures of risk, time series standard deviation and terminal wealth standard deviation, along with two weighting schemes.

Time series standard deviation

Both Evans and Archer (1968) and Campbell et al. (2001) use the time series standard deviation of equally-weighted portfolios. It is calculated as follows:

$$TSSD_N^i = \sqrt{\sum_{s=1}^{S} \frac{(R_s^i - \overline{R_N^i})^2}{S-1}},$$

where $TSSD_N^i$ is the time series standard deviation of an N-stock portfolio i, $R_s^i = \sum_{j=1}^{N} \frac{r_{j,s}^i}{N}$ is the return on portfolio i at time s, $r_{j,s}^i$ is the return on stock j, in portfolio i, at time s, and $\overline{R_N^i} = \sum_{s=1}^{S} \frac{R_s^i}{S}$ is the average time series return, over time, of portfolio i.

The average time series standard deviation of K portfolios, each of size N is given by:

$$\overline{TSSD_N} = \sum_{i=1}^{K} \frac{TSSD_N^i}{K}. \quad (1)$$

Newbould and Poon (1993) use market-value weights. The calculations are as above except that R_s^i is calculated using market value weights not equal weights:

$$R_s^i = \sum_{j=1}^{N} w_j r_{j,s}^i .$$

w_j represents market weight of stock *j* in portfolio *i*. The weight is determined in this paper according to the market value of each stock in the period preceding the formation of the portfolio. For example, if the period considered is 1991-1995, then the weight of each stock is determined directly from its market value in December 1990. The exact weight of a stock equals its market value divided by the portfolio's market value.

Standard deviation of terminal wealth

O'Neil (1997) uses this measure to investigate the size of a well diversified portfolio of mutual funds. The advantage of this measure is that it accounts for the variability across portfolios for a given investment horizon.

For equally-weighted portfolios, the terminal wealth of a portfolio of size *N* is the sum of the terminal wealth of all the stocks in the portfolio:

$$TW_N^i = \frac{1}{N} \sum_{j=1}^{N} TW_j^i ,$$

where, $TW_j^i = \prod_{s=1}^{S} (1 + r_{j,s}^i)$.

Average terminal wealth over K portfolios, each of size *N*, is given by:

$$\overline{TW_N} = \sum_{i=1}^{K} \frac{TW_N^i}{K} .$$

Terminal wealth standard deviation over *K* portfolios, each of size *N*, is given by:

$$TWSD_N = \sqrt{\sum_{i=1}^{K} \frac{(TW_N^i - \overline{TW_N})^2}{K-1}}. \quad (2)$$

For market-weighted portfolios, the calculations are similar except for TW_N^i that is calculated using market value weights not equal weights:

$$TW_N^i = \sum_{j=1}^{N} w_j TW_j^i .$$

Where w_j represents market weight of stock *j* in portfolio *i*.

Portfolios with longer investment horizons exhibit more variability in their terminal wealth and therefore may require more assets to achieve diversification. This is true because as the investment period increases the terminal wealth of each portfolio increases. This statement does not hold for average returns and thus time series standard deviation is independent of time.

3. Hypotheses formulation

In this section I use the notation and formulas defined in the previous section. The first hypothesis is borrowed from Evans and Archer (1968).

Hypothesis 1: Using equal weights, time series standard deviation decreases to an asymptote as portfolio size increases.

The second hypothesis is the same as the first one with the exception that I use market value weights instead of equal weights.

Hypothesis 2: Using market weights, time series standard deviation decreases to an asymptote as portfolio size increases.

Using market weights can be expected to change the results dramatically. Some firms have a market value substantially higher than others. Consequently, the addition of one such stock to a portfolio may have a substantial effect. For example, a large portfolio with a low standard deviation can become riskier once a risky security with high market value is added. Similarly, a large and risky portfolio can become less risky once a security with low standard deviation and high market value is added. Such an effect is not likely with equally weighted portfolios. Thus, as predicted by Newbould and Poon (1993), it may take more stocks to achieve diversification with a market weighting scheme. An example can illustrate this idea. Let us assume that the following companies have the following market values at a given point of time:

Table 1. Market value of 17 American companies in November 2001

Company name	Market value in millions, $	Ratio of GENERAL ELECTRIC CO Market value to company's market value
GENERAL ELECTRIC CO	382,204.16	1
DISNEY (WALT) COMPANY	42,849.87	9
GILLETTE CO	34,509.29	11
SYSCO CORP	16,389.75	23
WORTHINGTON INDUSTRIES	1,263.79	302

Table 1 (cont.). Market value of 17 American companies in November 2001

Company name	Market value in millions, $	Ratio of GENERAL ELECTRIC CO Market value to company's market value
RYDER SYSTEM INC	1,243.67	307
ALLEGHENY TECHNOLOGIES INC	1,239.23	308
GREAT LAKES CHEMICAL CORP	1,225.57	312
THOMAS & BETTS CORP	1,186.80	322
HERCULES INC	1,095.87	349
COOPER TIRE & RUBBER	1,075.55	355
BIG LOTS INC	1,070.02	357
AMERICAN GREETINGS -CL A	818.99	467
LOUISIANA-PACIFIC CORP	802.23	476
MCDERMOTT INTL INC	695.17	550
NATIONAL SERVICE INDS INC	655.73	583
US AIRWAYS GROUP INC	510.26	749

The largest company is GENERAL ELECTRIC CO. It has a market value 9 times higher than DISNEY (WALT) COMPANY, 302 times higher than WORTHINGTON INDUSTRIES and 749 times higher than US AIRWAYS GROUP INC. Let's assume an investor possesses a market value weighted portfolio containing the companies with the lowest 13 market values from the above table. The market value of GENERAL ELECTRIC CO is still 30 times the market value of the portfolio. If GENERAL ELECTRIC CO is added to this portfolio in proportion to its market value it is likely to dominate it. In that case the return and risk structure of this portfolio will be dominated by the General Electric Company and other stocks will become less relevant in regard to portfolio volatility. If GENERAL ELECTRIC CO is risky, the portfolio will become risky, if, on the other hand, GENERAL ELECTRIC CO has low volatility the portfolio will have low volatility. To eliminate such a dominating effect and thus stabilize risk, a portfolio must contain a large number of stocks. This leads to the third hypothesis:

Hypothesis 3: The time series standard deviation asymptote is reached with fewer stocks if equal weights are used than if market weights are used.

Elton and Gruber (1977) and Newbould and Poon (1993) argue that buying unequal amounts of each asset can lead to additional reduction of risk for any size of portfolio. One, therefore, might expect to achieve a lower level of risk if the market weighting scheme is adopted. However, none of the above papers explain why the average risk is higher for equal portfolio weights compared to market value weights. It seems to make sense, though, because small stocks are on average riskier than large stocks. Between 1920 and 2000, the annual standard deviation of the S&P Composite was 19.50 percent. For small stocks this figure was 37.53 percent. In an equally weighted portfolio the small stocks carry relatively higher weights than in a market weighted portfolio.

This leads to the fourth hypothesis:

Hypothesis 4: Using time series standard deviation, the asymptote is smaller for market weights than for equal weights.

Although portfolios of the same size are expected to yield the market return on average, some of them may substantially deviate from it. For diversification to be achieved, both *TSSD* and *TWSD* (defined earlier) must converge to an asymptote. O'Neil (1997) uses *TWSD* (terminal wealth standard deviation) to answer the question of how many funds constitute a diversified portfolio. *TWSD*, as argued by O'Neil, makes sense for investors who plan their investments for a pre-specified period of time because it is a risk measure that increases with the length of the investment horizon. *TSSD*, he adds, is not important for such investors because it is a risk measure that does not increase as the length of the investment horizon increases. What really matters for them is the variability of terminal wealth at the end of the period. As discussed in the previous section, his approach has many advantages. First, *TW* is obtained using times series returns and therefore accounts for the variations of returns across time. Second, *TW* measures the true dollar outcome of a portfolio. Third, *TWSD* accounts for the possibility that terminal wealth of a portfolio can deviate from the terminal wealth of the market portfolio. Finally, *TWSD* accounts for the investment horizon length. Samuelson (1990) shows that although the dispersion of returns converges toward the expected value with the passage of time, the dispersion of terminal wealth diverges away from its increasing expected value. This is why, an investor who plans to diversify for a long period may need more assets than an investor who plans to diversify for a short period. Hypotheses 5 through 8 use *TWSD* instead of *TSSD*.

Hypothesis 5: Using equal weights, TWSD decreases to an asymptote as portfolio size increases.

Hypothesis 6: Using market weights, TWSD decreases to an asymptote as portfolio size increases.

Hypothesis 7: Using TWSD, the asymptote is reached faster with equal weights than with market weights.

Hypothesis 8: Using TWSD, the asymptote is smaller for market weights than for equal weights.

4. Data and empirical results

The sample in this study consists of all the firms listed in the CRSP tape for the period ranging from 1980 to 2000. The data used consists of monthly returns and market values. Market values are used to determine market weights and are calculated by dividing the market value of the underlying stock by the market value of the portfolio.

The simulations are performed with replacement for every portfolio size. Ten thousand (K = 10,000) portfolios for every level of N are generated. N takes the values of 1, 10, 20, 30 and so on until 100. For every portfolio size, *TSSD* and *TWSD* are measured following equations (1) and (2). All hypotheses are evaluated using monthly returns. For every combination of time period, weighting scheme, and measure of portfolio risk the following regression is evaluated:

$$Y = A\frac{1}{N^2} + B,$$

where N is portfolio size as defined earlier and Y is one of the measures of risk described earlier (*TSSD* or *TWSD*). A is the slope, and B is the intercept. When portfolio size grows large that is when N becomes large Y converges toward B. B is therefore an estimate of the asymptote.

I declare a portfolio diversified when its risk is equal or smaller than B. The smallest portfolio with a risk less or equal to B is said to be diversified and the corresponding size is the size of a well diversified portfolio. Beyond B risk reduction is considered negligible. This kind of conclusion can only be validated if and only if the coefficient of determination is sufficiently high, that is if the regression equation fits well the outcome of the simulations. A close attention is therefore given to R-squares.

Tables 2 through 4 provide the results of the simulations and regressions.

Table 2. Average time series standard deviations of monthly portfolio returns

Portfolio size	Equally weighted			Market weighted		
	1981 to 1990	1991 to 2000	1981 to 2000	1981 to 1990	1991 to 2000	1981 to 2000
1	0.1144	0.1285	0.1056	0.1149	0.1297	0.1049
10	0.0609	0.0581	0.0529	0.0684	0.0617	0.0629
20	0.0554	0.0493	0.0476	0.0621	0.0534	0.0572
30	0.0535	0.0458	0.0456	0.0594	0.0498	0.0548
40	0.0526*	0.0440*	0.0446*	0.0580*	0.0473	0.0534*
50	0.0520	0.0428	0.0440	0.0569	0.0457*	0.0524
60	0.0516	0.0421	0.0436	0.0563	0.0442	0.0516
70	0.0513	0.0415	0.0433	0.0556	0.0432	0.0512
80	0.0510	0.0410	0.0430	0.0554	0.0423	0.0509
90	0.0509	0.0407	0.0429	0.0547	0.0416	0.0505
100	0.0507	0.0404	0.0428	0.0546	0.0410	0.0503
Goodness of fit (R^2)	97.78%	96.30%	97.69%	95.04%	94.69%	94.90%
Intercept	0.0529	0.0444	0.0449	0.0580	0.0469	0.0534
Slope	0.0616	0.0842	0.0608	0.0570	0.0830	0.0516

Notes: This table provides the results of the simulations. The average standard deviation is calculated using Formula (1). The last three rows are the outcome of the following regression: $Y = A\frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below B. The corresponding size is the size of a well diversified portfolio. This level is marked by an asterisk (*).

Table 3. Average time series standard deviation of monthly portfolio returns

Portfolio size	Equally weighted				Market weighted			
	1981-1985	1986-1990	1991-1995	1996-2000	1981-1985	1986-1990	1991-1995	1996-2000
1	0.1170	0.1314	0.1295	0.1499	0.1166	0.1323	0.1284	0.1508
10	0.0592	0.0695	0.0569	0.0717	0.0659	0.0714	0.0527	0.0804
20	0.0532	0.0625	0.0474	0.0613	0.0589	0.0648	0.0447	0.0692
30	0.0510	0.0603	0.0434	0.0572	0.0562	0.0619	0.0413	0.0642
40	0.0500*	0.0590*	0.0414*	0.0551*	0.0543*	0.0600*	0.0392	0.0609
50	0.0492	0.0582	0.0401	0.0539	0.0533	0.0588	0.0374*	0.0588*

Table 3 (cont.). Average time series standard deviation of monthly portfolio returns

Portfolio size	Equally weighted				Market weighted			
	1981-1985	1986-1990	1991-1995	1996-2000	1981-1985	1986-1990	1991-1995	1996-2000
60	0.0487	0.0577	0.0392	0.0529	0.0522	0.0580	0.0364	0.0571
70	0.0485	0.0573	0.0385	0.0524	0.0520	0.0573	0.0353	0.0557
80	0.0481	0.0570	0.0380	0.0519	0.0513	0.0568	0.0347	0.0547
90	0.0479	0.0567	0.0376	0.0516	0.0512	0.0564	0.0341	0.0537
100	0.0477	0.0566	0.0372	0.0511	0.0505	0.0561	0.0336	0.0528
Goodness of fit (R^2)	97.58%	97.37%	95.78%	96.03%	94.93%	96.15%	96.13%	92.24%
Intercept	0.0502	0.0594	0.0418	0.0557	0.0545	0.0600	0.0388	0.0606
Slope	0.0669	0.0721	0.0878	0.0943	0.0622	0.0724	0.0898	0.0904

Notes: This table provides the results of the simulations. The average standard deviation is calculated using Formula (1). The last three rows are the outcome of the following regression: $Y = A\frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below *B*. The corresponding size is the size of a well diversified portfolio. This level is marked by an asterisk (*).

Table 4. Standard deviation of terminal wealth for monthly returns

Portfolio size	Equally weighted			Market weighted		
	1981 to 1990	1991 to 2000	1981 to 2000	1981 to 1990	1991 to 2000	1981 to 2000
1	4.6976	15.8043	34.4385	4.9945	14.2489	33.3349
10	1.5285	4.6504	11.1564	2.0026	7.0612	13.6860
20	1.0769	3.3819	7.9023	1.5190	3.8534	10.5191
30	0.8775	2.7253	6.2903	1.2609	3.0315	8.7594
40	0.7568	2.4281	5.4014*	1.1155	2.3374*	8.0074
50	0.6696*	2.1755*	4.8184	0.9921*	2.1526	7.3416*
60	0.6113	1.9336	4.3977	0.9078	1.9154	6.6531
70	0.5675	1.7816	3.9948	0.8532	1.8049	6.2764
80	0.5303	1.6602	3.7732	0.7898	1.6597	5.9835
90	0.5046	1.5321	3.5553	0.7404	1.5542	5.6337
100	0.4738	1.4675	3.3704	0.7195	1.4815	5.4388
Goodness of fit (R^2)	93.98%	95.21%	93.87%	90.77%	82.99%	91.23%
Intercept	0.7527	2.3503	5.4150	1.0830	2.6627	7.7839
Slope	3.9529	13.4780	29.0835	3.9213	11.6311	25.6137

Notes: This table provides the results of the simulations. The standard deviation of terminal wealth is calculated using Formula (2). The last three rows are the outcome of the following regression: $Y = A\frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below *B*. The corresponding size is the size of a well diversified portfolio. This level is marked by an asterisk (*).

First, and as expected, risk decreases as portfolio size increases for all scenarios. Second, all R-squared except one are higher than 90 percent which means that the regression equation above is a good fit for all outcomes.

Hypotheses 1, 2, 5, and 6 are confirmed. In all situations in the three tables an asymptote is clearly reached. There is also limited support for hypotheses 3 and 7 as an asymptote is always reached at the same time (the majority of times) or faster when equal weights are used compared to market weights. Hypotheses 4 and 8 are not supported. Quite the opposite, the asymptote seems to be higher for market weights, which seems to imply that stocks with the higher market values seem to be more volatile than stocks with lower weights.

The striking result of course is that an asymptote is reached somewhere in the proximity of 40 to 50 stocks regardless of the risk measure, time period, or weighting scheme. This is an unexpected result as many well established papers, as mentioned earlier, predicted otherwise.

Conclusion

I conclude that forty to fifty stocks is all that is needed to achieve diversification. This contradicts several well established papers claiming that the number of stocks required to achieve diversification is in the hundreds. These papers argue that the method used by Evans and Archer (1968) looks at the time series variability of returns but ignores the cross sectional variability and the possibility of alternative weighting schemes. I resolve this matter by addressing all these issues simultaneously.

Evans and Archer (1968) is still a relevant paper and still deserve the attention of Finance textbooks readers as it provides a sound methodology to answer the important question of how large is a diversified portfolio.

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