Robert M. Hull (USA)

A capital structure model with growth

Abstract

Perpetuity gain to leverage (G_L) research originates in Modigliani and Miller (1963) and Miller (1977) who analyze the change in firm value from issuing debt to retire unlevered equity. Hull (2007) extends this research by developing the capital structure model (CSM) that shows how the costs of borrowing affect G_L. While this prior research is important in offering managers alternative ways to compute changes in values, it assumes no growth. This leads to our research question: "*How will the incorporation of growth (through the plowback-payout decision) affect G_L and thus influence the managerial decision concerning how much leverage is needed to maximize firm value?*". In answering this question, we use the Hull (2007) non-growth CSM framework. To incorporate growth within this framework that uses costs of borrowing, we must develop growth rates for both unlevered and levered equity. The unlevered equity growth rate (g_U) is the constant growth rate in cash paid to unlevered equity, while the levered equity growth rate (g_L) is that rate paid to levered equity. These rates are different because leverage not only alters the cash flow distribution but can also alter its amount. The break-through concept of g_L is shown to be a function of both the plowback-payout and debt-equity choices, thus establishing their interdependency and enabling us to derive growth-adjusted G_L equations. These equations not only show that managers of growth firms face different debt-equity choices than managers of non-growth firms, but also demonstrate how the plowback-payout and debt-equity choices together maximize firm value.

Keywords: capital structure, gain to leverage, leverage, growth rates, plowback-payout.
JEL Classification: G32.

Introduction

Prior perpetuity gain to leverage (G_L) research has failed to adequately deal with the influence of growth on a firm's capital structure choice. In particular, it has failed to analyze the relation between a firm's plowback-payout choice and its debt-equity choice. Building on the recent capital structure model (CSM) of Hull (2007)[1], this paper attempts this analysis by answering the research question: "*How will the incorporation of growth (through the plowback-payout decision) affect G_L and thus influence the managerial decision concerning how much leverage is needed to maximize firm value?*". Topics integrated into our analysis include: (1) the *minimum unlevered equity growth rate* that tells us when growth is profitable; and (2) the break-through concept of the *equilibrium levered equity growth rate*. The latter concept reveals the simultaneous influence of the plowback-payout and debt-equity choices on G_L and thus on the maximum firm value.

Besides the incorporation of growth, the need for this paper's analysis can also be seen from the contradiction found between theory and practice. Trade-off theorists (Baxter, 1967; Kraus and Litzenberger, 1973; DeAngelo and Masulis, 1980; Whited and Hennessy, 2005; Leary and Roberts, 2005; Hackbarth, Hennessy, and Leland, 2007; Korteweg, 2010) argue there is an optimal debt-equity mix that maximizes firm value. However, survey research (Pinegar and Wilbrecht, 1989; Hittle, Haddad, and Gitman, 1992) has found that practicing managers state they are more likely to follow a hierarchical approach consistent with the pecking order theory (POT) of Myers (1984) and Myers and Majluf (1984). The declared POT preference may be related to the fact that competing capital structure models touted by academic journals are too complicated mathematically for managers to understand[2].

Although managers have vocalized support for POT models, subsequent empirical research (Frank and Goyal, 2003; Fama and French, 2005) surprisingly find that just the opposite often occurs in practice. Furthermore, the factors advanced by POT advocates to explain managerial financing behavior have been challenged. For example, Harvey and Graham (2001) stated that asymmetric information does not appear to cause the importance of POT factors such as financial flexibility and equity undervaluation, as it should if POT provides the true model of capital structure choice. Could it be that the driving factor, when determining managerial practice about financing, is the greater costs that occur when internal equity is used instead of external equity? If so, these greater costs would be consistent with Fama and French (2005) who established that financing decisions violate the central predictions of POT because many firms issue shares of equity each year. If the POT does not hold, then the task is to firmly establish trade-off theory through a capital structure model that captures the advantages and disadvan-

[1] The acronym of CSM was coined by Hull (2007) who offered the most formal theoretical development, while other research by Hull (2005, 2008) focused more on practical and pedagogical applications.

[2] See Leland (1998), Whited and Hennessy (2005) and Strebulaev (2007) for just a few of the models that would be hard for a typical manager to fathom.

tages of debt in a way that leads to an optimal capital structure that managers can readily compute. This paper attempts not only to accomplish this task but also to incorporate growth so that tradeoff theory can integrate the debt-equity decision with the plowback-payout decision.

This paper has two major findings. First, this paper develops the break-through concept of the *equilibrium levered equity growth rate* (called *equilibrium g_L*) that ties together the plowback-payout and debt-equity choices. Second, this paper derives growth-adjusted (debt-for-equity and equity-for-debt) G_L equations capable of showing there is only one optimal plowback-payout choice and only one optimal debt-equity choice. Other important findings include showing that internal equity is typically much more expensive than external equity (due to the double corporate taxation associated with cash flows from internal equity) and the formulation of the *minimum unlevered equity growth rate*. The latter leads to the development of a critical point, where the plowback ratio must be greater than the effective corporate tax rate if growth from internal equity is to add value.

The remainder of the paper is organized as follows. Section 1 provides the background and motivation for this paper's research question. Section 2 illustrates why internal equity is more expensive than external equity due to its double corporate taxation. Section 3 describes the critical point for knowing when growth is profitable and introduces the break-through concept of the *levered equity growth rate*. Section 4 explains the interdependence of the plowback-payout and debt-equity decisions. Section 5 derives a CSM equation with growth for a *debt-for-equity* exchange; discusses the enigmatic cash flows associated with G_L; and, derives a CSM equation with growth for an *equity-for-debt* exchange. Section 6 describes a CSM equation in terms of two coefficients that multiply security factors. It also presents illustrations using this paper's CSM G_L equations for a debt-for-equity exchange. The final section provides summary statements and future research possibilities.

1. Background and motivation for the paper's research question

The perpetuity gain to leverage (G_L) research began with Modigliani and Miller (1963), referred to as MM, whose well-known simplifying assumptions yield:

$$G_L = T_C D, \tag{1}$$

where T_C is the exogenous corporate tax rate and D is the value of perpetual riskless debt (D). With no personal taxes and riskless perpetual interest payment (I), the value of D is:

$$D = \frac{I}{r_F}, \tag{2}$$

where r_F is the exogenous cost of capital on riskless debt[1]. Miller (1977) expanded equation (1) by including personal taxes so that

$$G_L = (1 - \alpha)D, \tag{3}$$

where $\alpha = \dfrac{(1 - T_E)(1 - T_C)}{(1 - T_D)}$; T_E and T_D are the respective personal tax rates applicable to income from equity and debt; D now includes personal taxes. With personal taxes, we have

$$D = \frac{(1 - T_D)I}{r_D}, \tag{4}$$

where $r_D > r_F$ if debt is not riskless.

The MM research that narrowly focuses only on a positive tax shield was criticized by those who argued for the influence of bankruptcy and agency effects (Baxter, 1967; Kraus and Litzenberger, 1973; Jensen and Meckling, 1976; Jensen, 1986). The empirical evidence on the valuation effects associated with debt has not yielded a consensus. For example, Miller (1977) and Warner (1977) argued that debt-related effects are trivial having no real impact on firm value, while others (Altman, 1984; Cutler and Summers, 1988; Fischer, Heinkel and Zechner, 1989; Kayhan and Titman, 2007) provided contrary evidence. Graham (2000) estimated that the corporate and personal tax benefits of debt can add as little as 4.3% to a firm's value. Korteweg (2010) found that the net benefit of leverage typically enhances firm value by 5.5%.

Most recently, Hull (2007) extended the MM and Miller non-growth research by developing the capital structure model (CSM) that shows how costs of borrowing affect the leverage decision. Hull (2007) claims that the CSM framework offers G_L equations with more practical potential than prior equations that fail to properly incorporate costs of borrowing and consist of variables that are often extraneously added (thus raising measurability concerns). However, this CSM research makes no mention of the role of the plowback or retained earnings decision and how this growth decision ties in with the leverage decision. The challenge of incorporating growth and its concomitant plowback ratio motivates this perpetuity G_L study and leads the following research question:

---

[1] The use of "equity" refers to common equity. Thus, for simplicity purposes, preferred equity is assumed to be zero.

"How will the incorporation of growth (through the plowback-payout decision) affect G_L and thus influence the managerial decision concerning how much leverage is needed to maximize firm value?"

2. Double corporate taxation makes internal equity more costly

This Section discusses the double corporate taxation on the use of internal equity and how this shapes the plowback ratio decision. It documents the greater cost of internal equity compared to external equity, thus explaining some recent empirical evidence against the POT.

2.1. Double corporate taxation on the use of internal equity. A firm with growth opportunities and sufficient internal cash flows from operating assets can consider reinvesting these cash flows. Let us refer to the before-tax cash flows from operating assets as CF_{BT}. In regard to that amount of CF_{BT} retained for growth, we refer to this amount as RE and define it as the before-tax perpetual cash flow earmarked for reinvestment. In regard to that amount of CF_{BT} earmarked for payment, we refer to this amount as C and define it as the before-tax perpetual cash earmarked for payment to equity. We define the before-tax plowback ratio as:

$$PBR = \frac{RE}{CF_{BT}}$$ and the before-tax payout ratio as

$$POR = \frac{C}{CF_{BT}},$$ where $CF_{BT} = RE + C$ and

$$PBR + POR = 1[1].$$

For what follows, we focus on the before-tax perpetual cash flow that results from growth and refer to this *before-tax* cash flow as R_U. For our first definition of R_U (that represents the change in C), we have:

$$R_U = g_U C, \tag{5}$$

where g_U is the constant growth rate in unlevered equity cash flows.

For our second definition (that represents the return on reinvested earnings), we have:

$$R_U = r_E(1 - T_C)RE,$$

where r_E is the expected rate of return on after-corporate tax retained earnings. This definition reveals that the *before-tax* cash flow of R_U is gener-

ated by investing *after-corporate* tax retained earnings given by $(1 - T_C)RE$. For an unlevered firm, we can view R_U as:

$$R_U = r_U(1 - T_C)RE, \tag{5a}$$

where, in the long-run, r_E is represented as the unlevered equity rate of r_U. For a levered firm, we must represent the change in C in a different fashion by changing the unlevered values to levered values. Doing this gives the perpetuity cash flow for levered equity from growth (R_L) as:

$$R_L = r_L(1 - T_C)RE, \tag{5b}$$

where R_L is the change in levered equity's cash flows and r_L is the levered equity rate.

We should emphasize that equations (5a) and (5b) reveal that the cash flows generated from internal earnings and retained for investment (e.g., RE) are not only taxed at the corporate level, but the cash flows to equity that it creates (R_U or R_L) will also be taxed a second time at the corporate level before it can be paid out. If an equivalent amount of RE was issued by external equity to generate R_U or R_L, then the firm would avoid the corporate taxation on internal RE but would have to pay flotation (or issuance) costs on the issuance of external equity.

2.2. Why internal equity is more costly than external equity. The analysis of taxes and flotation costs that follows is based on legislation and laws governing tax systems and flotation costs found for corporations in the USA. Our analysis could be adjusted to take into account any differences that might exist in other countries. Our corporation analysis could also be extended to other forms of businesses like proprietorships or partnerships that have dissimilar tax situations and different costs when raising funds.

Considering corporate taxes and flotation costs (while momentarily ignoring asymmetric information costs arising from external equity), the cost to equity owners to raise funds for growth can be represented as a negative cash outflow in one of two ways. These two ways are:

cost from using internal equity $= -(T_C)(GFR)$, or
cost from using external equity $= -(F)(GFR)$,

where GFR refers to *gross funds raised* before taxes or flotation costs are considered and F is the flotation costs per dollar raised. The expression representing the *cost from using internal equity* would be much more expensive than the *cost from using external equity*[2]

---

[1] Our use of *PBR* and *POR* is on a before-tax basis so we can better visualize and derive our formulas that account for the double taxation on income when using internal equity. This before-tax usage for *PBR* and *POR* should not be confused with the usage that would be on an after-corporate tax basis. The after-tax usage assumes net income (*NI*) represents the real cash flow (which it rarely does), such that $PBR = RE/NI$ and $POR = C/NI$.

[2] For simplicity, we are ignoring any deductible costs incurred in the external issuance process, which would only serve to bolster our argument that external equity is cheaper than internal equity.

because T_C should be close to five times greater than F. This is based on reported average estimates of 26% for T_C and 5.5% for F[1]. Thus, using external equity should be much cheaper than using internal equity.

We conclude that firms in countries with systems similar to the USA would want to avoid internal equity unless there is some other reason such as that related to barriers in raising external equity or to overvaluation of equity because of asymmetrical information. However, in regards to the latter, the typical fall in stock value attributable to overvaluation for a seasoned equity offering announcement has been shown by empirical studies to be largely (if not totally) explainable by flotation costs[2]. If this is true, then the POT overvaluation reason for issuing internal equity before external equity is lacking empirical verification.

3. Unlevered and levered equity growth rates

This Section sets the stage for deriving this paper's CSM equations by defining new variables affecting G_L when a firm chooses growth. The *minimum equity growth rate* leads to the development of the critical point for the plowback ratio (*PBR*), which is a point that guides managers when making their *PBR* choice. This Section also introduces the *equilibrating levered equity growth rate*, which is a break-through concept tying together the plowback-payout and debt-equity decisions.

3.1. The unlevered equity growth rate. For what follows, we assume that growth comes from internal equity funds. If a firm did choose to use external equity, then what follows could be modified by allowing a periodic infusion of external equity needed to make up any shortage of internal equity[3]. With this adjustment, what follows *remains applicable* even if growth is not assumed to result totally from the use of internal equity.

To begin, let us consider an unlevered growth firm with a long-term retention policy with internal equity funds sufficient to fund all growth. Rearranging the definition for R_U in equation (5), the constant growth rate in cash paid to equity (g_U) can be defined as:

$$g_U = \frac{R_U}{C},\qquad(6)$$

where g_U is the constant growth rate in unlevered cash dividend payments to equity owners (or, more simply put, g_U is the *unlevered equity growth rate*); R_U is the before-tax perpetual cash flow generated by an unlevered firm investing its after-corporate tax retained earnings; C is the before-tax perpetual cash flow earmarked for unlevered equity owners; and, R_U, C and RE all grow at g_U for far-reaching periods if the firm remains unlevered.

3.2. The minimum unlevered equity growth rate. What is the *minimum unlevered equity growth rate* (referred to as the *minimum* g_U) that a non-growth unlevered firm must attain so that its equity value will not fall when it chooses to grow by investing its retained earnings? The answer to this question is shown below.

Consider the after-tax unlevered firm value with no growth that we define as:

$$E_U \text{ (no growth)} = \frac{(1-T_E)(1-T_C)CF_{BT}}{r_U},$$

where E_U is the same as V_U since $D = 0$; T_E is the effective personal tax rate paid by equity; T_C is the effective corporate tax rate; CF_{BT} is the before-tax cash flow earmarked for unlevered equity owners; and r_U is the unlevered equity cost of borrowing. With growth, one minus the plowback ratio, ($1-PBR$), times the numerator of $(1 - T_E)(1 - T_C)(CF_{BT})$ determines the after-tax perpetual cash flow paid to equity. This implies that r_U must also be lowered by at least ($1 - PBR$) if E_U is not to decrease when the firm takes on growth. For this lowered discount rate, we have a denominator of

$$(1 - PBR)r_U = r_U - r_U PBR,$$

where the *minimum* g_U must equal $r_U PBR$ if the denominator of $r_U - r_U PBR$ is equal to the growth-adjusted discount rate of $r_U - g_U$. Thus,

E_U (no growth) $= E_U$ (with growth) or

$$\frac{(1-T_E)(1-T_C)CF_{BT}}{r_U} = \frac{(1-PBR)(1-T_E)(1-T_C)CF_{BT}}{r_U - g_U}$$

---

only when $g_U = r_U PBR$. We call this latter g_U value by the name of the *minimum* g_U[1]. Similarly, for a levered firm where g_U changes to g_L, we have *minimum* $g_L = r_L PBR$. This means that if a growth firm becomes levered, then it can only be profitable if $g_L > g_U$. Such profitability is always the case for reasonable leverage choices that avoid financial ruin.

3.3. The critical point for a plowback ratio. Let us assume all growth comes from internal equity and recall that the *minimum* $g_U = r_U PBR$. Using the latter equation along with (5) and (5a), we can show that the *minimum* g_U implies that:

$$PBR = T_C{}^2,$$

where T_C is the *minimum PBR* needed to insure growth does not decrease value. Thus, if the *minimum* g_U is attained, R_U in equation (5a) equals R_U in equation (5) only when $PBR = T_C$. The critical point of T_C for PBR results from the double corporate taxation when internal equity is used. This point gives the minimum starting value for setting the *PBR* because managers should not undertake growth unless its *PBR* is at least equal to its critical point.

For an unlevered firm growing strictly from internal equity, managers should refrain from growth if the critical point of $PBR = T_C$ is unsustainable. This managerial action holds even if a firm becomes

levered with T_C changing in a favorable manner so as to increase firm value. Whatever the value of T_C, *PBR* cannot be below it. The main point is that T_C is an important reference for managers considering growth through the use of internal equity funds. Relaxing the assumption of using internal equity, we can achieve a critical point of $PBR = F$ by strictly using external equity. This is because external equity avoids the double corporate taxation from using internal equity while taking on the flotation costs of external equity[3].

3.4. The break-through concept of the levered equity growth rate. Equation (6), which defines g_U, must be altered when the firm becomes levered because debt brings other cash flows that affect the growth of equity cash flows. Below we develop the equation for the *levered equity growth rate* (g_L) to account for these other cash flows.

If the unlevered firm with growth becomes levered such as through a debt-for-equity exchange, equity owners not only have their ownership proportion altered but they also lose the cash flow equal to interest payment of I paid to debt owners. Leverage can also give equity owners a positive cash flow if $G_L > 0$ holds but increases the riskiness of their cash flows. Of importance, a debt-for-equity exchange alters both the make-up and the amount of the perpetual before-tax cash flows that, prior to the addition of debt, had been fixed at CF_{BT} but which after the debt-for-equity exchange can be greater than CF_{BT}, albeit some of it is paid as interest (I).

Let us view the value associated with a positive G_L in terms of a positive perpetual before-tax cash flow and call it G where G must be discounted at a rate to make it equal to G_L[4]. Given that I is not taxed at the corporate level, we adjust G, R_L, and C for corporate taxes multiplying by $(1 - T_C)$ and define the *levered equity growth rate* (g_L) as:

$$g_L = \frac{(1 - T_C)R_L}{(1 - T_C)(C + G) - I}, \tag{7}$$

where R_L, as given earlier in equation (5b), is the perpetual cash flow generated by a levered firm plowing back its after-corporate tax retained earnings; I (unlike C or G) is not subject to corporate taxes; and, the amount of debt issued must be rea-

---

[1] We can illustrate this equality by letting $(1 - T_E)(1 - T_C)EBT = \$1.2B$ (B = billions), $r_U = 0.12$, and $PBR = 0.3$ if a firm chooses growth. With no growth (e.g., $PBR = 0$), and we have:

E_U (no growth) = $\dfrac{(1 - T_E)(1 - T_C)CF_{BT}}{r_U}$ = $\dfrac{\$1.2B}{0.12}$ = \$10B. To main-

tain equality between the growth and non-growth situations, we must have *minimum* $g_U = r_U PBR = 0.12(0.3) = 0.0360$. This is seen below by noting that with growth, we have:

E_U (growth) = $\dfrac{(1 - PBR)(1 - T_E)(1 - T_C)CF_{BT}}{r_U - g_U}$ = $\dfrac{(1 - 0.3)\$1.2B}{0.12 - 0.036}$ =

$\dfrac{\$0.84B}{0.0840}$ = \$10B.

[2] The proof is as follows. Inserting $r_U PBR$ into equation (5) for g_U, we get $R_U = r_U PBR(C)$. Equating this expression for R_U with R_U in equation (5a), we have: $r_U PBR(C) = r_U(1 - T_C)RE$. Canceling r_U from both sides of the latter equality, we get $PBR(C) = (1 - T_C)RE$. Noting that:

$PBR = \dfrac{RE}{CF_{BT}} = \dfrac{RE}{RE + C}$ and inserting $\dfrac{RE}{RE + C}$ for *PBR*, our

equality becomes: $\left(\dfrac{RE}{RE + C}\right)C = (1 - T_C)RE$. Dividing both sides by

RE, we get: $\dfrac{C}{C + RE} = (1 - T_C)$. Solving for T_C, we get: $T_C =$

$1 - \dfrac{C}{RE + C}$. Noting that $\dfrac{C}{RE + C} = \dfrac{C}{CF_{BT}} = POR$, we have: $T_C =$

$(1 - POR)$. Noting that $(1 - POR) = PBR$, we have: $PBR = T_C$. Similarly, if we use external equity where equation (5a) becomes $R_U = r_U(1 - F)RE$, then we would get: $PBR = F$.

---

[3] A marginal critical point (where $PBR = T_C - F$) results if we look at the cost of financing on a marginal basis that compares the cost of internal equity with the cost external equity. Based on empirical data given earlier for T_C and F, the marginal critical point for a typical firm using internal equity would be $PBR = T_C - F = 0.26 - 0.055 = 0.205$.

[4] This paper's analysis of growth includes the role of *PBR* and *RE* and thus sets straight and expands the explanation given by Hull (2005) for growth, where G is not included in his denominator when computing g_L. Definitions for computing G will be given later.

sonable. By "reasonable", we mean a debt value that could not be chosen because it would cause a large I and thus set the targeted levered equity growth rate at a large and unsustainable rate. But, most noteworthy, large I values will eventually lead to negative g_L values that coincide with negative G_L values when using equation (12). Thus, *the break-through concept of g_L indicates that a growth firm is limited in its debt-equity choices if it wants to avoid financial ruin.*

3.5. The equilibrating unlevered and levered growth rates. Equations (5) and (5a) for R_U can be used to get what we call an equilibrating unlevered equity growth rate (*equilibrating g_U*), which is the rate that balances the two formulations for R_U. From equation (5), we have $R_U = g_U C$ and from equation (5a), we have $R_U = r_U(1 - T_C)RE$. Equation (5) and (5a) and solving for g_U, we get:

$$equilibrating \ g_U = \frac{r_U(1 - T_C)RE}{C}, \qquad (6a)$$

where equation (6a) gives a g_U value such that equations (5) and (5a) will give the same R_U value.

We have two equations involving R_L that can be used to get what we call an equilibrating levered equity growth rate (*equilibrating g_L*), which is a rate that balances the two R_L formulations. We have:

$$R_L = g_L \left[C + G - \frac{I}{(1 - T_C)} \right], \qquad (6b)$$

where R_L in equation (6b) is derived from equation (7). We have equation (5b), where

$R_L = r_L(1 - T_C)RE$. Equating these equations and solving for our *equilibrating g_L*, we get:

$$equilibrating \ g_L = \frac{r_L(1 - T_C)RE}{C + G - \frac{I}{(1 - T_C)}} \qquad (7a)$$

where equation (7a) gives a g_L value such equations (5b) and (6b) give the same R_L. The *equilibrating g_L* is important as this is the g_L used in this paper's growth-adjusted discount rates.

4. Plowback-payout and debt-equity decisions

In this Section, we discuss how a target levered equity growth rate is determined by the interlinked plowback-payout and target debt-equity decisions. We also comment on how a firm can maintain its target debt-equity ratio with growth considered.

4.1. Impact of plowback and leverage decisions on the levered equity growth rate. Once a growth firm targets a debt-equity ratio that it believes is optimal, then its "target" levered equity growth rate

(g_L^T) can be formulated based on its targeted amount of interest (I^T) and other relevant variables given in equation (7a). For example, substituting I^T for I in equation (7a) gives:

$$g_L^T = \frac{r_L(1 - T_C)RE}{C + G - \frac{I^T}{(1 - T_C)}}, \qquad (7b)$$

where we can see that g_L^T increases (1) as the managerial chosen targeted debt level (and thus I^T) rises relative to $C + G$, and (2) as RE goes up. We can note that whenever RE increases, it simultaneously means that C must fall since CF_{BT} (e.g., $RE + C$) is fixed at the time of a debt-for-equity transaction. Thus, an increase in RE not only increases g_L^T by increasing the numerator but also leads to the amount of C going down in the denominator further increasing g_L^T. Similarly, an increase in I, relative to $(1 - T_C)G$, increases g_L^T by decreasing the denominator. In conclusion, both the plowback-payout decision and the debt-equity decision have a significant impact on g_L^T.

It is important to emphasize that the plowback-payout choice affects g_L^T through RE and C, and the financing choice influences g_L^T through I^T and G. The role of the plowback-payout choice can be more visibly seen in equation (7b) if we note that C and RE are a function of plowback-payout decision. In a like fashion, the amount of I^T is determined by the debt-equity choice. For example, if an unlevered equity firm decides to retire ¼ of its equity value through a debt offering, then:

amount of debt $= D = (¼)E_U$,

where E_U is unlevered equity value at the time of the debt-for-equity exchange. I^T in equation (7b) equals:

$r_D D = r_D(¼)E_U$,

where r_D is the cost of debt. The ¼ visibly indicates the impact of the debt-equity choice on g_L^T through I^T.

4.2. Interdependence of plowback-payout and debt-equity decisions. With no growth, assets are not expanding and the cash dividends per share equals the available cash earnings per share. This means that $PBR = 0$ and $POR = 1$. This is not the case with growth, such as caused by investing internally generated funds, where $PBR > 0$ holds. For an unlevered firm financing with internal equity funds to achieve a specified level of expansion, the plowback-payout decision determines the amounts of RE and C, where these two amounts in turn establish R_U

and g_U. Thus, C, RE, R_U and g_U are determined endogenously (subject to finite operating cash flows) when an unlevered firm makes its plowback-payout decision.

What if managers of an unlevered firm decide it can maximize its value by becoming levered because $G_L > 0$ holds for at least one debt level choice for a *PBR* choice greater than its critical point? If so, we must now consider how the change from g_U to g_L intrinsically leads to maximizing firm value in such a way that the plowback-payout decision would be determined based on consideration of debt-equity choices. In other words, as indicated from equations (7), (7a), and (7b) for the levered equity growth rate, when a firm chooses a plowback-payout choice, it would simultaneously consider this choice in conjunction with a debt-equity choice. Because the plowback-payout decision determines the growth rate, it must on some level determine the optimizing G_L which, as will be shown in equation (12), is a function of both g_U and g_L. If follows that the ultimate determinant of an optimal G_L value (and thus optimal firm value), which implies one and only one growth-adjusted discount rate for levered equity, cannot be separated from its plowback-payout choice. Thus, the discovery of an *equilibrating levered equity growth rate* reveals an important finding on how managers go about maximizing value:

A firm's plowback-payout and debt-equity decisions are interlinked, and even inseparable, and a G_L model dependent on the usage of a levered equity growth rate reveals that firm maximization involves making both decisions jointly[1].

4.3. Maintaining the optimal debt-equity ratio with growth considered. A perpetuity formula approximates the expected long-term life value of a security. Any growth rate used in a perpetuity formula is chosen because it best captures the effective growth for a long horizon (that also includes expectations about greater growth rates for short periods). With this in mind, we discuss the maintenance of a firm's target debt-equity ratio for a long horizon, where the target is assumed to be optimal.

Unlike equityholders who experience growth in residual payments, debtholders' payments are viewed as fixed due to the fixed number of bonds and/or fixed amount of the bank loans. Growth in debt can come externally by increasing the dollar amount of debt. Based on (7b), the target levered equity growth rate,

g_L^T is established by I^T through attainment of the target debt-equity ratio. To maintain this ratio, debt would have to grow at the same tax-adjusted rate as given in equation (7b) for equity. Thus, to maintain a target (and thus optimal) debt-equity ratio over time, debt would need to grow at a rate commensurate to the growth in equity value.

Absent any speculation on the existence of an optimal firm size, the optimal debt-equity ratio could be maintained over time without issuing additional debt. For example, instead of using cash to pay out dividends, the cash could be used to periodically buy back shares and thus move the firm back to its optimal debt-equity ratio. Even if we ignore any personal tax advantage of repurchase over dividends, the cash (used to buyback equity) can be more valuable to equity holders *as a whole* because there would not only be the value of the dividends no longer given to those who have sold equity, but there would also be the value of going back to the optimal debt-equity ratio. Thus, in terms of per share value, shareholders can be as well off retiring equity (as issuing debt) since the desired optimal debt-equity ratio is maintained. However, suppose the firm decides to periodically add debt at a rate that maintains its optimal debt-equity ratio. Even though on a per share basis shareholders should be no better off than when retiring equity, the increased debt would add its own gain to leverage.

5. CSM G_L equations with growth

This Section develops the paper's capital structure model (CSM) with growth. After defining firm value before and after a *debt-for-equity* exchange, we derive a CSM G_L with tax rates, borrowing rates, and growth rates. This Section also discusses G, the enigmatic perpetual cash flow created with leverage and derives a CSM G_L equation with growth for an *equity-for-debt* exchange.

5.1. Derivation of G_L for debt-for-equity exchange with growth. To derive G_L for a debt-for-equity exchange given an unlevered firm with fixed tax rates[2] and constant growth, we begin with the definition that G_L is:

$$G_L = V_L - V_U, \tag{8}$$

where V_L is levered firm value with growth and V_U is unlevered firm value with growth, where growth

---

[1] This conclusion should hold (as described earlier) even if external equity is a surrogate of internal equity. Either way, a firm would still have a g_L that increases with debt (until it can become negative if I^T becomes too large) and thus is influenced by the leverage decision.

[2] Fixed tax rates mean that T_C, T_E, and T_D do not change with the change in the security mix. However, increased debt can jeopardize any corporate shield tax advantages as well as alter investor clienteles and their equity and debt tax rates. It is likely that α increases with more debt and thus gravitates toward the Miller (1977) value of $\alpha = 1$.

means the plowback ratio (PBR) is greater than zero[1]. Noting that unlevered firm value (V_U) when $D = 0$ is the same as unlevered equity value (E_U), we have:

$$V_U \text{ (with growth)} = E_U \text{ (with growth)} =$$
$$= \frac{(1-T_E)(1-T_C)C}{r_{Ug}}, \qquad (9)$$

where C is the perpetual cash flow belonging to unlevered equity with $C = (1-PBR)(CF_{BT})$ or $C = POR(CF_{BT})$; r_{Ug} is the growth-adjusted discount rate on unlevered equity given as $r_{Ug} = r_U - g_U$; r_U is the unlevered cost of equity; and, g_U is the equilibrating unlevered equity growth rate given in equation (6a).

To get levered firm value (V_L), we first define levered equity (E_L). The definition for E_L assumes that any change in firm value from the debt-for-equity exchange is captured by the growth-adjusted levered equity rate (r_{Lg}) and not by a change in cash flows. Later, we will consider an equation for E_L that includes the cash flow of G (that can result when $G_L > 0$) and comment on how this increases the discount rate. But for now, we define levered equity as:

$$E_L = \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}}, \qquad (10)$$

where r_{Lg} is the growth-adjusted discount rate on levered equity given as $r_{Lg} = r_L - g_L$; r_L is the levered cost of equity; and, g_L is the equilibrating levered equity growth rate given previously in equation (7a). Because there is no growth-adjusted rate on interest paid to debt owners[2], debt has the same definition given previously in equation (4). Given E_L and D, we have:

$$V_L = E_L + D = \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} + \frac{(1-T_D)I}{r_D}, \quad (11)$$

where for now any increase in value beyond V_U is associated with the mix of securities lowering the overall cost of borrowing making perpetual cash flows more valuable for security owners.

Given the above definitions, Appendix A derives G_L for an unlevered growth firm that undergoes a debt-for-equity exchange. We have:

$$G_L = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right]D - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]E_U, \qquad (12)$$

where $r_{Lg} = r_L - g_L$ and $r_{Ug} = r_U - g_U$. With no growth, equation (12) reduces to the Hull (2007) CSM equation of:

$$G_L = \left[1 - \frac{\alpha r_D}{r_L}\right]D - \left[1 - \frac{r_U}{r_L}\right]E_U, \qquad (13)$$

where E_U (no growth) $= \dfrac{(1-T_E)(1-T_C)C}{r_U}$ with

$C = CF_{BT}$ since $PBR = 0$ (and $RE = 0$).

Besides no growth where equation (12) becomes (13), let us further assume discount rates are equal (i.e. $r_D = r_U = r_L$). For this situation, equation (13) reduces to the Miller equation given in equation (3), where $G_L = [1-\alpha]D$. Further assuming that personal taxes are zero and debt is riskless, we get the MM equation given in equation (1), where $G_L = T_C D$[3]. Equation (1) can be further reduced to the Modigliani and Miller (1958) no tax model with $T_C = 0$, which causes $G_L = 0$ to hold. By including growth-adjusted discount rates, we conclude that equation (12) is more complete than equation (13), which is more complete than equations (1) and (3) by capturing the effects from discount rates changing.

5.2. Treating the value of G_L as a perpetual cash flow belonging to levered equity. We now analyze the perpetual cash flow of G resulting from G_L, where G was used in the denominators of equations (7), (7a), and (7b)[4]. The end product of G reflects the outcome that cash flows to equity have been changed by the debt-for-equity transaction. Any net positive change (e.g., $G_L > 0$) can result from a number of considerations including tax and agency effects. Because it is difficult to know the exact make-up of G's value, we term it as an "enigmatic" variable. In other words, the perpetual cash flow of G can be represented by any number of perpetual cash flows combined with any number of possible discount rates as long as the final discounted value of

---

[1] Except for modifications on how terms are expressed and arranged, this paper's derivation is like that of Hull (2005) in that any change in value is not overtly expressed in terms of G. Algebraically, it is also like the non-growth derivation found in Hull (2007). See Hull (2007, 2008) for analyses on how changes in values for the two CSM components are consistent with mainline capital structure theories and thus integrate these models within its domain.

[2] Growth in debt is not generated internally but, as discussed at the end of the previous Section, equity can be retired or debt can be added periodically to maintain the optimal leverage ratio (subject to frictions such as transaction costs that allow temporary straying from the optimal).

[3] See page 14 in Hull (2007) for a proof of the latter two statements.

[4] Hull (2005) did not include G in his denominator when computing g_L. Based on Graham (2000) and Korteweg (2010), leverage increases firm value from 4.3% to 5.5% suggesting that G could be small if most of the positive value from G_L is captured by a relatively lower overall cost of borrowing associated with the addition of debt. Tests we have conducted indicate that leaving out G will lead to an overvaluation of the maximum G_L.

G yields the actual G_L. Below we attempt to describe G and offer two definitions for it where these definitions make it possible to compute G, g_L, and G_L.

Let us assume that the perpetuity cash flow of G falls within the domain of the equity owners and thus is discounted at the same rate as C. If so, G_L given by equation (12) can be expressed as:

$$G_L = \frac{(1 - T_E)(1 - T_C)G}{r_{Lg}}, \quad (14)$$

where G is a perpetuity with a present value that equals G_L given by equation (12) and the value of r_{Lg} in equation (14) is the same value as that for r_{Lg} in equation (12). Solving for G in equation (14), we get:

$$G = \frac{r_{Lg}G_L}{(1 - T_E)(1 - T_C)}, \quad (15)$$

where G is the before-tax perpetual cash flow created from the debt-for-equity exchange.

Besides equation (15), we can create another expression for G. Let us begin by viewing the after-tax cash flows paid to levered equity as $(1 - T_E)(1 - T_C)(C - I + G)$ when $G > 0$. For the perpetuity value of this cash flow to equal the perpetuity value of $(1 - T_E)(1 - T_C)(C - I)$ in equation (12), the discount rate for $(1 - T_E)(1 - T_C)(C - I + G)$ would have to be greater than r_{Lg} given in equation (12). Referring to this greater growth-adjusted discount rate as r_{Lg}', we can solve for it by dividing $(1 - T_E)(1 - T_C)(C - I + G)$ by E_L where E_L is computed from $E_L = E_U + G_L - D$ after G_L is calculated using equation (12). Given r_{Lg}', we define a second expression for levered equity value as:

$$E_L' = \frac{(1 - T_E)(1 - T_C)(C - I + G)}{r_{Lg}'}, \quad (10a)$$

where r_{Lg}' $> r_{Lg}$ if $G > 0$ and E_L' given in equation (10a) equals E_L given in (10). Rearranging the expression for E_L' in (10a), we get our second expression for G of:

$$G = \frac{r_{Lg}'E_L'}{(1 - T_E)(1 - T_C)} - C + I, \quad (15a)$$

where G in equation (15a) equals that found in equation (15). We can use this G value to compute our *equilibrating* g_L given in equation (7a), where g_L in turn is used to compute G_L in equation (12).

There are interdependencies among G, g_L, and G_L as these three variables are mutually dependent. This means that a reiterative process is needed because

(1) we must know G before we can compute the *equilibrating* g_L in equation (7a); (2) we must know *equilibrating* g_L before we can compute G_L in equation (12); and, (3) we must know G_L before we can compute G in equation (15). This interdependent situation creates a circular reference (when using Excel) because a formula refers back to its own cell, either directly or indirectly. To overcome this problem, one must enable iterative or repeated recalculations within Excel so that the precise value for a variable in question can be computed. In the process, E_L in equation (10) equals E_L' in equation (10a) with r_{Lg}' $> r_{Lg}$ if $G > 0$; the G equations of (15) and (15a) give the same G value, and the *equilibrating* g_L in equation (7a) can be computed based on this G value to get our end product of G_L; and, an optimal G_L value can be determined from all combinations of plowback and debt choices tested so that we know the optimal plowback-payout ratio and the optimal debt-equity ratio.

5.3. CSM G_L equation with growth for an equity-for-debt exchange. We now derive G_L for an equity-for-debt exchange when growth is considered. Suppose a firm is overlevered and can increase its value through an equity-for-debt exchange where it retires all of its debt and becomes unlevered[1]. For this equity-for-debt scenario, we refer to the gain to leverage as $G_L^{Equity-for-Debt}$ and define it as:

$$G_L^{Equity-for-Debt} = V_U - V_L, \quad (16)$$

where $V_U > V_L$ holds for an overlevered firm seeking to maximize its value by retiring its debt. Using equation (16) and definitions given previously for D, E_U, E_L and V_L, Appendix B shows:

$$G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]E_U - \left[1 - \frac{\alpha r_D}{r_{Lg}}\right]D, \quad (17)$$

where the components found in equation (17) would be reversed from those in equation (12) taking on different signs.

For a firm that becomes unlevered through an equity-for-debt exchange, equation (17) would render a positive value from reducing financial distress caused by too much debt. When a positive value occurs, the 1st component of equation (17) would represent a positive change in value that dominates any negative effect from the 2nd component of equation (17). The 1st component can be shown to directly depend on the percentage change in the

---

[1] To avoid creating new variables, we assume the firm becomes unlevered. However, it is not necessary to assume the firm becomes unlevered as an equation similar to equation (17) could result even if only part of the debt was retired. We leave it to future research to analyze in detail leverage increases and decreases for a levered firm.

growth-adjusted rate of return on equity[1]. Thus, the rate of change in the growth-adjusted discount rate on equity is a key factor in determining the impact on firm value for a leverage change. The negative effect from the 2nd component can come from lowering both a positive tax shield effect and a positive agency shield effect. In conclusion, equation (17) is consistent with the logic of trade-off theory that suggests that a firm will undergo a leverage decrease if the positive effect from reducing the financial distress costs dominates the negative effect from reducing the positive tax-agency effect.

6. Coefficients in a CSM G_L equation and illustrations

This Section describes this paper's CSM G_L equation with growth in terms of two coefficients that multiply security factors. It also offers illustrations of this equation.

6.1. Coefficients in CSM equations. CSM equations for G_L can be represented by positive and negative coefficients that multiply security factors. For example, we can represent equation (12) as

$$G_L = n_1 D - n_2 E_U, \qquad (18)$$

where $n_1 = \left[1 - \dfrac{\alpha r_D}{r_{Lg}}\right]$ and $n_2 = \left[1 - \dfrac{r_{Ug}}{r_{Lg}}\right]$. Tests of equation (18) indicate that $n_1 > n_2$ will hold until a large leverage ratio or overlevered situation is reached. This is because the initial large gap between $n_1 - n_2$ narrows as debt increases due to the fact that n_1 decreases with debt while n_2 increases with debt. We also found that values for n_1 and n_2 fall as a firm's plowback ratio increases with the gap of $n_1 - n_2$ narrowing as the firm nears its optimal *PBR* that maximized G_L. The changes in n_1 and n_2 as both debt-equity and plowback-payout choices change reflect the dependence of G_L values on these two choices.

Together, n_1 and n_2 emphasize that G_L is a function of tax rates, growth rates, and discount rates for debt and equity. Thus, G_L is determined by any factors that affect these rates. Going beyond the general leverage and plowback-payout categories, such factors can include current tax legislation, investor clientele tax rates, nondebt tax shields, tax credits, employee stock options, industry factors, expected growth rate in GNP, need for financial slack, government security rate, expected market return, outstanding debt, opportunity costs (such as a firm's future ability to borrow based on its debt-equity choice), financial risk, business risk, nondiversified risk, expectations about investment shocks, free cash flows, managerial autonomy, inside ownership level, and so forth.

We can describe two divergent scenarios for the two security size factors of D and E_U. For a younger and growing firm, we expect E_U values to be greater than D values, while for an older and mature firm, we expect D and E_U values to be more similar. The costs of borrowing (like tax and levered equity growth rates) are a function of the total amount of debt and not just the last issue of debt. For an unlevered firm to issue an amount of D that is small relative to its E_U, equation (18) suggests that n_1 must be sizeable compared to n_2 for G_L to be positive. If an unlevered firm issued a large amount of debt (such that D approaches E_U), then n_1 would no longer have to be sizeable compared to n_2 for G_L to be positive. Our tests indicate that increased debt would make n_2 approach n_1 but would not likely surpass n_1 except for extreme debt levels.

6.2. Illustrations using a CSM G_L equation with growth. Appendix C and Appendix D give illustrations using equations (12) and (18) when growth uses internal equity. Details (including all computed values for variables) are in an Excel spreadsheet available on request. The illustrations attempt to use reasonable values for tax rates (e.g., we use $T_C = 0.26$) and borrowing costs based on risk-free rate of 4% and market return of 10%. Our initial α value of about 0.8 is allowed to gravitate towards 1.0 as debt increases, which is a value for α believed by Miller (1977). The costs of borrowing are influenced by Hull (2005, 2007) and Pratt and Grabowski (2008).

The three gray-shaded cells above the graph in Appendix C correspond to the maximum G_L for the three *PBR* choices. Two of the gray-shaded cells correspond to a debt choice of 0.30. This Appendix illustrates that a firm' debt choice for a non-growth firm can differ from that for a growth firm. Each debt choice reflects the proportion of E_U being retired. Appendix C also reveals the great risk that occurs when a high debt choice combines with a *PBR* greater than the critical point. This is illustrated for when we set *PBR* at 0.30, which is above the critical point of 0.26. For this *PBR*, the optimal G_L is given for a debt choice of 0.6. But, if we jump by only 0.1 to 0.7, then G_L becomes negative (as seen by the negative G_L/E_U). Thus, a high *PBR* combined with high debt leads to ruin if unruly and unexpected events cause movement past a firm's *ODE*.

---

[1] To illustrate, we have:

$$\left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U = \left[\frac{r_{Lg}}{r_{Lg}} - \frac{r_{Ug}}{r_{Lg}}\right] E_U = \left[\frac{r_{Lg} - r_{Ug}}{r_{Lg}}\right] E_U = \left[\frac{\Delta r_{Lg}}{r_{Lg}}\right] E_U.$$ Thus,

we see that the 1st component of equation (12) depends directly on the percentage change in equity's growth-adjusted rate of return.

The two gray-shaded columns above the graph in Appendix D give variable values for (1) the critical point of *PBR* = 0.26; and (2) the maximum increase in firm value (as captured by G_L as a fraction of E_U). This appendix reveals the following. First, once we get past a *PBR* of 0.42, positive G_L values no longer occur for any debt choices and so we must choose zero debt when *PBR* > 0.42. Second, the debt choice remains constant at 0.30 for low *PBR* values but once we get near the critical point of *PBR* = 0.26, the debt choice jumps from 0.30 to 0.50 and peaks at 0.60 before falling back to zero. Thus, given our estimates of key variables, low debt choices occur for either lower *PBRs* or higher *PBRs*. *Third,* greater increases in G_L (and thus firm value) occur for lower values of the coefficient differential of $n_1 - n_2$. Fourth, the optimal debt-to-equity (*ODE*) choice changes with the plowback-payout decision. This is consistent with our analysis of the breakthrough concept of g_L where we concluded that a firm's plowback-payout and debt-equity decisions are interlinked revealing that firm maximization involves making both decisions jointly. Fifth, greater increases in G_L and firm value occur for higher debt choices (that are not too extreme).

Summary and future research

This paper has two major findings. First, we develop the breakthrough concept of the leverage growth rate that ties together the interdependency of the plowback-payout and debt-equity choices. Second, with this concept in place, we are able to broaden the perpetuity gain to leverage (G_L) research by incorporating growth within the capital structure model (CSM) framework formalized by Hull (2007) and show there is only one optimal plowback-payout choice and only one optimal debt-equity choice. In the process, we show that managers of growth firms (compared to non-growth firms) can make different optimal debt-to-equity choices.

In addition to the two major findings, this paper contrasts the cost of using internal versus external equity when expanding a firm's assets. Contrary to pecking order theory (POT), we argue that internal equity is typically more expensive than external equity (at least for countries like the USA). This argument is consistent with the recent empirical evidence against the central predictions of POT.

From the development of the *minimum unlevered equity growth rate*, we show that the plowback ratio must exceed a critical point if a firm is to profitably choose growth. For a firm seeking growth from the use of internal equity, the critical point is a firm's corporate tax rate. While a firm's unlevered equity growth rate (g_U) depends on its plowback-payout decision, a firm's levered equity growth rate (g_L) depends on both its plowback-payout and debt-equity decisions. Thus, these two decisions are intertwined in determining a firm's *equilibrating g_L*, which has a pronounced effect on G_L and firm maximization.

The CSM is vital to capital structure research as it provides a robust set of G_L equations that include a set of essential variables covering an array of situations. Such versatility offers potential for future CSM research to tie together the pieces from major capital structure theories by accounting for and quantifying their hypothesized effects. The CSM can shed light on topics of debate within the literature such as the existence of the rate at which to discount the tax shield, the role of growth, the trade-off between positive tax-agency benefits and financial distress costs, different empirical findings concerning capital structure, and the uncertain effect on firm value surrounding high leverage ratios. The continued expansion of CSM research is important as the CSM offers G_L equations with more practical potential to measure the impact of managerial choices. To the extent changes in growth-adjusted equity rates and debt rates are more accurate to estimate than the nearly impossible task of directly measuring the many hypothesized bankruptcy and agency costs, this paper's G_L equations overcome measurability problems.

Future G_L research can extend this paper by further exploring the theoretical implications, practical applications, and pedagogical exercises that are inherent in the CSM. Extension of CSM research can expand on the growth aspect of this paper by considering wealth transfer effects, changes in tax rates as the debt-equity ratio changes, and a levered situation from which a firm can optimize its value by increasing or decreasing its debt-equity ratio. Additionally, a practical exercise along the lines of Hull (2008), but with growth incorporated, can be developed. This exercise would expand on the illustrations given in Appendix C and Appendix D.

References

1. Altman, Edward (1984). A Further Empirical Investigation of the Bankruptcy Cost Question, *Journal of Finance*, 39, 1067-1087.
2. Baxter, Nevins (1967). Leverage Risk of Ruin and the Cost of Capital, *Journal of Finance*, 22, 395-403.
3. Cutler, David, and Lawrence Summers (1988). The Costs of Conflict Resolution and Financial Distress: Evidence from the Texaco-Pennzoil Litigation, *Rand Journal of Economics*, 19, 157-172.
4. DeAngelo, Harry, and Ronald Masulis (1980). Optimal capital structure under corporate and personal taxation, *Journal of Financial Economic*, 8, 3-29.

5. Fama, Eugene F., and French, Kenneth R. (2005). Financing Decisions: Who Issues Stock? *Journal of Financial Economics*, 76, 549-582.
6. Fischer, Edwin O., Robert Heinkel, and Josef Zechner (1989). Optimal Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance*, 44, 19-40.
7. Frank, M. and V. Goyal (2003). Testing the Pecking Order Theory of Capital Structure, *Journal of Financial Economics*, 67, 217-248.
8. Graham, John (2000). How Big Are the Tax Benefits of Debt? *Journal of Finance*, 55, 1901-1941.
9. Hackbarth, Dirk, Christopher Hennessy, and Hayne Leland (2007). Can the Trade-off Theory Explain Debt Structure? *Review of Financial Studies*, 20, 1389-1428.
10. Harvey, Campbell, and John Graham (2001). The Theory and Practice of Corporate Finance: Evidence from the Field, *Journal of Financial Economics*, 60, 187-243.
11. Hennessy, Christopher, and Toni Whited (2005). Debt Dynamics, *Journal of Finance*, 60, 1129-1165.
12. Hittle, L., K. Haddad, and L. Gitman (1992). Over-the-Counter Firms, Asymmetric Information, and Financing Preferences, *Review of Financial Economics*, 2 (Fall), 81-92.
13. Hull, Robert M., and Richard Fortin (1993/1994). Issuance Expenses and Common Stock Offerings for Over-the-Counter Firms, *Journal of Small Business Finance*, 3 (1), 1-17.
14. Hull, Robert M., and Robert Kerchner (1996). Issue Costs and Common Stock Offering, *Financial Management*, 25 (4), 54-66.
15. Hull, Robert M. (2005). Firm Value and the Debt-Equity Choice, *Regional Business Review*, 24 (May), 50-75.
16. Hull, Robert M. (2007). A Capital Structure Model, *Investment Management and Financial Innovations*, 4 (3), 8-24.
17. Hull, Robert M. (2008). Capital Structure Decision-Making: a Pedagogical Application, *Journal of Financial Education*, 34 (Fall), 88-111.
18. Jensen, Michael C., and William H. Meckling (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics*, 3, 305-360.
19. Jensen, Michael C. (1986). Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers, *American Economics Review*, 76, 323-329.
20. Kayhan, Ayla, and Sheridan Titman (2007). Firms' Histories and their Capital Structure, *Journal of Financial Economics*, 83, 1-32.
21. Korteweg, Arthur (2010). The Net Benefits of Leverage, *Journal of Finance* (Forthcoming).
22. Kraus, Alan, and Robert H. Litzenberger (1973). A State-Preference Model of Optimal Financial Leverage, *Journal of Finance*, 28, 911-922.
23. Leary, Mark, and Michael Roberts (2005). Do Firms Rebalance Their Capital Structures? *Journal of Finance*, 60, 2575-2619.
24. Leland, Hayne (1998). Agency Costs, Risk Management, and Capital Structure, *Journal of Finance*, 53, 1213-1243.
25. Miller, Merton (1977). Debt and Taxes, *Journal of Finance*, 32, 261-275.
26. Modigliani, Fred, and Merton Miller (1958). The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economics Review*, 48, 261-297.
27. Modigliani, Fred, and Merton Miller (1963). Corporate Income Taxes and Cost of Capital: a Correction, *American Economics Review*, 53, 433-443.
28. Myers, Stewart (1984). The Capital Structure Puzzle, *Journal of Finance*, 39, 575-592.
29. Myers, Stewart, and N. Majluf (1984). Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have, *Journal of Financial Economics*, 13, 187-221.
30. Pinegar, J. Michael, and Lisa Wilbrecht (1989). What Managers Think of Capital Structure Theory: a Survey, *Financial Management*, 18, 82-91.
31. Pratt, Shannon, and Roger Grabowski (2008). Cost of Capital: Applications and Examples, Hoboken, NJ: John Wiley & Sons, Inc.
32. Smith, Clifford (1977). Alternative Methods for Raising Capital, *Journal of Financial Economics*, 5, 273-307.
33. Strebulaev, Ilya (2007). Do Tests of Capital Structure Theory Mean What They Say? *Journal of Finance*, 62 (August), 1747-1787.
34. Warner, J. (1977). Bankruptcy Costs: Some Evidence, *Journal of Finance*, 32, 337-348.

Appendix A. Proof of equation (12)

Proof of equation (12) for an unlevered firm with constant growth undergoing a debt-for-equity exchange when tax rates do not change. Using equation (8) for G_L while noting from equation (11) that $V_L = E_L + D$ and further noting that V_U is the same as E_U (because $D = 0$):

$$G_L = V_L - V_U = E_L + D - E_U.$$

Inserting for E_L using equation (10):

$$G_L = \frac{(1 - T_E)(1 - T_C)(C - I)}{r_{Lg}} + D - E_U.$$

Multiplying out $\dfrac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}}$ and rearranging to get two components inside two brackets:

$$G_L = \left[D - \frac{(1-T_E)(1-T_C)I}{r_{Lg}} \right] - \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}} \right].$$

Multiplying $\dfrac{(1-T_E)(1-T_C)I}{r_{Lg}}$ by $\dfrac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $\left[\dfrac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}} \right] \dfrac{(1-T_D)I}{r_D}$, which is $\left[\dfrac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}} \right] D$,

setting $\alpha = \dfrac{(1-T_E)(1-T_C)}{(1-T_D)}$, and factoring out D:

$$G_L = \left[1 - \frac{\alpha r_D}{r_{Lg}} \right] D - \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}} \right].$$

Multiplying $\dfrac{(1-T_E)(1-T_C)C}{r_{Lg}}$ by $\dfrac{r_{Ug}}{r_{Ug}}$ to get $\left(\dfrac{r_{Ug}}{r_{Lg}} \right) \dfrac{(1-T_E)(1-T_C)C}{r_{Ug}}$, which is $\left(\dfrac{r_{Ug}}{r_{Lg}} \right) E_U$, and factoring out E_U:

$$G_L = \left[1 - \frac{\alpha r_D}{r_{Lg}} \right] D - \left[1 - \frac{r_{Ug}}{r_{Lg}} \right] E_U. \tag{12}$$

We can express G_L in equation (12), like Hull (2005), by rearranging the 2nd component and substituting definitions for r_{Ug} and r_{Lg}. Doing this gives: $G_L = \left[1 - \dfrac{\alpha r_D}{r_L - g_L} \right] D + \left[\dfrac{r_U - g_U}{r_L - g_L} - 1 \right] E_U$ even though g_L is defined differently by Hull (2005) who leaves out G.

Appendix B. Proof of equation (17)

Proof of equation (17) for a levered firm with constant growth that becomes unlevered by undergoing an equity-for-debt exchange when tax rates do not change. Using equation (16) for $G_L^{Equity-for-Debt}$ while noting V_U is the same as E_U (because $D = 0$) and $-V_L = -E_L - D$:

$$G_L^{Equity-for-Debt} = V_U - V_L = E_U - E_L - D.$$

Inserting for E_L using equation (10):

$$G_L^{Equity-for-Debt} = E_U - \frac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}} - D.$$

Multiplying out $\dfrac{(1-T_E)(1-T_C)(C-I)}{r_{Lg}}$ and rearranging to get two components inside two brackets:

$$G_L^{Equity-for-Debt} = \left[E_U - \frac{(1-T_E)(1-T_C)C}{r_{Lg}} \right] - \left[D - \frac{(1-T_E)(1-T_C)I}{r_{Lg}} \right].$$

Multiplying $\dfrac{(1-T_E)(1-T_C)C}{r_{Lg}}$ by $\dfrac{r_{Ug}}{r_{Ug}} = 1$ to get $\left(\dfrac{r_{Ug}}{r_{Lg}} \right) \dfrac{(1-T_E)(1-T_C)C}{r_{Ug}}$, which is $\left(\dfrac{r_{Ug}}{r_{Lg}} \right) E_U$, and factoring out E_U:

$$G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}} \right] E_U - \left[D - \frac{(1-T_E)(1-T_C)I}{r_{Lg}} \right].$$

Multiplying $\dfrac{(1-T_E)(1-T_C)I}{r_{Lg}}$ by $\dfrac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $\left[\dfrac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}} \right] \dfrac{(1-T_D)I}{r_D}$, which is

$\left[\dfrac{(1-T_E)(1-T_C)r_D}{(1-T_D)r_{Lg}} \right] D$, setting $\alpha = \dfrac{(1-T_E)(1-T_C)}{(1-T_D)}$, and factoring out D:

$$G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]E_U - \left[1 - \frac{\alpha r_D}{r_{Lg}}\right]D^1. \tag{17}$$

Appendix C. Illustration using equation (12)

For the graph in this Appendix, the critical point is $T_C = 0.26$ and PBR is fixed at 0, 0.15 and 0.3 while the *debt choice* (DC) increases from 0 to 0.7 for each PBR. Each debt choice reflects the proportion of E_U being retired. The graph illustrates that a non-growth firm ($PBR=0$) can make a different debt choice than a growth firm ($PBR=0.3$). The graph also reveals a steep drop-off in G_L/E_U with too much debt when PBR is greater than its critical point of 0.26. The highest G_L/E_U value occurs when $DC = 0.6$ and $PBR = 0.3$. G_L/E_U becomes very negative if DC increases to 0.7 revealing great risk when too much debt is chosen.

Debt choice (DC)	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700
G_L/E_U (PBR=0.00)	0.000	0.043	0.062	0.065	0.057	0.045	0.012	−0.032
G_L/E_U (PBR=0.15)	0.000	0.042	0.063	0.069	0.065	0.056	0.032	0.005
G_L/E_U (PBR=0.30)	0.000	0.049	0.080	0.098	0.111	0.126	0.156	−0.244
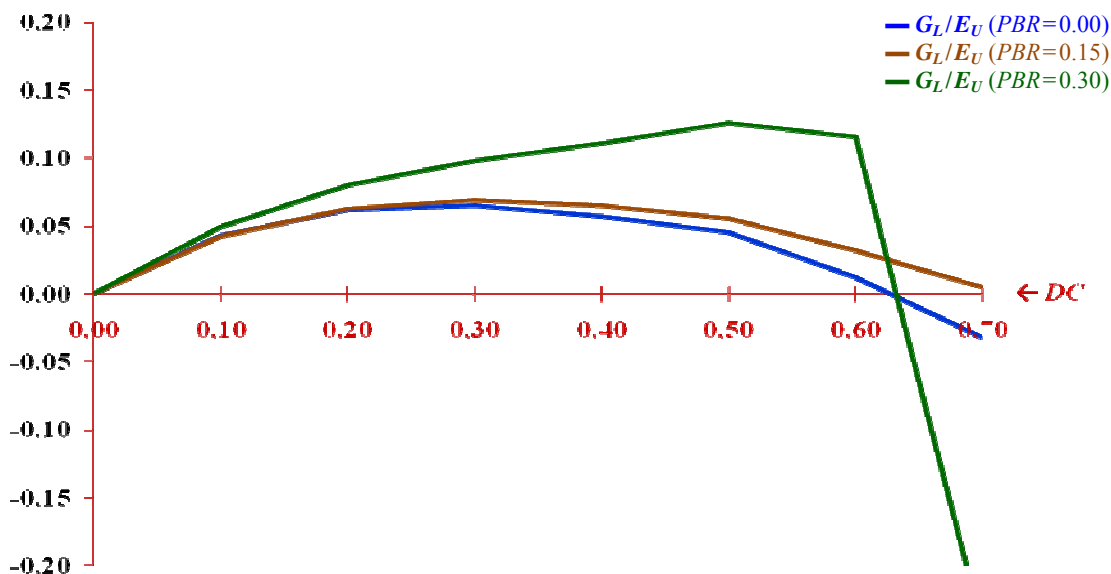
Appendix D. First illustration using equation (18)

For the graph in this appendix, the critical point is $T_C = 0.26$. The graph illustrates what happens as the plowback ratio (PBR) increases. The debt choice (DC) represents the fraction of unlevered equity (E_U) that is being exchanged for debt. G_L/E_U is the gain in leverage as a fraction of E_U. The coefficient differential from equation (18) is $n_1 - n_2$. ODE is the optimal debt-to-equity ratio that corresponds to the maximum G_L/E_U or maximum firm value. The major point illustrated is that plowback-payout and debt-equity decisions are interlinked with firm maximization involve both decisions operating in unison.

PBR	0.000	0.050	0.100	0.150	0.200	0.250	0.260	0.270	0.300	0.330	0.360	0.390	0.400	0.420
DC	0.300	0.300	0.300	0.300	0.300	0.500	0.500	0.600	0.600	0.500	0.500	0.300	0.200	0.000
G_L/E_U	0.065	0.065	0.066	0.069	0.073	0.087	0.093	0.104	0.156	0.170	0.321	0.183	0.138	0.000
$n_1 - n_2$	0.521	0.511	0.498	0.482	0.462	0.294	0.289	0.188	0.161	0.230	0.167	0.266	0.300	0.312
ODE	0.392	0.392	0.391	0.389	0.387	0.851	0.844	1.192	1.086	0.757	0.631	0.349	0.218	0.000

---

[1] We could also express equation (17) as $G_L^{Equity-for-Debt} = \left[1 - \frac{r_{Ug}}{r_{Lg}}\right]E_U + \left[\frac{ar_D}{n_{Lg}} - 1\right]D$ or, if substituting definitions for r_{Ug} and r_{Lg}, we could

further express equation (17) as $G_L^{Equity-for-Debt} = \left[1 - \frac{r_U - g_U}{r_L - g_L}\right]E_U + \left[\frac{\alpha r_D}{r_L - g_L} - 1\right]D$.