

Sanjay K. Nawalkha (USA), J. Alberto de Luca (Spain), Gloria M. Soto (Spain)

Simple formulas for financial analysts for pricing zero-dividend and positive-dividend stocks

Abstract

This paper derives simple stock valuation formulas for pricing zero-dividend and positive-dividend stocks using alternative wealth creation models given as: (1) the EVA model of Stewart (1991); (2) the residual income model of Edwards and Bell (1961) and Ohlson (1990; 1991; 1995); and (3) the franchise factor model of Leibowitz and Kogelman (1990; 1992; 1994). An advantage of these models over the wealth distribution models (such as, Gordon's (1963) dividend discount model) is that dividends are obtained endogenously under these models. The authors derive formulas both for dividend paying as well as zero-dividend paying stocks under multiple growth rates. These formulas are easy to use and allow a variety of assumptions that can be input by financial analysts for pricing stocks.

Keywords: stock valuation, dividend discount model, franchise factor model, residual income model, EVA model.

JEL Classification: G10, G11, G12.

Introduction

The dividend discount model (DDM) of Gordon's (1963) provides a widely used rule of thumb formula for stock valuation based on how the wealth of a company is distributed to its stockholders via dividend payments. Although the academic literature typically presents this formula as the key formula for stock valuation, the DDM has serious limitations. It is common for firms to pay out a constant level of dollar dividends over many time periods and then increase the dividends in a step function to a higher level over future time periods. Also, the dividend growth rate being under managerial discretion can be manipulated in such a way that it becomes too noisy to be used for stock valuation. Hence, stock valuation cannot be done accurately because the estimates of dividend growth rates are too noisy. Also, the traditional dividend discount models cannot be used for pricing zero dividend stocks.

This paper offers two alternatives to the DDM formula which can be used to value zero-dividend and positive-dividend stocks straightforwardly. These alternatives focus on how the wealth is created instead of how the wealth is distributed. From a purely theoretical view, this change in perspective should not alter how stocks are valued. However, since the exogenous parameters required by wealth creation models are different from those required by the DDM, these models can lead to better stock valuation, whenever the parameters associated with these models have less noise and then can be measured more accurately.

Three types of wealth creation models are investigated in this paper:

1. The EVA model of Stewart (1991),
2. The residual income model of Edwards and Bell

(1961) and Ohlson (1990; 1991; 1995).

3. The franchise factor model of Leibowitz and Kogelman (1990; 1992; 1994).

The EVA model, when specialized to equity valuation, is identical and the residual income model, thus leading to two alternative stock valuation formulas, one under the EVA/residual income model and other under the franchise factor model. In these models dividends are derived endogenously (in contrast to the DDM model) and hence, the models can price zero-dividend stocks.

Finally, since firms belonging to different phases of the product life cycle and industry life cycle may experience different growth rates along time, our formulas have flexibility to capture a variety of growth rate assumptions by allowing for up to three different growth rates. Even for the most complex cases, our formulas are simple enough to use, with the benefit that they do not sacrifice the realism of the more sophisticated models.

1. A brief review of valuation approaches

Fundamental analysis focusing on the earning power of the firm is generally viewed as a better approach to stock valuation vis-à-vis the technical approach. The various steps of fundamental analysis include intuitive judgments of managerial skills, financial ratio analysis, earnings projections using econometric techniques, and equilibrium risk/return analysis. The various outputs from the above analysis are the building blocks for stock valuation.

Among them, the most widely known model is the Gordon's (1963) dividend discount model (DDM). According to the DDM, the price of a stock is the present value of all its future dividends. Although the DDM is intuitively appealing, its usage among practitioners is quite limited. This is because the DDM requires the growth rate of dividends as an exogenous input, which is hard to estimate. Divi-

dividends are under managerial discretion and can be manipulated becoming too noisy and scarcely informative to be used for stock valuation. In this regard, Shiller (1981) shows that market price volatility is far too high to be explained by changes in dividends. Fama and French (2001) and DeAngelo et al. (2004) argue that the tendency of firms to pay less dividends in recent times is not justified by changes in firm characteristics. Given that a single growth rate (or at most two growth rates) of dividends is the main input to the DDM, the valuation cannot be obtained accurately because the estimates of dividend growth rates are inaccurate. Further, Miller and Modigliani (1961) demonstrate that dividend policy is irrelevant for stock valuation and Black (1976) demonstrates how the advantages and disadvantages of dividends offset each other under various market imperfections such as taxes, floatation costs, etc. These papers further undermine the DDM noting that stock valuation should be a function of the firm's investment policy and not the financing policy related to its dividends.

In the past two decades, many alternatives have been proposed to the traditional DDM and other extensions of this model. These alternatives include other approaches which, similar to the DDM, try to estimate the intrinsic value of the stock based on its capacity to generate cash flow in the future (discounted cash flow valuation), but where the focus is on earnings or returns instead of dividends¹; relative valuation approaches (or price multiple models) where stock valuation is based upon how the stocks of similar firms are priced in the market; and option valuation approaches, which use the recent theory of real options to take into account the uncertainty and choices available in firm's investments and projects.

Damodaran (2007) and Poitras (2010) review these approaches. As Damodaran points out, discounted cash flow valuation 'gets the most play in the academia and comes with the best theoretical credentials'. This assertion justifies our interest on this type of models. Moreover, discounted cash flow valuation is supported by empirical analysis. For example, the evidence in Kaplan and Ruback (1995) and Berkman et al. (2000) points out that discounted cash flow valuation and relative valuation have similar accuracy, and the results of Lee et al. (1999) reveal a better performance of discounted cash flow valuation.

¹ The equivalence of discounted cash flow firm valuation and excess return models is proven in several papers. For example, see Hartman (2000), Lundholm and O'Keefe (2001) or Fernandez (2002). Provided that coherent assumptions are applied, all these models are internally consistent because they analyze the same reality but under different perspectives (cash flows, excess earnings, residual income, etc.).

The three models investigated in this paper are the EVA model of Stewart (1991), the residual income model of Edwards and Bell (1961) and Ohlson (1990; 1991; 1995), and the franchise factor model of Leibowitz and Kogelman (1990; 1992; 1994). The Economic Value Added (EVA) model of Stewart (1991), popularly known as just the EVA model, has gained significant attention by the practitioners and academics over the last two decades. There are many books specialized on the topic, such as Stewart (1991) and Young and O'Byrne (2000)².

The EVA model is based on the principle that the value of a firm should depend upon the size and the timing of its after-tax operating cash flows, and not on when these cash flows are distributed to its financiers. The EVA is defined as the excess (economic) after-tax earnings of an asset over the cost of financing that asset over any period. The present value of all future EVAs is defined as the MVA (i.e., market value added). As shown by Stewart (1991), the MVA represents the differential the market is willing to pay for the value of a firm over its current book value. Hence, the value of a firm is given as the sum of its current book value and its MVA. As shown later, when the concepts of EVA and MVA are specialized to stock valuation, the market value of a firm's stock equals the sum of the current book value per share and the equity-MVA per share. We call this model the equity-EVA model.

The second wealth creation model we consider is the residual income model of Edwards and Bell (1961) and Ohlson (1990; 1991; 1995). The residual income is defined as the excess earnings available to stock holders over the cost of equity financing over any period. The present value of equity is shown to equal the sum of the equity's current book value and the present value of all the future residual incomes from equity. It is easy to see that the residual income model is based upon essentially the same principle as the equity-EVA model. In fact, under general assumptions the residual income of equity over any period is the same as the equity-EVA over that period. For this reason, the stock pricing formulas obtained later in this paper are the same for these two wealth creation models, which are jointly termed as the EVA/residual income model.

² The main selling point used by Stern Stewart and Co. is that EVA users are able to register superior stock returns due to the higher capacity of EVA to deliver superior metrics. However, empirical research does not always substantiate this assertion. For example, Chen and Dodd (2001) show that EVA is empirically comparable to residual income and, more recently, Palliam (2006) find that EVA firms do not necessarily have superior stock returns compared to firms that do not use EVA.

The franchise factor model of Leibowitz and Kogelman (1990; 1992; 1994) is the third wealth creation model considered in this paper. Similar to the residual income and the EVA models, the franchise model explicitly considers the excess returns available on the future investment opportunities. However, there is a subtle difference between the EVA/residual income model and the franchise factor model in the way the returns are projected into the future. The EVA/residual income model requires the projections of future returns on the total value of book equity. In contrast, the franchise factor model requires the projections of future franchise returns only on the increments to book equity.

As noted before, the DDM and these three wealth creation models are consistent and should lead theoretically to the same stock value. However, the exogenous parameters required by the models are different. Since the quality of the estimates of the relevant parameters might differ, there might appear significant differences in valuation. In the previous discussion we pointed out serious data deficiencies of the DDM, which are expected to be overcome by wealth creation based models such as the EVA/residual income model and the franchise factor model. In fact, the empirical evidence provided in Bernard (1995), Peman and Sougiannis (1998), Frankel and Lee (1998) and Francis et al. (2000), among others, shows that excess return models, in general, outperform DDM. However, the search of the best model has not finished, even it might be argued that there not exist such a model, but a best-suited-model which depends on the circumstances. As noted by Stowe et al. (2007), the most suitable model for equity valuation should be consistent with the characteristics of the firm whose stocks are being valued, appropriate given the availability and characteristics of data, and consistent with the analyst's valuation purpose and perspective. In this context, the contribution of this paper is to provide simple formulas for these alternative models which can promote a better evaluation of the models by making it easier to analyze their empirical performance using market data. The formulas can be used to price stocks at any point in time and across time and to analyze how quickly market prices converge to model prices. As pointed out by Damodaran (2007), these are indicators of the true measure of a valuation model.

2. Rule of thumb formulas under the dividend discount model

The two most popular wealth distribution models are the constant-growth-rate DDM and the two-

growth-rate DDM. The stock price under the constant-growth-rate DDM is given as (see Gordon, 1963):

$$P_0 = D_1 / (1+k)^1 + D_1(1+g)/(1+k)^2 + D_1(1+g)^2 / (1+k)^3 + \dots, \quad (1)$$

where D_1 is the dividend per share to be paid in period 1, g is the growth rate of dividends and k is the firm's capitalization rate.

The above equation is a growing perpetuity and its closed-form solution is given as:

$$P_0 = D_1 / (k - g), \text{ when } k > g, \text{ and} \quad (2a)$$

$$P_0 = \infty, \text{ when } k \leq g. \quad (2b)$$

Hence, k must be strictly greater than g in order to get a finite stock price.

The two-growth-rate DDM is given as follows. The time zero price P_0 of one share equals the discounted value of all future dividends per share:

$$P_0 = D_1 / (1+k)^1 + D_1(1+g_1)/(1+k)^2 + \dots + D_1(1+g_1)^{T-1} / (1+k)^T + D_1(1+g_1)^{T-1} \cdot (1+g_2)^1 / (1+k)^{T+1} + D_1(1+g_1)^{T-1} \cdot (1+g_2)^2 / (1+k)^{T+2} + \dots, \quad (3)$$

where dividends grow at the rate g_1 for the first T periods and g_2 after the T periods. Equation (3) can be written in closed-form as follows¹:

$$P_0 = D_1 / (k - g_1) \left[1 - \left[(1+g_1) / (1+k) \right]^T \right] + P_T / (1+k)^T, \text{ when } k > g_2, \text{ and} \quad (4a)$$

$$P_0 = \infty, \text{ when } k \leq g_2, \quad (4b)$$

where:

$$P_T = D_T(1+g_2)/(k - g_2) = \text{price of the stock at time } T.$$

Hence, k must be strictly greater than g_2 in order to get a finite stock price.

3. Rule of thumb formulas under the EVA/residual income model

The residual income model suggests that the price of a firm's share is consisting of two components: its book value and an infinite sum of discounted residual incomes. The pricing valuation of the model can be expressed as follows²:

¹ It can be shown that the limit value of the expression $A/(x-y)[1 - ((1+y)/(1+x))^N]$ when x tends to y is equal to $NA/(1+x)$. This limit solution will be useful in a number of equations in this paper. For example, the limit value of the expression $D_1/(k-g_1)[1 - ((1+g_1)/(1+k))^T]$ in equation (4a), when k equals g_1 is equal to $TD_1/(1+k)$.

² For expositional simplicity this paper ignores the expectation operators in the numerators of equations (5) and (6). Ohlson (1990; 1991; 1995) models equations (5) and (6) with the expectations operators.

$$P_0 = BV_0 + (ROE_1 - k) \cdot BV_0 / (1+k)^1 + (ROE_2 - k) \times BV_1 / (1+k)^2 + (ROE_3 - k) \cdot BV_2 / (1+k)^3 + \dots + \quad (5)$$

where BV_t is the book value of the firm's share at time t and ROE_t is the return on equity ratio in period t .

As shown by Ohlson (1990; 1991; 1995), as long as the firm's earnings and book value are forecast in a manner consistent with clean surplus accounting (i.e., "all inclusive" concept of income accounting), the price of the stock defined in equation (5) is equal to the infinite sum of discounted dividends:

$$P_0 = D_1 / (1+k)^1 + D_2 / (1+k)^2 + D_3 / (1+k)^3 + \dots + .$$

Note that the dividends in this equation do not necessarily assume any specific growth rates for the dividends. Equation (5) is an insightful description of the price of a firm's stock. The stock price equals its book value (the capital invested) plus the sum of the present values of excess earnings net of their cost in all future periods.

The residual income model is also consistent with the EVA model of Stewart (1991), when the latter is specialized to equity valuation. As noted earlier, the EVA is defined as the excess (economic) earnings of an asset over the cost of financing that asset over any period. The present value of all future EVAs is defined as the MVA (i.e., market value added). Since the MVA represents the differential the market is willing to pay for the value of a firm over its current book value, the value of a firm is given as the sum of its current book value and its MVA. In the following we specialize the concepts of EVA and MVA to *equity valuation* instead of total firm valuation.

The equity-EVA per share can be defined as the excess (economic) earnings per share over the cost of financing the share over any period. The present value of all future equity-EVAs per share is defined as the equity-MVA per share (i.e., market value added per share), or:

$$\begin{aligned} \text{Equity-MVA per share} &= \\ &= \text{Equity-EVA}_1 \text{ per share} / (1+k)^1 + \quad (6) \\ &+ \text{Equity-EVA}_2 \text{ per share} / (1+k)^2 + \dots +, \end{aligned}$$

where equity-EVAs are defined as follows:

$$\text{Equity-EVA}_t \text{ per share} = (ROE_t - k) BV_{t-1}, \quad (6a)$$

for all $t = 1, 2, 3, \dots$

Under the Stewart (1991) approach, the price of the firm's share is given as the sum of equity's book value per share and its equity-MVA per share, or:

$$P_0 = BV_0 + \text{Equity-MVA per share}. \quad (6b)$$

By substituting the values of *Equity-EVA_t per share* from equation (6a) into equation (6) and then substituting equation (6) into equation (6b), the price of the stock can be easily seen to be identical to the price of the stock given in equation (5) under the residual income model. Since the two models are identical for equity valuation, we refer to both these models as EVA/residual income model.

In the following we develop simple equity valuation formulas that are consistent with the EVA/residual income model by making specific assumptions about how the firm's ROE and dividend payout ratio change over time. These models are simple enough to be used by practitioners for equity valuation. We derive the EVA/residual income model assuming two growth rates. Appendix A extends the derivation of the EVA/residual income model to three growth rates.

Consider a firm with a ROE equal to R_1 for the first T periods, followed by a ROE equal to R_2 after the T periods. Further, assume that the firm's earnings retention ratio is b_1 for the first T periods and b_2 after the T periods. Hence, the firm's dividend payout ratio changes from $1 - b_1$ to $1 - b_2$, when the ROE changes from R_1 to R_2 after the T^{th} period. The growth of book value per share for this firm is as follows. If BV_0 is the time zero book value per share, then the time t book value per share under the above assumptions is given as:

$$BV_t = BV_0(1 + g_1)^t, \quad (7)$$

for all $t = 0, 1, 2, \dots, T$, and

$$BV_t = BV_T(1 + g_2)^{t-T}, \text{ for all } t = T + 1, T + 2, \dots,$$

where:

$$g_1 = b_1 R_1,$$

$$g_2 = b_2 R_2.$$

Hence, the book value per share grows at a growth rate g_1 until period T , followed by a growth rate g_2 , forever. The earnings per share grow over time as follows:

$$E_t = BV_0 (1 + g_1)^{t-1} R_1, \quad (8)$$

for all $t = 1, 2, \dots, T$, and

$$E_t = BV_T (1 + g_2)^{t-(T+1)} R_2, \text{ for all } t = T + 1, T + 2, \dots$$

Since the firm's earnings retention ratio is b_1 for the first T periods and b_2 after the T periods, its dividends per share (D_t) and the change in the retained earnings per share (ΔRE_t) grow as follows:

$$D_t = E_t (1 - b_1), \Delta RE_t = E_t b_1, \quad (9)$$

for all $t = 1, 2, \dots, T$, and

$$D_t = E_t (1 - b_2), \Delta RE_t = E_t b_2, \text{ for all}$$

$$t = T + 1, T + 2, \dots$$

Obviously, $E_t = D_t + \Delta RE_t$, for all $t = 1, 2, \dots$

By substituting the book value from equation (7) into equation (5), the stock price can be given in a closed-form under the EVA/residual income model. Note that the exogenous variables needed to compute the stock price are the growth rates g_1 and g_2 (of the book value of the equity), and the corresponding ROEs given as R_1 and R_2 . These variables are determined by the *investment opportunities* available to the firm and may be estimated with less noise than the growth rate of dividends needed to value stocks in the traditional DDMs. Though in theory the growth rates in the book value of equity under the EVA/residual income model are the same as the growth rates of dividends in the traditional DDMs, in practice these growth rates will be estimated differently since *dividend policy may be adjusted with a lag*. Since managers may be less inclined to pursue non-optimal investment policies (as opposed to adjusting dividends slowly), it is hypothesized that growth rates under the EVA/residual income model may be estimated more accurately, and hence this model may lead to better stock valuation. Finally, since dividends are obtained exogenously under the EVA/residual income model, this model can price even zero dividend stocks. In the following, we are going to derive the price of the stock under two scenarios of dividend payouts: positive dividends and zero dividends.

3.1. Positive dividends. Under this case, the earnings retention ratio b_2 is strictly less than one implying positive dividends after T periods. If the earnings retention ratio b_1 is less than one, then dividends are also positive in the first T periods. If b_1 equals one, then the dividends are zero in the first T periods. The price of the stock is given as¹:

$$P_0 = BV_0 + BV_0 \left[\frac{(R_1 - k)}{(k - g_1)} \right] \times \left[1 - \frac{(1 + g_1)^T}{(1 + k)^T} \right] + BV_0 \left[\frac{(1 + g_1)^T (1 + k)^T}{(1 + g_2)^T} \right] \times \left[\frac{(R_2 - k)}{(k - g_2)} \right], \quad (10a)$$

when $k > g_2$, and

$$P_0 = \infty, \text{ when } k \leq g_2. \quad (10b)$$

Equation (10a) gives a simple closed-form solution for the stock price with two growth rates for the book value, using the EVA/residual income valuation framework. In general, if both R_1 and R_2 (the two ROEs) are greater than k , the stock price is

greater than its book value. From equation (10b) it can be seen that the capitalization rate k must be strictly greater than g_2 in order to get a finite stock price. Equation (10a) gives the stock price consistent with a *single* growth rate if $R_1 = R_2$ and $g_1 = g_2$. Of course, to get a finite price then would require $k > g_1 = g_2$.

The earnings, dividends, and additions to retained earnings all grow at the growth rate g_1 until period T , followed by an adjustment between period T and $T + 1$, after which these variables grow at the growth rate g_2 forever (from period $T + 1$ onwards).

3.2. Zero dividends. Under this case both b_1 and b_2 are equal to one. This implies that $g_1 = R_1$ and $g_2 = R_2$. The price of the stock is given as:

$$P_0 = 0, \text{ when } k > g_2, \quad (10c)$$

$$P_0 = BV_0 \left[\frac{(1 + g_1)^T}{(1 + k)^T} \right], \quad (10d)$$

when $k = g_2$, and

$$P_0 = \infty, \text{ when } k < g_2. \quad (10e)$$

Unlike the earlier case, the capitalization rate k must be *equal* to g_2 in order to get a finite and a positive stock price. If k is strictly greater (less) than g_2 , then the stock price is zero (infinite). The best way to understand equation (10c) through (10e) is by considering the liquidation of the firm's assets at infinity, at which time all of the accumulated capital is returned to the shareholders. If $k > g_2$, then the capital invested from T onwards is being compounded at a rate g_2 which is less than the capitalization rate k at which it will be discounted back from infinity to time T . This gives a zero stock price at time T , which together with the fact that dividends are zero until time T , gives a zero stock price today. If $k = g_2$, then the capital invested from time T onwards is being compounded at a rate g_2 that is exactly equal to the capitalization rate k at which it will be discounted back from infinity to time T . Since the compounding and discounting cancel each other out exactly after period T , the stock price at time T equals its accumulated capital $BV_T = BV_0(1 + g_1)^T$. The current stock price is then obtained by discounting BV_T by the discount factor equal to $1/(1 + k)^T$, which gives equation (10d). Finally, if $k < g_2$, then the capital invested from time T onwards is being compounded at a rate g_2 that is greater than the capitalization rate k at which it will be discounted back from infinity to time T . This gives an infinite stock price at time T , implying an infinite stock price today.

4. Rule of thumb formulas under the franchise factor model

Similar to the EVA/residual income model, the franchise model explicitly considers the excess returns available on the future investment opportuni-

¹ See footnote 1 (p. 159) on how to obtain the solution of (10a) when $k = g_1$.

ties. However, in contrast to the EVA/residual income model, which requires the projections of future returns on the book, the franchise factor model requires the projections of future franchise returns on the increments to book equity.

The franchise factor model can be summarized as follows. The P/E ratio of a company depends upon the sum of two components: the inverse of the capitalization rate k (the base P/E) and all future franchise investment opportunities. As shown by Leibowitz and Kogelman (1990, 1994), the P/E ratio can be formulated as follows:

$$P_0 / E_1 = 1/k + FF_1 G_1 + FF_2 G_2 + FF_3 G_3 + \dots + FF_J G_J, \tag{11}$$

where:

$$FF_i = (R_i - k)/rk, \text{ for all } i = 1, 2, \dots, J,$$

G_i is a present value of all future investments that provide a return equal to R_i in perpetuity, expressed as a proportion of the original book value of equity, BV_0 , for all $i = 1, 2, \dots, J$, and r is perpetual return provided by the original book value of equity, BV_0 .

It can be seen that a higher P/E ratio results only if the future investment opportunities offer a return that is higher than the firm's capitalization rate k . Interestingly, the P/E ratio does not depend upon the already existing investments, even if these investments generate a return (i.e., the perpetual return r) that is higher than the capitalization rate k .

In the following analysis we identify the franchise factor model with two growth rates and develop simple closed-form solutions that may be used to price stocks with zero as well as non-zero dividends. Appendix B extends the franchise factor model to three growth rates.

Assume that the original book value BV_0 generates a perpetual return, r , every period in the first T periods. The increments to book equity generate a perpetual supernormal return, R_1 , every period. The increments to book equity beginning $T + 1$ period onwards generate a perpetual normal return, R_2 , every period. Also, we assume that the firm's earnings retention ratio is b_1 for the first T periods and b_2 after the T periods. Hence, the firm's dividend payout ratio is $1 - b_1$ for the first T periods and $1 - b_2$ after the T periods. This model is similar to the model of Leibowitz and Kogelman (1994) with two franchise factors.

The growth of book value per share for the firm given above is as follows:

$$BV_t = BV_0 + BV_0 (r/R_1) [(1+g_1)^t - 1], \tag{12}$$

for all $t = 0, 1, 2, \dots, T$, and

$$BV_t = BV_T + BV_0 (r/R_2) (1+g_1)^T [(1+g_2)^{t-T} - 1],$$

for all $t = T + 1, T + 2, \dots$,

where:

$$g_1 = b_1 R_1,$$

$$g_2 = b_2 R_2.$$

The earnings per share of the firm grow as follows:

$$E_t = BV_0 r (1 + g_1)^{t-1}, \text{ for all } t = 1, 2, \dots, T, \text{ and} \tag{13}$$

$$E_t = BV_0 r (1 + g_1)^T (1+g_2)^{t-(T+1)},$$

for all $t = T + 1, T + 2, \dots$

Since the firm's earnings retention ratio is b_1 for the first T periods and b_2 after the T periods, its dividends per share and retained earnings per share are given as follows:

$$D_t = E_t (1 - b_1), \Delta RE_t = E_t b_1, \tag{14}$$

for all $t = 1, 2, \dots, T$, and

$$D_t = E_t (1 - b_2), \Delta RE_t = E_t b_2,$$

for all $t = T + 1, T + 2, \dots$

Since the above firm has two franchise factors, its stock price is given by equation (11) with $J = 2$. The two franchise factors FF_1 and FF_2 are easy to compute (using the definition in equation (11)) since the values of R_1, R_2, r , and k are exogenously given. G_1 and G_2 can be obtained as follows:

$$G_1 = [\Delta RE_1 / (1+k) + \Delta RE_2 / (1+k)^2 + \dots + \Delta RE_T / (1+k)^T] / BV_0, \tag{15a}$$

$$G_2 = [\Delta RE_{T+1} / (1+k)^{T+1} + \Delta RE_{T+2} / (1+k)^{T+2} + \dots] / BV_0, \tag{15b}$$

where ΔRE_t (for all $t = 1, 2, \dots$) is defined by equations (13) and (14).

Note that under clean surplus accounting, ΔRE_t gives the increment in book equity for the period t . Hence, G_1 (G_2) represents the present value of *all increments in book equity* that generate a perpetual return equal to R_1 (R_2), as a proportion of the original book value of the equity BV_0 . By solving the values of G_1 and G_2 in a closed-form solution and substituting these solutions in equation (11), a closed-form formula of the stock price can be obtained. In the following we derive the price of the stock under two scenarios of dividend payouts: positive dividends and zero dividends.

4.1. Positive dividends. Under this positive dividend case, the earnings retention ratio b_2 is

strictly less than one implying positive dividends after T periods. If the earnings retention ratio b_1 is less than one, then dividends are also positive in the first T periods. If b_1 equals one, then the dividends are zero in the first T periods. By making all of the appropriate substitutions from equations (13), (14), (15a) and (15b) into equation (11), the price of the stock is given as:

$$P_0 = E_1 [1/k + FF_1 \cdot G_1 + FF_2 \cdot G_2], \quad (16a)$$

when $k > g_2$, and

$$P_0 = \infty, \text{ when } k \leq g_2, \quad (16b)$$

where:

$E_1 = r \cdot BV_0$ = the first period earnings,

$G_1 = (r/R_1) [g_1/(k - g_1)] [1 - (1+g_1)^T/(1+k)^T]$,

$G_2 = (r/R_2) [g_2/(k - g_2)] [(1+g_1)^T/(1+k)^T]$, and

$FF_i = (R_i - k)/rk$, for $i = 1, 2$.

Equation (16a) gives a closed-form solution for the stock price with two franchise factors. In general, if both R_1 and R_2 (the two returns on increments to book equity) are greater than k , the stock price is greater than its base price equal to E_1/k . From equation (16b) it can be seen that the capitalization rate k must be strictly greater than g_2 in order to get a finite stock price. Equation (16a) gives the stock price consistent with only one franchise factor if $R_1 = R_2$ and $g_1 = g_2$. Of course, to get a finite price then would require $k > g_1 = g_2$.

The earnings grow at a supernormal growth rate g_1 until period $T + 1$, followed by a normal growth rate g_2 forever. Dividends and additions to retained earnings grow at a supernormal growth rate g_1 until period T , followed by an adjustment between period T and $T + 1$, after which these variables grow at a normal growth rate g_2 forever (from period $T + 1$ onwards).

4.2. Zero dividends. Under this case both b_1 and b_2 are equal to one. This implies that $g_1 = R_1$ and $g_2 = R_2$. The price of the stock is given as:

$$P_0 = 0, \text{ when } k > g_2, \quad (16c)$$

$$P_0 = (E_1/k) [(1+g_1)^T/(1+k)^T], \quad (16d)$$

when $k = g_2$, and

$$P_0 = \infty, \text{ when } k < g_2. \quad (16e)$$

Unlike the earlier case, the capitalization rate k must be *equal* to g_2 in order to get a finite and a positive stock price. If k is strictly greater (less) than g_2 , then the stock price is zero (infinite). The earnings which always equal additions to retained earnings under this case grow at the growth rate $g_1 = R_1$, until period $T + 1$, followed by a growth rate $g_2 = R_2$, forever. The dividends are zero until infinity under this case.

Conclusions

This paper synthesizes a number of equity valuation approaches in the accounting and finance literature and derives simple rule of thumb stock valuation formulas under the EVA/residual income model and the franchise factor model. These models have different underlying assumptions. The dividend discount model assumes the value of an equity firm is the present value of all distributed dividends. Thus, the model focuses on the wealth distribution of the firm's resources in the valuation equation. The EVA/residual income model suggests that the price of the equity is the sum of the book value of equity and the present value of the residual incomes generated by the firm's returns exceeding its cost of equity. In contrast to the EVA/residual income model that requires the projections of future returns on the book equity, the franchise factor model requires the projections of future franchise returns on the increments to book equity.

Though the dividend discount model, the EVA/residual income model, and the franchise factor model are all internally consistent, the techniques of estimating the growth variables under these models are different. Since book value growth variables may reflect fundamentals more accurately than the dividend growth variable (which can be idiosyncratic due to managerial control over the dividend policy), the EVA/residual income model and the franchise factor model may lead to better stock valuation than the standard dividend discount model.

References

1. Berkman, H., M.E. Bradbury and J. Ferguson (2000). "The Accuracy of Price-Earnings and Discounted Cash Flow Methods of IPO Equity Valuation", *Journal of International Financial Management and Accounting*, Vol. 11, 71-83.
2. Bernard, V.L. (1995). "The Feltham-Ohlson Framework: Implications for Empiricists", *Contemporary Accounting Research*, 733-747.
3. Chen, S. and J.L. Dodd (2001). "Operating Income, Residual Income and EVA: Which Metric is More Relevant?", *Journal of Managerial Issues*, Vol. 13, 65-86.
4. Damodaran, A. (2007). *Valuation Approaches and Metrics: A Survey of the Theory and Evidence, Foundations and Trends in Finance*, Now Publishers Inc.
5. DeAngelo, H., L. DeAngelo, and D. Skinner (2004). "Are Dividends Disappearing? Dividend Concentration and the Consolidation of Earnings", *Journal of Financial Economics*, Vol. 72, 425-456.

6. Edwards, E. and P. Bell (1961). *The Theory and Measurement of Business Income*, Berkeley, CA: University of California Press.
7. Fama, E.F. and K.R. French (2001). "Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay?", *Journal of Financial Economics*, 60, 3-44.
8. Fernandez, P. (2002). "Three Residual Income Valuation Models and Discounted Cash Flow Valuation, Working Paper. Available at SSRN: <http://ssrn.com/abstract=296945>.
9. Francis, J., P. Olsson and D. Oswald (2000). "Comparing the Accuracy and Explainability of Dividend, Free Cash Flow and Abnormal Earnings Equity Value Estimates", *Journal of Accounting Research*, Vol. 38, 45-70.
10. Frankel, R. and C.M.C. Lee (1998). "Accounting Valuation, Market Expectation, and Cross-Sectional Stock Returns", *Journal of Accounting and Economics*, 25, 283-320.
11. Gordon, M. (1963). "Optimal Investment and Financing Policy", *Journal of Finance*, May, 264-272.
12. Hartman, J.C. (2000). "On the Equivalence of Net Present Value and Economic Value Added as Measures of a Project's Economic Worth", *Engineering Economist*, Vol. 45, 158-165.
13. Kaplan, S.N. and R.S. Ruback (1995). "The Valuation of Cash Flow Forecasts: An Empirical Analysis", *Journal of Finance*, Vol. 50, 1059-1093.
14. Lee, C.M.C., J. Myers and B. Swaminathan (1999). "What is the Intrinsic Value of the Dow?", *Journal of Finance*, Vol. 54, 1693-1741.
15. Leibowitz, M. and S. Kogelman (1990). "Inside the P/E Ratio: The Franchise Factor", *Financial Analysts Journal*, November/December, 17-35.
16. Leibowitz, M. and S. Kogelman (1992). "Franchise Factor and the Growth Process", *Financial Analysts Journal*, January/February, 53-62.
17. Leibowitz, M. and S. Kogelman (1994). "The Growth Illusion: The P/E 'Cost' of Earnings Growth", *Financial Analysts Journal*, March/April, 36-48.
18. Lundholm, R. and T. O'Keefe (2001). "Reconciling Value Estimates from the Discounted Cash Flow Model and the Residual Income Model", *Contemporary Accounting Research*, Vol. 18, 311-335.
19. Miller, M. and F. Modigliani (1961). "Dividend Policy, Growth and the Valuation of Shares", *Journal of Business*, October, 411-433.
20. Ohlson, J.A. (1990). "A Synthesis of Security Valuation Theory and the Role of Dividends, Cash Flows, and Earnings", *Contemporary Accounting Research*, 648-676.
21. Ohlson, J.A. (1991). "The Theory of Value and Earnings, and an Introduction to the Ball-Brown Analysis," *Contemporary Accounting Research*, Fall, 1-19.
22. Ohlson, J.A. (1995). "Earnings, Book Values, and Dividends in Security Valuation", *Contemporary Accounting Research*, Vol. 11, 661-687.
23. Palliam, R. (2006). "Further Evidence on the Information Content of Economic Value Added", *Review of Accounting and Finance*, Vol. 5 (3), 204-215.
24. Penman, S. and T. Sougiannis (1998). "A Comparison of Dividend, Cash Flow, and Earnings Approaches to Equity Valuation", *Contemporary Accounting Research*, Vol. 15, 343-383.
25. Poitras, G. (2010). *Valuation of Equity Securities: History, Theory and Application*, World Scientific Publishing Company.
26. Shiller, R. (1981). "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", *American Economic Review*, Vol. 71, 421-436.
27. Stewart III, G.B. (1991). *The Quest for Value: A Guide for Senior Managers*, New York, NY, Harper Business.
28. Stowe, J.D., T.R. Robinson, J.E. Pinto and D.W. Mcleavey (2007). *Equity Asset Valuation*, John Wiley and Sons.
29. Young, S.D. and S.F. O'Byrne (2000). *EVA and Value-Based Management: A Practical Guide to Implementation*, McGraw Hill.

Appendix A. Rule of thumb formulas under the EVA/residual income model with three growth rates

In the following, we consider a firm with three types of growth rates, i.e., its ROE equal to R_1 for the first T periods, its second ROE equal to R_2 for the next $S - T$ periods, and finally its ROE equal to R_3 after the S th period, forever. Let the earnings retention ratios corresponding to these three ROEs be given as b_1 , b_2 , and b_3 . The growth of book value per share for this firm is as follows:

$$BV_t = BV_0(1 + g_1)^t, \text{ for all } t = 0, 1, 2, \dots, T, \quad (\text{A1})$$

$$BV_t = BV_T(1 + g_2)^{t-T}, \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$BV_t = BV_S(1 + g_3)^{t-S}, \text{ for all } t = S + 1, S + 2, \dots,$$

where:

$$g_1 = b_1 R_1,$$

$$g_2 = b_2 R_2,$$

$$g_3 = b_3 R_3.$$

The earnings per share of the firm grow over time as follows:

$$E_t = BV_0 (1 + g_1)^{t-1} R_1, \text{ for all } t = 1, 2, \dots, T, \quad (\text{A2})$$

$$E_t = BV_T (1 + g_2)^{t-(T+1)} R_2, \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$E_t = BV_S (1 + g_3)^{t-(S+1)} R_3, \text{ for all } t = S + 1, S + 2, \dots$$

Also, the firm's dividends per share and retained earnings per share grow over time as follows:

$$D_t = E_t (1 - b_1), \Delta RE_t = E_t b_1, \text{ for all } t = 1, 2, \dots, T, \quad (\text{A3})$$

$$D_t = E_t (1 - b_2), \Delta RE_t = E_t b_2, \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$D_t = E_t (1 - b_3), \Delta RE_t = E_t b_3, \text{ for all } t = S + 1, S + 2, \dots$$

By substituting the book value from equation (A1) into equation (5), the stock price can be given in a closed-form. The closed-form solutions for the valuation of the stocks are given assuming positive dividends and zero dividends in the following sections.

1. Positive dividends. Under this case, the earnings retention ratio b_3 is strictly less than one implying positive dividends after S periods. If the earnings retention ratios b_1 and b_2 are both less than one, then the dividends are also positive in the first S periods. If b_1 equals one and b_2 is less than one, then the dividends are zero in the first T periods but positive afterwards. Finally, if both b_1 and b_2 are equal to one, then the dividends are zero in the first S periods and positive afterwards. The price of the stock is given as¹:

$$P_0 = BV_0 + BV_0 \left[\frac{(R_1 - k)}{(k - g_1)} \right] \left[1 - \frac{(1 + g_1)^T}{(1 + k)^T} \right] + BV_0 \left[\frac{(1 + g_1)^T (1 + k)^T}{(1 + k)^T} \right] \left[\frac{(R_2 - k)}{(k - g_2)} \right] \times \left[1 - \frac{(1 + g_2)^{S-T}}{(1 + k)^{S-T}} \right] + BV_0 \left[\frac{(1 + g_1)^T (1 + g_2)^{S-T}}{(1 + k)^S} \right] \left[\frac{(R_3 - k)}{(k - g_3)} \right], \text{ when } k > g_3, \text{ and} \quad (\text{A4a})$$

$$P_0 = \infty, \text{ when } k \leq g_3. \quad (\text{A4b})$$

Note that the capitalization rate k must be strictly greater than g_3 in order to get a finite stock price. The earnings, dividends, and additions to retained earnings all grow at the growth rate g_1 until period T , followed by an adjustment between period T and $T + 1$, after which these variables grow at the growth rate g_2 until period S . There is another adjustment between period S and $S + 1$ after which these variables grow at the growth rate g_3 forever.

2. Zero dividends. Under this case, the earnings retention ratios b_1 , b_2 , and b_3 , are all equal to one. This implies that $g_1 = R_1$, $g_2 = R_2$ and $g_3 = R_3$. The price of the stock is given as:

$$P_0 = 0, \text{ when } k > g_3, \quad (\text{A4c})$$

$$P_0 = BV_0 \left[\frac{(1 + g_1)^T (1 + g_2)^{S-T}}{(1 + k)^S} \right], \text{ when } k = g_3, \text{ and} \quad (\text{A4d})$$

$$P_0 = \infty, \text{ when } k < g_3. \quad (\text{A4e})$$

Unlike the earlier case, the capitalization rate k must be *equal* to g_3 in order to get a finite and a positive stock price. If k is strictly greater (less) than g_3 , then the stock price is zero (infinite). The intuition behind equations (A4c) through (A4e) is similar to the intuition described for equations (10c) through (10e) in Section 3.2.

The earnings (which always equal additions to retained earnings under this case) grow at the growth rate $g_1 = R_1$ until period T , followed by an adjustment between period T and $T + 1$, after which earnings grow at the growth rate $g_2 = R_2$ until period S . There is another adjustment between period S and $S + 1$ after which earnings grow at the growth rate $g_3 = R_3$, forever. The dividends are zero until infinity under this case.

Appendix B. Rule of thumb formulas under the franchise factor model with three growth rates

In the following, we consider a firm whose original book value BV_0 generates a perpetual return, r , every period in the first T periods. The additions to retained earnings in the first T periods generate a return in perpetuity, R_1 . The additions to retained earnings in the next $S - T$ periods generate a return in perpetuity, R_2 . Finally, the additions to retained earnings from the $S + 1$ th period onwards generate a return in perpetuity, R_3 . Also, we assume that the firm's earnings retention ratio is b_1 for the first T periods, b_2 for the next $S - T$ periods, and b_3 after the S periods.

The growth of book value per share for the firm given above is as follows.

$$BV_t = BV_0 + BV_0 (r/R_1) [(1 + g_1)^t - 1], \text{ for all } t = 0, 1, 2, \dots, T, \quad (\text{B1})$$

¹ See footnote 1 (p. 159) on how to obtain the solution of (A4a) when $k = g_1$ and/or $k = g_2$.

$$BV_t = BV_T + BV_0 (r/R_2) (1 + g_1)^T [(1 + g_2)^{t-T} - 1], \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$BV_t = BV_S + BV_0 (r/R_3) (1 + g_1)^T (1 + g_2)^{S-T} [(1 + g_3)^{t-S} - 1], \text{ for all } t = S + 1, S + 2, \dots,$$

where:

$$g_1 = b_1 R_1,$$

$$g_2 = b_2 R_2,$$

$$g_3 = b_3 R_3.$$

The earnings per share of the firm grow as follows:

$$E_t = BV_0 r (1 + g_1)^{t-1}, \text{ for all } t = 1, 2, \dots, T, \tag{B2}$$

$$E_t = BV_0 r (1 + g_1)^T (1 + g_2)^{t-(T+1)}, \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$E_t = BV_0 r (1 + g_1)^T (1 + g_2)^{S-T} (1 + g_3)^{t-(S+1)}, \text{ for all } t = S + 1, S + 2, \dots$$

Since the firm's earnings retention ratio is b_1 for the first T periods, b_2 for the next $S - T$ periods, and b_3 after the S periods, its dividends per share and retained earnings per share are given as follows:

$$D_t = E_t (1 - b_1), \Delta RE_t = E_t b_1, \text{ for all } t = 1, 2, \dots, T, \tag{B3}$$

$$D_t = E_t (1 - b_2), \Delta RE_t = E_t b_2, \text{ for all } t = T + 1, T + 2, \dots, S, \text{ and}$$

$$D_t = E_t (1 - b_3), \Delta RE_t = E_t b_3, \text{ for all } t = S + 1, S + 2, \dots$$

Since the above firm has three franchise factors, its stock price is given by equation (11) with $J = 3$. The variables G_1 , G_2 , and G_3 in equation (11) can be obtained as follows:

$$G_1 = [\Delta RE_1 / (1 + k) + \Delta RE_2 / (1 + k)^2 + \dots + \Delta RE_T / (1 + k)^T] / BV_0, \tag{B4a}$$

$$G_2 = [\Delta RE_{T+1} / (1 + k)^{T+1} + \Delta RE_{T+2} / (1 + k)^{T+2} + \dots + \Delta RE_S / (1 + k)^S] / BV_0, \text{ and} \tag{B4b}$$

$$G_3 = [\Delta RE_{S+1} / (1 + k)^{S+1} + \Delta RE_{S+2} / (1 + k)^{S+2} + \dots] / BV_0, \tag{B4c}$$

where ΔRE_t (for all $t = 1, 2, \dots$) is defined by equations (B2) and (B3).

By solving the values of G_1 , G_2 , and G_3 in closed-form and substituting these in equation (11), a closed-form solution of the stock price can be obtained. In the following we derive the price of the stock under two scenarios of dividend payouts: positive dividends and zero dividends.

1. Positive dividends. Under this case, the earnings retention ratio b_3 is strictly less than one implying positive dividends after S periods. If the earnings retention ratios b_1 and b_2 are both less than one, then the dividends are also positive in the first S periods. If b_1 equals one and b_2 is less than one, then the dividends are zero in the first T periods but positive afterwards. Finally, if both b_1 and b_2 are equal to one, then the dividends are zero in the first S periods and positive afterwards. By making all of the appropriate substitutions from equations (B2), (B3), (B4a), (B4b), and (B4c) into equation (11), the price of the stock is given as:

$$P_0 = E_1 [1/k + FF_1 \cdot G_1 + FF_2 \cdot G_2 + FF_3 \cdot G_3], \text{ when } k > g_3, \text{ and} \tag{B5a}$$

$$P_0 = \infty, \text{ when } k \leq g_3, \tag{B5b}$$

where:

$$G_1 = (r/R_1) [g_1 / (k - g_1)] [1 - (1 + g_1)^T / (1 + k)^T],$$

$$G_2 = (r/R_2) [g_2 / (k - g_2)] [(1 + g_1)^T / (1 + k)^T] [1 - (1 + g_2)^{S-T} / (1 + k)^{S-T}],$$

$$G_3 = (r/R_3) [g_3 / (k - g_3)] [(1 + g_1)^T (1 + g_2)^{S-T} / (1 + k)^S], \text{ and}$$

$$FF_i = (R_i - k) / rk, \text{ for } i = 1, 2, \text{ and } 3.$$

Equation (B5a) gives a closed-form solution for the stock price with three franchise factors. In general, if R_1 , R_2 , and R_3 (the three returns on increments to book equity) are greater than k , the stock price is greater than its base price equal to E_1 / k . From equation (B5b) it can be seen that the capitalization rate k must be strictly greater than g_3 in order to get a finite stock price.

The earnings grow at a growth rate g_1 until period $T + 1$, followed by a growth rate g_2 until period $S + 1$, followed by a growth rate equal to g_3 , forever. The dividends and additions to retained earnings all grow at the growth rate g_1 until period T , followed by an adjustment between period T and $T + 1$, after which these variables grow at the growth rate g_2 until period S . There is another adjustment between period S and $S + 1$ after which these variables grow at the growth rate g_3 forever.

2. Zero dividends. Under this case b_1 , b_2 , and b_3 are all equal to one. This implies that $g_1 = R_1$, $g_2 = R_2$, and $g_3 = R_3$. The price of the stock is given as:

$$P_0 = 0, \text{ when } k > g_3, \quad (\text{B5c})$$

$$P_0 = (E_1/k) [(1 + g_1)^T(1 + g_2)^{S-T} / (1 + k)^S], \text{ when } k = g_3, \text{ and} \quad (\text{B5d})$$

$$P_0 = \infty, \text{ when } k < g_3. \quad (\text{B5e})$$

Unlike the earlier case, the capitalization rate k must be *equal* to g_3 in order to get a finite and a positive stock price. If k is strictly greater (less) than g_3 , then the stock price is zero (infinite). The earnings (which always equal additions to retained earnings under this case) grow at the growth rate $g_1 = R_1$, until period $T + 1$, followed by a growth rate $g_2 = R_2$, until period $S + 1$, followed by a growth rate $g_3 = R_3 = k$, forever. The dividends are zero until infinity under this case.