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The pension separation theorem

Abstract

The Tobin's separation theorem, a pillar of classic portfolio theory asserts that single-period mean variance efficient (MVE) investment portfolios, consist of combinations of the risk-free asset (the single period zero coupon bond ("ZCB")) and the market portfolio. In this paper the theorem is generalised to combinations of the market portfolio and risk-free assets such as bonds and annuities; securities with cash payoffs before end-of-term over the same single period. This apparently simple extension has immediate and far-reaching application to world's multi-trillion dollar pension industry, for retirees exiting defined contribution (DC) funds with their individual lump sum which they must convert to a retirement income stream (RIS). An optimal pension portfolio under the mean variance criterion (MVC) is formed by splitting the lump sum into two parts. With one part, a riskless income asset (government-issued annuity bond for instance) is purchased, while the residual is invested in the market portfolio over the annuity term to reinstate capital. MVE pension portfolios can be used to mitigate investment, inflation, liquidity and longevity risk and are preferable under criteria other than mean-variance to account-based drawdown currently favoured by many lump sum retirees. A case study in the Australian pension context is provided. Existence of efficient RIS portfolios has policy implications for government infrastructure provision to support lump sum retirees in the pension phase.

Keywords: mean-variance criterion, separation theorem, lump sum conversion, retirement income stream, capital reinstatement, government pension phase infrastructure.
JEL Classification: C61, G11, G12.

Introduction

Conversion of a retirement lump sum to an income stream is a problem of increasing significance in organization for economic co-operation and development (OECD) countries. The Watson Wyatt global pension asset study for 2010 estimated pension assets in 13 major pension markets of the study to total 23.3 trillion USD, representing a weighted average 70% of GDP for the countries surveyed.

1. State sponsored private pension schemes

In countries in which nationally coordinated private pension schemes are in place for employees, a large and increasing proportion of pension assets are in DC funds, necessitating retirees to convert their nest egg into an income stream for the indefinite period of their residual lives. "Participant-directed defined contribution plans have become the cornerstone of the private sector retirement sector around the world" (Mitchell and Utkus, 2003).

In DC funds investment risk is transferred to the individual. Usually a statutory proportion of gross income is contributed regularly (often compulsorily) into an employee member's fund account. The member has some choice in the sort of investment portfolio into which contributions are paid. Over working life the account balance is expected to grow providing a termination lump sum used to fund retirement. At cessation of employment beyond some preservation age the lump sum becomes available to generate the employee's pension. Retiree members, who exit DC funds, will always be faced with decisions about converting a lump sum benefit into a secure income stream for their residual lives. How can this best be achieved?

1.1. Optimal retirement income stream literature. Ever since Yaari (1965) propounded that in the absence of a bequest motive, rational persons would annuitize all their wealth at actuarially fair prices, various authors have attempted to explain why such a small proportion actually do so (see Blake and Hudson, 2000; Milevsky and Young, 2001). In the literature of retirement income streams, the MVC of Markowitz portfolio theory has been ignored. Considering its paradigm standing in finance, at the very least its implications for lump sum conversion should be plainly available for all to see. All sorts of criteria other than mean-variance have been tried in the quest to find an optimal way to convert a retirement lump sum into an income stream in the face of the various hazards to which lump sum retirees are exposed in the pension phase, none with conspicuous success. There is a large literature in financial economics concerned with "mutual fund theorems" in which the problem of managing risky asset holdings of an investment portfolio while consuming income from it is investigated in continuous time. Perplexingly, the problem has been taken out of the mean-variance framework, for which there is no literature, into a utility function setting. Starting with Merton (1969; 1971) mutual fund theorems like Tobin's separation theorem are derived. For developments in this vein see the references canvassed in the review papers by Sethi (1995), Cadenillas (2000), Pratelli (2005) and Bayraktar and Young (2008). While such studies shed light on investor/consumer behavior, they are dependent on and limited by assumptions about investor preferences as captured by classes of utility functions, and are also dependent on (often elaborate) theories of market structure.

Tobin's original result simply involves a choice about how much cash and how much market portfolio to hold. The pension separation theorem provides a solution to the optimal RIS problem in precisely this vein. It determines credible, simple and applicable strategies that accord both with observed evidence about how retirees are funding their retirement and with general expert opinion about fundamentals which should underpin lump sum conversion. In particular, it identifies strategies that defer full annuitization of lump sums in favor of taking a minimum consumption income, while taking advantage of the expected high returns from medium to long-term investment in shares. If retirees have a bequest motive, so that at older ages they are indifferent (apart from consumption income) as to whether their remaining retirement monies are directed to themselves or accrue to their estate, they will never fully annuitize. Eventual discovery of these strategies was foreseen by Trott (2005) and Updegrave (2006). The very existence of a mean-variance efficient class of RIS strategies has policy implications for government. Instead of abandoning lump sum retirees at the completion of the accumulation phase of the scheme, as happens at the moment, governments could take an active role in the pension phase providing infrastructure by issuing part of public debt in the form of rolling annuity bonds of medium term, to assist retirees with their pension portfolios.

1.2. Life annuities and drawdown. What evidence exists suggests that lump sum retirees are, for the main part, providing themselves with income by regular drawdown from managed funds. In many countries governments provide in the pension phase, if not guidance, at least tax incentives to use account-based pensions. In Australia, where the total funds under management in the Superannuation Guarantee Scheme (SGS) is just over one trillion dollars (ASFA, 2009), new pensions offer tax-free income streams if minimal annual drawdown conditions are fulfilled. Over the twelve months to March 2007, sales of account-based pensions and annuities totaled about $ 1.39 billion compared with sales of life annuities of just $ 29 million.

"Basically, Australian consumers are not very interested in financial products which offer a low implicit rate of return, have high fees and which have nil capital value on the death of the primary or reversionary beneficiary" (Clare, 2007, p. 7).

These figures are affirmed in other studies. In US, over the years of 1992-2002 only 8 percent of retiring health workers on DC schemes chose to annuitize (Glickman and Kuehneman, 2006). Lump sum retirees certainly want to retain control of their capital. In Australia, in 2009, for the first time, the self-managed superannuation funds sector emerged as the largest sector in the superannuation industry, in terms of consolidated assets. The largest segment of this sector resides in the retirement phase (Australian Government Publication: Statistical Summary of Self-Managed Funds, 2009).

2. Tobin's separation theorem

Nobel laureate James Tobin (1958) demonstrated that the construction of a MVE investment portfolio can be separated into two distinct operations:

- selection of an efficient portfolio of risky assets that does not depend on preference; and
- combination of this portfolio with a riskless investment.

Only the per dollar allocation between the risky portfolio and the riskless investment depends on investor preference. The result is also known as the mutual fund theorem.

Within the mean-variance framework, the efficient portfolio is the market portfolio.

"If all investors have the same view of the market and seek mean-variance efficiency, then it follows that all investors mix the same portfolio of risky assets and this must be the market portfolio" (Markowitz, 2007).

The Tobin separation theorem underpins the capital market line (CML).

2.1. The pension separation theorem. For pension provision from a lump sum a result, precisely analogous to Tobin's can be proved essentially as a theorem in classic mathematical analysis. MVE retirement income stream portfolios are constructed by combining:

- an efficient portfolio of risky assets (the "market portfolio"); with
- a riskless income product (a government-issued annuity bond or indexed annuity bond).

Only the allocation between the market portfolio and the riskless investment depends on investor preference. Portfolio risk is measured by the proportion f of each dollar invested in risky assets.

At the extremes, if $f = 1$, pension is taken by drawdown with the entire lump sum invested in the market portfolio; if $f = 0$, the entire lump sum is annuitized.

As with the Tobin result, the precise investment term remains unspecified. Retirees must adopt pragmatic criteria to determine a suitable initial term. These include size of the lump sum, required annual pension, annuity cost, long-term expected return on the market index, or with longevity risk in mind, the probability that the market investment reinstates the entire original lump sum in real terms, at the end of the guaranteed income years.

Under this last criterion, for a suitably chosen term, if income consumption is modest MVE portfolios dominate account-based drawdown. That is, a MVE portfolio is more likely to reinstate the original real capital at end-of-term than drawdown from the market account.

But if consumption is heavy (the annual pension is large relative to the original lump sum) then account-based drawdown is more likely to deliver the original capital at end-of-term. However with drawdown, risk of bankrupting the account over the initial term also increases with consumption, and this quickly becomes more probable than capital reinstatement. These matters are dealt with in detail in Section 5, but a glance at Figure 4 makes clear how the probability of capital reinstatement under drawdown converges with the probability of insolvency.

But their widespread implementation will generally require governments to issue part of public debt in the form of suitable annuity bonds. In the light of the global financial crisis (GFC) during which many governments have borrowed heavily to finance "stimulus packages" for their economies, this strategy could be politically as well as economically attractive, given the increasing number of pensioner votes to be considered.

An approximation to a MVE strategy, currently available in most countries, is to purchase a term-certain indexed annuity of about twelve years from a life office or bank, together with investment in the national bourse accumulation index over the guaranteed income years. It is assumed that the expected time to reinstate original capital is a function of the particular bourse in which the lump sum is invested, and this expected time is consulted in selection of the term of the initial annuity purchased. Initial, because the retiree expects to repeat the strategy (or a modified version of it) when the first annuity term expires.

2.2. Establishing the separation theorem for pensions. The generalization of Tobin's result is underpinned by a rather bland result in mathematical analysis, and is not intuitively helpful. More helpful is to appreciate how the generalization arises. Consequently, what follows is heuristic development in a finance setting which it is hoped will assist intuition. A proof of the theorem is provided in the Appendix.

2.3. The capital market line. It is necessary to be clear about what is assumed about the two securities which feature in the CML derivation. If proportion $(1-f)$ of each dollar is invested in a riskless security with yield i and the remainder f is invested in the market with mean μ_m and variance σ_m^2 then the expected yield on the portfolio μ_p is given by:

$$1+\mu_p=(1-f)\times(1+i)+f\times(1+\mu_m) \text{ or}$$

$$\mu_p - i = f(\mu - i). \tag{1}$$

And since portfolio variance $\sigma_p^2 = f^2 \times \sigma_m^2$, whence $f=\sigma_p/\sigma_m$ the equation to the CML is obtained:

$$\mu_p - i = \frac{\sigma_p}{\sigma_m}(\mu_m - i). \tag{2}$$

Assumed:

1. The investment period is of fixed but unspecified term.
2. The riskless security returns guaranteed fixed amount i at end of the period for each dollar invested at the outset.
3. The market portfolio has random return R_m at the end of the period. Each dollar, invested at the outset, provides a terminal amount $1 + R_m$ with $E[R_m] = \mu_m$ and $Var[R_m] = \sigma_m^2$ in obvious notation.

Not assumed:

1. That the market portfolio is passive; out-performing stocks will be re-weighted and under-performing stocks downgraded over the investment term.
2. That the efficient frontier of risky assets or the CML is static.

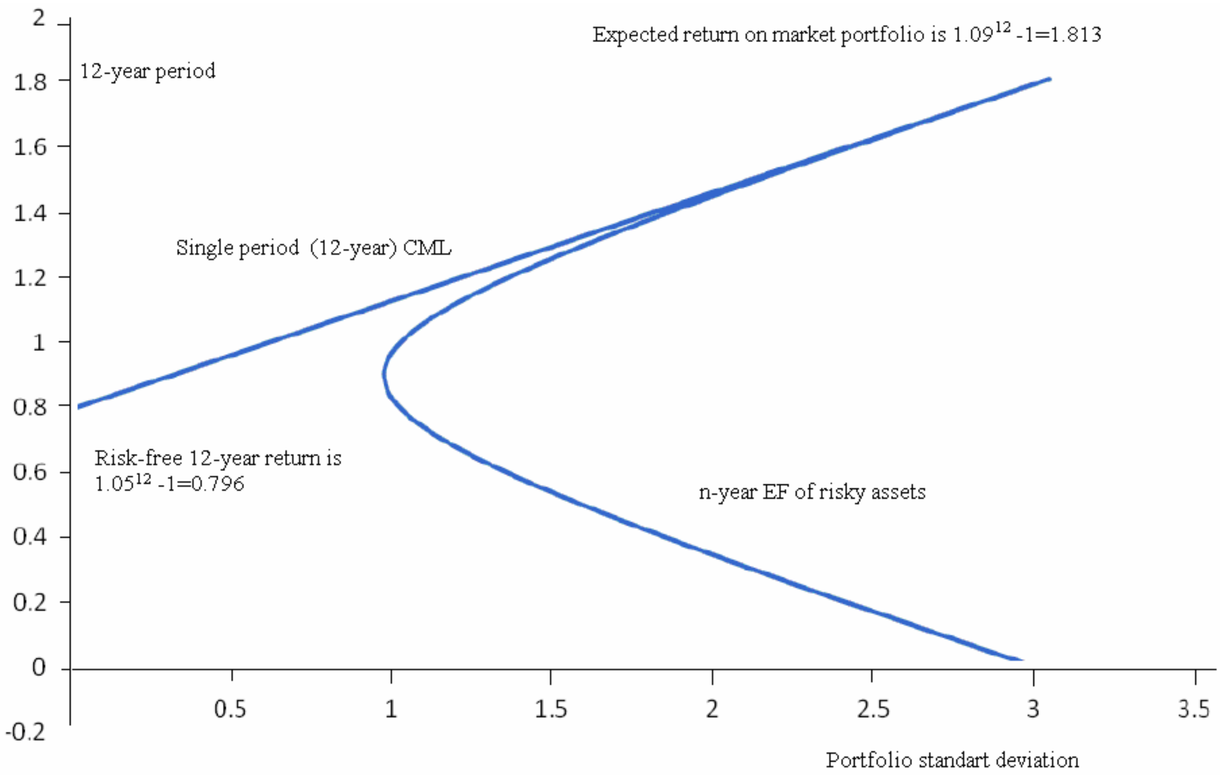
Characteristics of the market return (μ_m and σ_m) change stochastically over time. The risk-free rate i changes either stochastically or by Central Bank intervention. Notwithstanding, given their values at any instant over any specified term, equations (1) and (2) identify the MVE portfolios.

2.4. Pension separation theorem heuristics. The starting point of the pension theorem is the original theorem underpinning the one-period CML. The first change required is one of perspective. The single investment period is considered to be a number of years (n).

We start with portfolios which consist of:

1. Risk-free n-year zero-coupon bonds (ZCBs) purchased at the start of the period and held till end-of-term; and
2. The market portfolio held for n years; annual market returns are assumed to be uncorrelated (see discussion below, Section 4.1).

The situation is depicted in Figure 1 for a single 12-year investment period, with a riskless ZCB yielding 5 percent per annum (r = 0.05), and the market portfolio yielding an expected 9.0 percent per annum (μ_m = 0.09). Note that in Figure 1 the 12-year return, not the annual return, is plotted on the vertical axis.



Notes: n = 12, r = 0.05, and μ_m = 0.09 (so that 12-year risk-free return is 1.0512-1 = 0.7959 and 12-year expected market return 1.0912-1 = 1.8127). The n-year expected return is plotted on the vertical axis, and the n-year standard deviation on the horizontal axis. For convenience, the efficient frontier is depicted here as a hyperbola. In fact, it is easy to show that unrestricted short sales of risky assets must be allowed for this to be the case.

Fig. 1. The *n*-year CML and efficient frontier of risky assets

In order to accommodate risk-free investments which provide payoffs before end-of-term (annuity bonds, indexed annuity bonds, coupon bonds, capital indexed bonds, etc.), two changes are made to the n-year CML diagram in Figure 1:

1. The vertical axis is changed to measure annualized yield rather than n-year yield.
2. The horizontal axis is employed to measure risk as the fraction f of each dollar invested in the market index rather than portfolio standard deviation. For an n-year term it remains true that $\sigma_p(n) = f\times\sigma_m(n)$ where the standard deviations refer to standard deviation of n-year returns.

If $\mu(f)$ is the annualized CML portfolio expected yield, r the annual yield on the ZCB and μ_m the expected annual yield on the market portfolio when proportion f of each dollar is invested in the market portfolio then:

$$\{1+\mu(f)\}^n = (1-f)\times(1+r)^n \times f \times (1+\mu_m)^n ,$$

so that (3)

$$\mu(f) = \left\{(1-f)\times(1+r)^n \times f \times (1+\mu_m)^n\right\}^{1/n} - 1 .$$

Equation (3) is a version of the n-year CML. The curve resulting from plotting $\mu(f)$ against f exhibits slight convexity.

When the efficient frontier consists of portfolios mixing a n-year ZCB with n-year investment in the market portfolio, we call it the n-year fundamental capital market curve (FCMC).

Figure 2 depicts the n-year FCMC with a one-year CML with $r = 0.05$, $\mu_m = 0.09$ and $n = 12$.

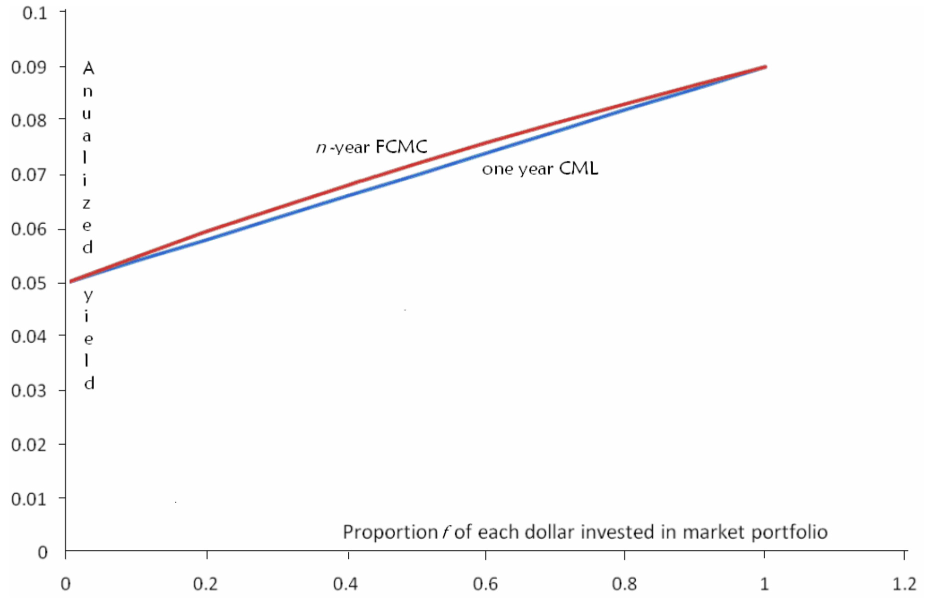


Fig. 2. The *n*-year FCMC for a 12-year risk-free ZCB, yielding 5 per cent per annum, and the market portfolio, yielding 9 per cent per annum

Now suppose that instead of ZCBs, risk-free investments are available that have payoffs before end-of-term. Combinations of the market portfolio and other riskless n-year assets will also have CML analogues described more generally as mean-variance capital market curves (MVCMCs).

The main result established in this paper can now be stated as follows: the MVCMC for any portfolio consisting of the market portfolio and a risk-free asset with payoffs before end-of-term lies above the FCMC for a ZCB with the same yield as the risk-free asset except at the points, where $f = 0$ and $f = 1$, where the two curves coincide.

Relative positions of the MVCMC and the FCMC are depicted in Figure 3 below (the risk-free security used is a 12-year annuity bond payable monthly, indexed at 3 percent at the start of each new year).

This is the sense in which such portfolios are optimal; they provide higher yields than the FCMC.

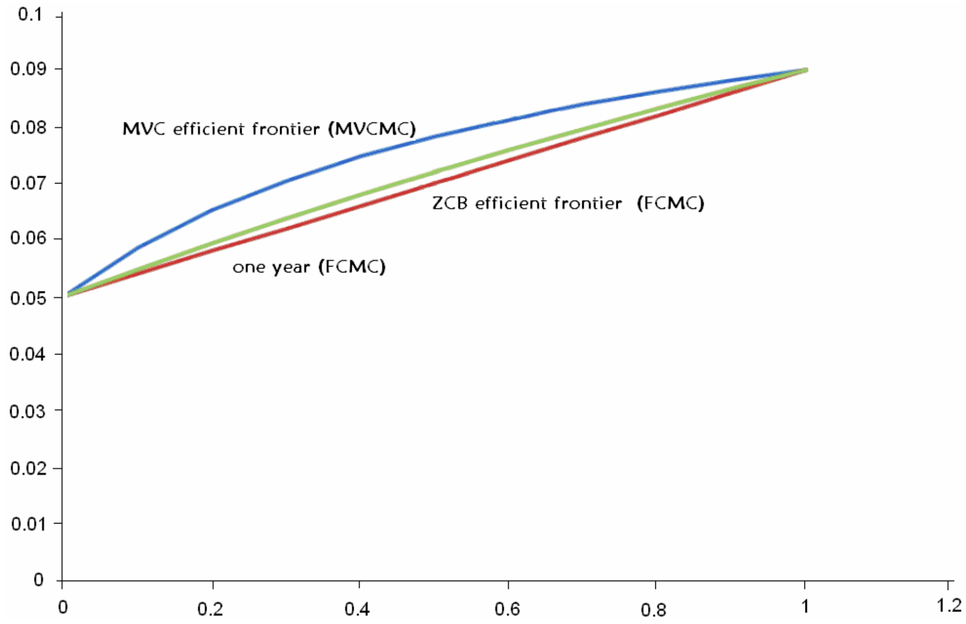


Fig. 3 The *n*-year FCMC for a ZCB, yielding 5 per cent per annum, and a risk-free security, yielding 5 per cent per annum with cash flows before end-of-term (the MVC efficient frontier or MVCMC).

3. The pension separation theorem

An income stream consists of non-negative cash flows $C_0, C_1, \cdots C_m$ which occur at times $t_0 = 0 < t_1 < \cdots < t_{m-1} < t_m = n$ years. At least one of the $C_1, C_2, \cdots C_{m-1}$ is strictly positive.

A n-year pension is generated from lump sum L by:

- purchasing the cash flows at time $t = 0$ with lump sum $(1 - f)L$, $(0 \leq f \leq 1)$ at risk-free yield i per annum; and
- investing the remaining $f\,L$ for n years in the market portfolio which has expected annual return j ($> i$) and expected n-year accumulation $(1 + j)n$. Annual returns on the market portfolio are assumed to be uncorrelated.

Then for every $f \varepsilon (0,1)$ the yield curve $r = r(f)$ for this strategy lies above the corresponding yield curve $\mu = \mu(f)$ for the FCMC, i.e., the yield curve:

$$(1+\mu)^n = (1-f)(1+i)^n + f \times (1+j)^n \text{ or}$$

$$\mu(f) = \left\{(1-f)(1+i)^n + f \times (1+j)^n\right\}^{1/n} - 1$$

arising when proportion $(1 - f)$ of each dollar is invested at $t = 0$ in an n-year ZCB at riskless annual yield i.

The curves coincide at the endpoints, where $f = 0, f = 1$.

Proof. See Gay (2010).

4. Preservation of capital in the pension phase

Drawdown from a managed fund is an income stream method favored by lump sum retirees wishing to retain control of capital. How likely is it that a self-funding retiree will preserve sufficient capital to provide stable inflation-adjusted income throughout residual life? This question is tackled via an example and simulation set in an Australian investment and retirement context.

Analogous studies have been carried out for DC funds in their accumulation phase. Blake, Cairns and Dowd (2001) used stochastic simulation to compare end-of-term accumulation under a number of investment strategies across six asset classes over the forty years of contributions of a salaried UK worker. One of their main conclusions was that over the long investment horizon, a static asset allocation strategy with high equity weightings outperformed any of the dynamic (switching, rebalancing) strategies that they investigated. The significance of this is that use of “lifestyle strategies”, the cornerstone of many DC plans, in which high equity holdings are gradually replaced by bonds and cash as fund members near retirement, is contra-indicated.

More recent research by Basu and Drew (2009) has substantially reinforced this finding in Australian markets. They concluded that by switching to conservative assets in the later years of a DC plan, lifecycle strategies sacrifice significant growth opportunity and prove counterproductive to the participant’s wealth accumulation objective. They concluded that this sacrifice does not seem to be compensated adequately in terms of reducing the risk of potentially adverse outcomes to which portfolios with high equity weightings may be subject.

4.1. Modeling sharemarket returns. Extensive econophysics research has shown that stock index returns have heavy tails when returns are calculated over periods of up to four days (Gopikrishnan et al., 1999). For individual stocks the period is about sixteen days (Plerou et al., 1999). Further these short-term returns have long-range dependence and intermittency properties. Over longer periods, stock and stock index returns exhibit “aggregational Gaussianity” – the returns progressively assume properties of normal variates – and are either uncorrelated or autocorrelations are insignificant (Cont, 2001). Thus, a reasonable assumption about annual index returns is that they are independently and normally distributed. This fact is used as a basis for simulation investigation of RIS portfolios in this study.

4.2. Data sets used for modeling and stress testing. Two sets of data have been used:

1. Data provided by AXA Australia. The long-term expected average for the Australian all ordinaries accumulation index on the Australian stock exchange (ASX) is in excess of 13.3 per cent with a standard deviation of 17.6 per cent (http://www.crcfs.com.au/uploads/file/109 20years 20of 20All 20Ords 20returns.pdf-accessed 19/03/2010).

This annual returns data (depicted in Figure 1 in the Appendix) exhibits increasing volatility especially over the last four decades. Its normality over the entire 109-year history is rejected by a simple chi-square test with eight classes.

The AXA data is used to stress-test rolling interval returns in the sequel.

2. Vanguard Australia data. Vanguard’s web site lists ASX annual returns since 1971 with an average of 12.2 per cent and a standard deviation of 20.7 per cent (annual returns need to be downloaded individually from this site). A chi-square test with five classes does not reject the normality of this data (http://www.vanguard.com.au/personal_investors/knowledge-centre/indexing/en/interactive-index-chart.cfm-accessed 16/11/2010).

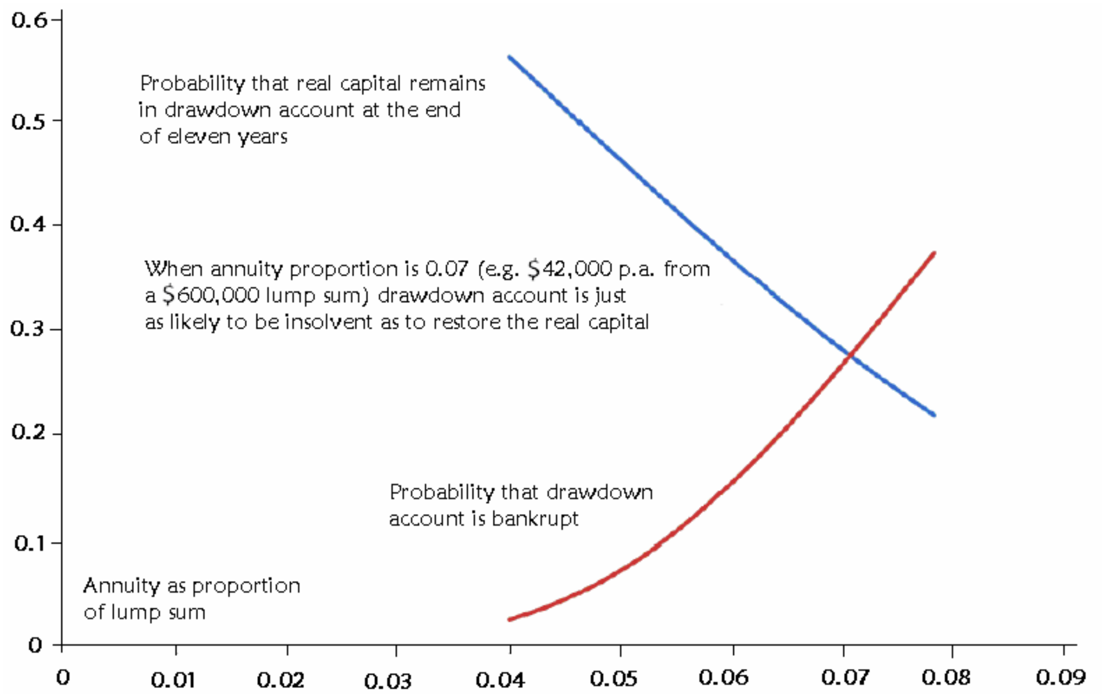
Thus, when a normality assumption for annual returns is needed, the Vanguard data (mean and standard deviation) is used.

5. Case study: an Australian self-funding retiree

A retiree aged of 65, a homeowner with no substantial outstanding debt, has a lump sum of $600000 with which to provide for retirement.

The national bourse accumulation index (the all ordinaries index) is supposed to provide long-term annual returns which are approximately normal with mean 12.2 per cent and standard deviation 20.7 per cent derived from the Vanguard reference above. Inflation is expected to average 3 per cent per annum long term. This is the upper limit of the Reserve Bank of Australia's inflation target. The retiree intends to take an annual pension which is chosen to commence in the range $24,000 to $47,000. Once chosen, the selected annual income is indexed at 3 per cent per annum at the start of each new year and is payed monthly. What are the retiree's prospects of surviving on this drawdown pension long term? Evidently the more modest the pension, the better the chances that the lump sum will provide income for the residual life of the retiree.

5.1. Capital preservation under drawdown. In the Figure 4 the upper curve depicts the probability that at the end of an eleven year period the indexed lump sum ($\$600000 \times (1.03)^{11} = \$830540$) or more, remains in the drawdown account when varying amounts of annual starting pension are taken from the account. The lower curve represents the probability that the drawdown account becomes insolvent over the eleven year term. Each data point is based on a simulation sample of end-of-term accumulation after drawdown, of one million eleven-year terms. For this calculation, the following approximation is used. Over any year of the eleven-year term, the previous year's account balance less half the indexed annual pension of that year is exposed to market risk, then reduced by the indexed annual pension consumed that year. This is analogous to methodology of both simulation papers mentioned above.



Note: The reinstatement probability rate of decrease against starting indexed pension, is virtually linear until it reaches insolvency probability curve. The probability that the original real capital (at least) is intact in the fund equals the probability of account insolvency when annual indexed pension starts at 7% of the lump sum ($42000 for a $600000 lump sum). For capital reinstatement, annual market index returns are assumed to be uncorrelated with mean 12.2 per cent and standard deviation 20.7 per cent.

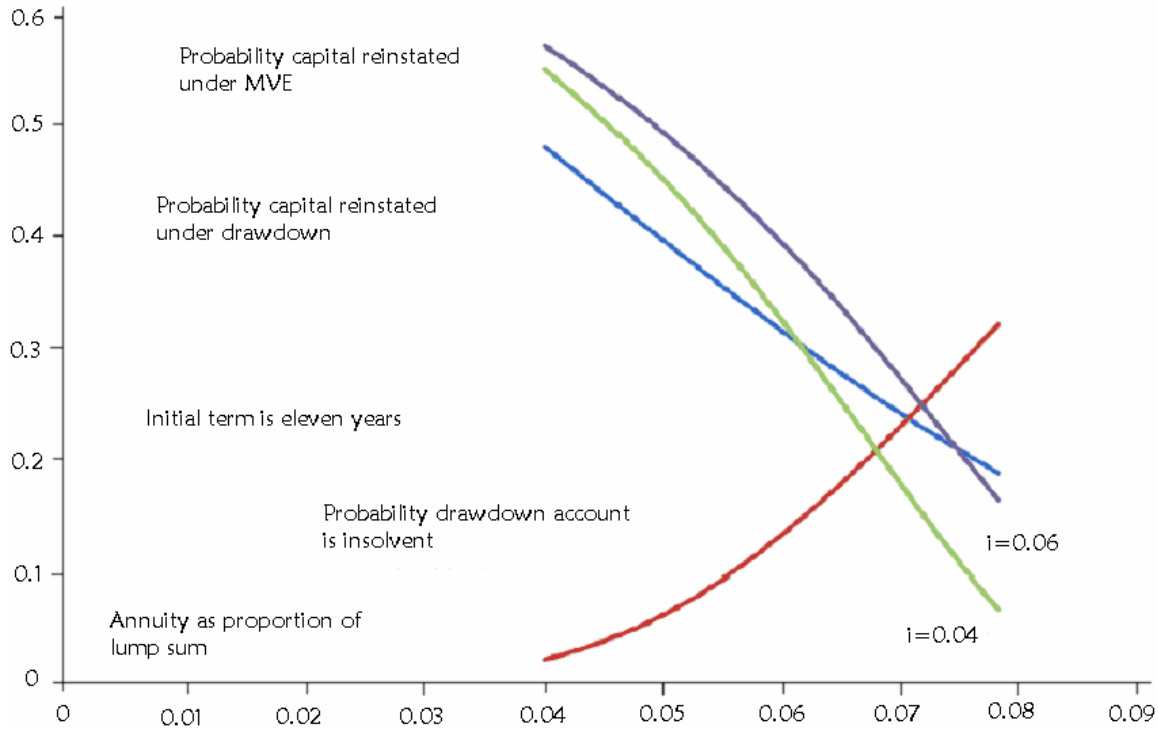
Fig. 4. Probability of reinstating real capital (upper curve) versus probability of bankrupting the account during drawdown over an eleven-year term

To calculate the probability of insolvency, these returns are further assumed to be normally distributed.

5.2. Capital preservation using MVE. If instead of using drawdown, a MVE strategy is employed, the retirement lump sum is split into an income component and an investment component. For a retiree with $600000 able to purchase an eleven-year annuity at a yield of 4 per cent per annum it would cost $10.26635 per starting dollar of indexed annuity and $246392 for an 11-year annuity of $24000 per annum payable monthly, indexed at 3 percent per annum (see Appendix). Based on a large simulation sample of eleven-year terms, the remaining $353608 invested in the index will replace the real capital ($830540) in about 64 percent of cases.

If the eleven-year indexed annuity can be purchased at a yield of 6 per cent per annum, the annuity cost is reduced to $9.27239 per starting dollar, total cost is $222537 for $24000 per annum leaving $377463 to invest in the index. The proportion of such eleven-year terms, in which the indexed capital is replaced, increases to about 66 per cent.

In Figure 5, the probabilities of capital reinstatement, using an MVE strategy, are superimposed on the drawdown probabilities depicted in Figure 4.



Note: Probability of capital reinstatement under drawdown (the decreasing straight line) and under two annuity/market investment strategies which are optimal under the mean-variance criterion. If annuities can be purchased at 4 per cent per annum then drawdown is preferable to MVE only if starting annual indexed pension exceeds \$37000 (= 0.06167 × \$600,000 – the proportion 0.06167 of the lump sum – plotted on the horizontal axis) That is, the MVE capital reinstatement probability curve is above the decreasing straight line until annual pension taken reaches \$37000.

Fig. 5. Preservation of capital under drawdown and MVE

If annuities can be purchased at a yield of 6 percent, drawdown is only preferable to MVE if annual indexed pension exceeds proportion 0.075 of the lump sum (\$45000 in the case of a \$600000 lump sum). The lower upward-seeking curve is the probability of bankruptcy and applies only to drawdown. The MVE portfolios cannot become insolvent because they are not drawn upon; income derives from the annuity.

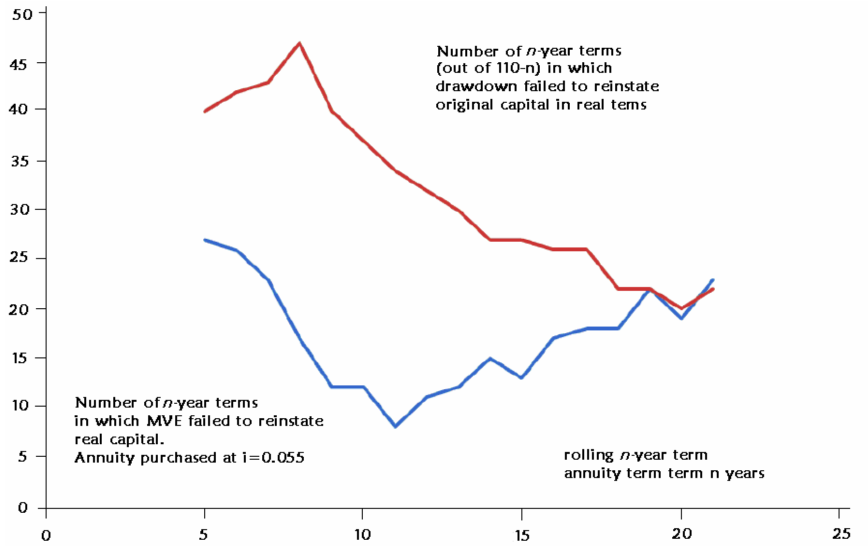
5.3. Choice of term for MVE annuities. If an MVE strategy is undertaken, which annuity term should be chosen? The investment horizon needs to be long enough to smooth out market volatility, and enable the portfolio to achieve something like the long term expected market return.

However, the longer is the term the more expensive is the annuity. There will be less residual to invest in the market. Is there an optimal MVE annuity term if other annuity parameters have been fixed? Australian retirees over the age of 65 must consume at least 5 percent of their lump sum in order to attract certain tax concessions on income and fund earnings.

Figure 6 depicts the effect of investing for varying annuity terms using 109 years of all ordinaries returns, assuming a starting annual annuity of 5 percent (\$30000 for a retiree with \$600000 account balance at the start of the financial year). It is assumed that the annuity can be purchased at a yield of 5.5 percent. This is the current yield on ten-year commonwealth government bonds.

The lower polygonal line shows the number of reinstatement failures under MVE when an n-year annuity is chosen and the strategy is implemented over (110-n) rolling n-year terms. That is, the number of times the residual lump sum invested in the market index fund failed to reinstate the entire original capital indexed at 3 percent per annum.

This is a minimum when term n is eleven years. This stochastic minimization indicates that terms between 9 and 14 years would be acceptable. The upper line depicts the number of reinstatement failures when taking the indexed pension using drawdown over n years.



Note: Over 109 years of ASX returns, there are 99 possible periods of eleven years of rolling returns. Use of MVE would have only failed to restore the entire real capital in eight of these periods. It is assumed that the eleven-year indexed annuity can be purchased at a yield of $i = 0.055$ (5.5 percent per annum) and a starting annuity equal to five percent of the lump sum is consumed.

Fig. 6. Failure of retirement strategy to reinstate real capital from 109 years of ASX data

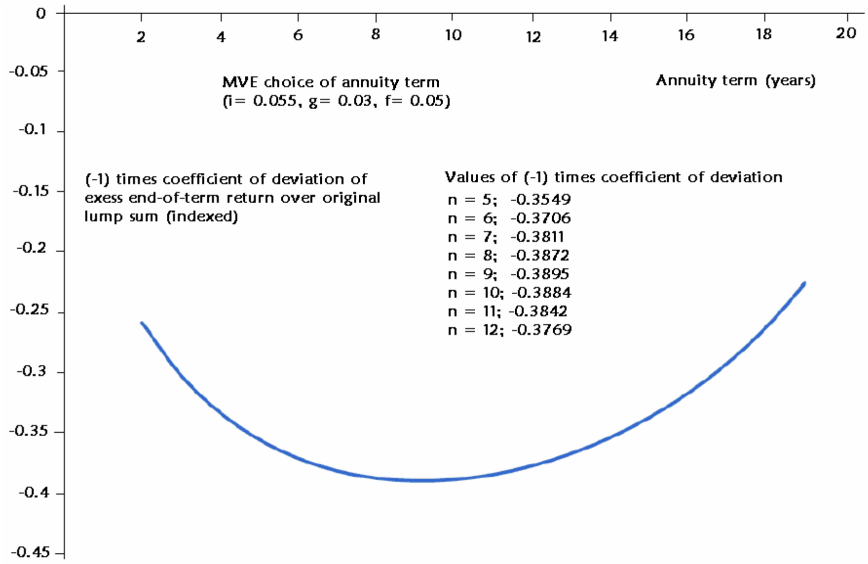
Table 1. Severity of MVE capital reinstatement failures

Year	Shortfall (dollars)	Shortfall (percent)
1939	32316	3.9
1945	13265	1.6
1948	14473	1.7
1974	333497	40.2
1975	70987	8.5
1978	28271	3.4
1979	6877	0.8
2008	178689	21.5

Note: Actual and percentage shortfall below the eleven-year indexed capital ($600000 \times (1.03)^{11} = \$830540$) and the end-of-period years in which the shortfall occurred. It would seem that only the period, ending in 1974, would have derailed the MVE strategy (certainly) and the period ending in 2008 (probably).

Table 1 shows the actual and percentage shortfall below the indexed capital and the eight end-of-term years in which this occurred.

Analytic optimization reinforces these results. A criterion by which the annuity term can be selected is to minimize (-1) times the coefficient of deviation of excess return above the indexed lump sum. Figure 7 indicates that when the annuity can be purchased at a yield of 5.5 percent per annum and an indexed annuity starting at five percent of the lump sum is consumed, the coefficient of deviation is quite flat near the absolute minimum of $n = 9$ years. This suggests that any annuity term in the range seven to eleven years is acceptable.



Note: If annual market returns are approximately normal, the probability of reinstating real capital is maximized if (-1) times coefficient of deviation of excess return is a minimum. The graph shows the minimum to be quite flat in the vicinity of $n = 9$ (the theoretical minimum).

Fig. 7. Theoretical answer to selection of optimal MVE annuity term

6. Government infrastructure and MVE portfolios

For retirees interested in retaining control of capital, MVE portfolios have a lot to recommend them, particularly if government is prepared to issue and create a market in rolling indexed annuity bonds. Assuming the bonds can be purchased at acceptable yields, MVE portfolios provide:

- a guaranteed indexed income stream in medium term;
- access to emergency capital at any time in the index fund and via the annuity bond market
- the highest expected return for any given level of risk (i.e., for amount selected for investment in the index);
- high probability of reinstating real capital in the index fund at the end of the guaranteed income years if prudent income is consumed, in particular, a better chance of this, than by taking pension by drawdown. For further detail see Gay and Duns (2009).

6.1. Economic efficiency of MVE portfolios. Since MVE portfolios deliver the highest expected return for any given level of risk, they are economically efficient in that retirees obtain the biggest bang for their retirement dollar within their comfort zone. This feature very much aligns them with government objectives. In the wake of the GFC, governments are generally issuing more public debt to pay for stimulus packages. Appropriately structured bonds issued as a small part of public debt funding, in lieu of a fraction of the coupon bond issue, could be of considerable service to DC retirees.

Longevity risk presents funding problems for retirees; it presents different challenges for governments. But provision of appropriate pension phase infrastructure for lump sum retirees would seem to present governments with a unique opportunity to deliver economically sound policy with obvious political and social benefits. There is little doubt that an innovation of this nature would attract considerable antipathy from sectors of the pension industry. Notwithstanding, it has the potential to furnish manifest benefits to DC retirees, and should certainly be subjected to the wider scrutiny of industry participants and pension academics.

Conclusion

A pension separation theorem similar to the classic Tobin theorem can be established for retirement income stream provision. MVE strategies involve splitting a retirement lump sum into a risk-free income component, and a pure investment component. The theorem identifies simple and effective strategies for lump sum conversion which fall between account-based drawdown and full annuitization.

Given prudent pension consumption, MVE strategies largely preserve capital and so are an effective hedge against longevity risk. They also help manage investment, inflation and liquidity risks. They are more likely to reinstate the entire original lump sum in real terms at the end of guaranteed income years, than account-based drawdown favoured by many retirees. If, however, pension consumption is heavy, reinstatement of initial capital is more likely under account based drawdown. But with heavy drawdown bankrupting the account is a more probable outcome.

The existence of an economically efficient method of lump sum conversion suggests a government role for support of lump sum retirees in the pension phase.

If governments issued part of public debt in the form of rolling annuity bonds or indexed annuity bonds of medium term, lump sum retirees could use these bonds to provide themselves with pension income, while investing in a national bourse accumulation index fund to replenish the capital used to purchase the annuity.

References

1. The Association of Superannuation funds of Australia Ltd., www.superannuation.asn.au/Super-Statistics/default.aspx (accessed 16/11/10).
2. Australian Government Publications: Review into the governance efficiency structure and operation of Australia's Superannuation System. A statistical summary of self-managed superannuation funds. Available at: http://www.supersystemreview.gov.au/content/downloads/statistical_summary_smsf/SMSF_statistical_summary_report.pdf (accessed 16/11/10).
3. Basu, Anup K. and Drew, Michael E. (2009). Portfolio size effect in retirement accounts: what does it imply for lifecycle asset allocation funds? *Journal of Portfolio Management*, 35, No. 3, pp. 61-72.
4. Blake, David. Cairns, Andrew J.G. and Dowd, Kevin (2001). Pensionmetrics: stochastic pension plan design and value-at-risk during the accumulation phase, *Insurance, Mathematics and Economics*, 29, pp. 187-215.
5. Bayraktar, Erhan and Young, Virginia, R. (2008). Mutual fund theorems when minimizing the probability of lifetime ruin", *Finance Research Letters*, 5, pp. 69-78.
6. Blake, D. and Hudson, R. (2000). Improving security and flexibility in retirement: full technical report, Retirement Income Working Party, Pensions Institute, Birbeck College, London, March.
7. Clare, Ross (2007). Coping with the burden of longevity – do superannuation fund members really need or want new financial products? Paper presented to 15th Annual Colloquium of Superannuation Researchers,

July, 2007 held at University of New South Wales Centre for Pensions and Superannuation. Available at: http://wwwdocs.fce.unsw.edu.au/fce/Research/CLARE_Ross.pdf (accessed 16/11/2010).
8. Cont, Rama (2001). Empirical properties of asset returns: stylized facts and statistical issues, *Quantitative Finance*, 1, pp. 223-236.
9. Gay, Roger (2010). The pension separation theorem, Website paper: Available at: www.rogergaysuper.com (click on title).
10. Gay, Roger and Duns, John (2009). Completing the pensions task: infrastructure for nationally coordinated private schemes, *Journal of Economic and Social Policy*, 13, Issue 1, Article 4.
11. Glickman, Mark. and Kuehneman Jr. Gene (2006). Retiree pension payout decisions – evidence from the health and retirement study, 1992-2002, US Government Accountability Office Working Paper Series. Available at: http://www.aeaweb.org/annual_mtg_papers/2006/0108_0800_0401.pdf (accessed 16/11/10).
12. Gopikrishnan, P. Plerou, V. Amaral, L.A.N. Meyer, M. and Stanley, H.E. (1999). Scaling of the distribution of fluctuations of financial market indices, *Physical Review E*, 60, pp. 5305-5316.
13. Markowitz, Harry M. (2007). Investors do not get paid for bearing risk", published in The Fundamental Index: A Better Way To Invest, Arnott, R.D. and Hsu, J.C. Available at http://www.researchaffiliates.com/ideas/pdf/bearing_risk.pdf (accessed 16/11/10).
14. Merton, Robert C. (1969). Lifetime portfolio selection under uncertainty: the continuous-time case, *Review of Economics and Statistics*, 51, pp.247-257.
15. Merton, Robert C. (1971). Optimal consumption and portfolio rules in a continuous time model, *Journal of Economic Theory*, 3, pp. 373-413.
16. Milevsky, M. and Young, V. (2001). The real option to delay annuitization: its not now-or-never, *York-Schulich-Finance*, Working Paper, No. MM11-1, November 2001.
17. Mitchell, Olivia.S. and Utkus, Stephen P. (2003). Lessons from behavioral finance for retirement plan design, Pension Research Council Working Paper, 2003-2006, The Wharton School, University of Pennsylvania.
18. Pratelli, Maurizio (2005). Generalizations of Merton's mutual fund theorem in infinite dimensional financial models, *Progress in Probability*, 59, pp. 507-519.
19. Plerou, V. Gopikrishnan, P. Amaral, L.A.N. Meyer, M. and Stanley, H.E. (1999). Scaling of the distribution of fluctuations of individual companies, *Physical Review E*, 60, pp. 6519-6529.
20. Sethi, Suresh P. (1995). Optimal consumption-investment decisions allowing for bankruptcy: a survey, Social Science Research Network publication. Available at http://ssrn.com/abstract=1327996.
21. Tobin, James (1958). Liquidity preferences as behavior towards risk, *The Review of Economic Studies*, 67, pp. 65-86.
22. Trott, Matt (2005). Various options to choose from, Pensions Management, June, 2005. Available at: http://www.pensionsmanagement.co.uk/news/fullstory.php/aid/2016/Matt_Trott.html (accessed 16/11/10).
23. Updegrave, Walter (2006). Income for life? Lump sum? A better option, Money Magazine, October 9, 2006, Available at: http://money.cnn.com/2006/10/09/pf/expert/expert.moneymag.moneymag/index.htm (accessed 16/11/10).
24. Yaari, M. (1965). Uncertain lifetime, life insurance and the theory of the consumer, *Review of Economic Studies*, 32 (2), pp.137-150.

Appendix

1. 109 years of ASX all ordinaries annual returns

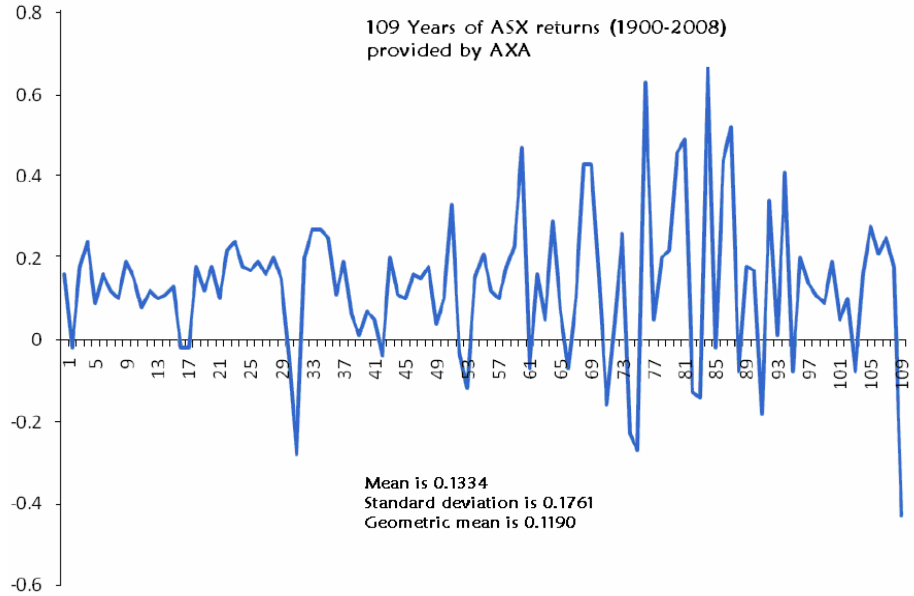


Fig. 1. ASX accumulation index annual returns 1900-2008 (data supplied by AXA) showing stochastic volatility of returns

2. Present value of an annually indexed annuity certain payable pthly

The cost of \$1.00 of annuity for n-years indexed at a rate g at the start of each new year, purchased at a yield r when the \$1.00 is payable pthly (i.e., each annual dollar is divided into 1/p and paid regularly in arrears (at p intervals of $1/p$ years) is from standard mathematics of finance:

$$a_{n:r:g}^{(p)} = \left(r / r^{(p)}\right) \times \left\{1 - \left(\frac{1+g}{1+r}\right)^{n}\right\} / (r - g),$$

here $r^{(p)} = p\left\{(1+r)^{1/p} - 1\right\}$,

when $n = 11$, $r = 0.04$, $g = 0.03$, $p = 12$ the cost is \$10.26635. Total annuity cost is, thus, 24000 × \$10.26635 = \$246,392. When $n = 11$, $r = 0.06$, $g = 0.03$, $p = 12$ the cost is \$9.27239.