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Extension of break-even analysis for payment default prediction: evidence from small firms

Abstract

Break-even analysis (BEA) is widely used as a management method to analyze the relationship between the sales volume and the profit of the firm. In this paper BEA is extended and applied for payment default prediction. First, break-even point is defined as the point, firstly, at which the net profit is zero and, secondly, at which the loss makes the equity critical. Second, the margins of safety (MOSs) are derived for both the sales volume and the cost of debt. Three hypotheses for prediction ability of MOSs are drawn. MOSs are used as predictors of payment default in a logistic regression model (LRM) in a sample of Finnish small firms. It was found that MOS for net sales drawn for the critical equity target is a very powerful predictor of payment default. The prediction ability of traditional financial ratios and also of familiar non-financial predictors was outperformed by the margin.

Keywords: break-even analysis, margin of safety, financial distress, default prediction.
JEL Classification: G30, G32, M40, M41.

Introduction

Financial distress may cause large economic and social losses for each stakeholder of the firm. Therefore, financial distress prediction has played an important role in financial research over many decades (Jones & Hensher, 2004; Altman & Hotchkiss, 2006; Balcaen & Ooghe, 2006; Lensberg et al., 2006). Prediction models can be applied for example by managers of distressed firms, lending specialists, accounts receivable managers, investors, security analysts, auditors, bankruptcy & reorganization lawyers, and judges (Altman & Hotchkiss, 2006, pp. 281-296). The prediction models may include both financial and non-financial variables (Laitinen, 1999; Back, 2005). However, the traditional models are based on financial ratios extracted from annual financial statements, income statement and balance sheet. The main problem in this traditional approach is that the predictors are drawn purely empirical grounds without any reference to the theory (Balcaen & Ooghe, 2006). The objective of this study is to introduce a new financial measure for distress prediction originally based on the financial theory for the break-even analysis (BEA).

BEA is a technique widely used by management accountants for at least one hundred years (Dow & Johnson, 1969). Henry Hess (1903) introduced the technique for managerial use as a "costs, receipts, and profits" chart. In its basic form, it is based on classifying costs into fixed and variable parts, and solving for the point at which costs equal revenues so that profit is zero (point of crisis, point of profit, profitless point). However, in the passage of time it has widely diffused and developed to play an important role in accounting and economic theory (Dow & Johnson, 1969, p. 30): "Ultimately, no assessment of the contributions of the break-even concept can be complete without recognition both of its role in the development of accounting theory and of its potential for applications to broader areas of economic and political life." It is clear that BEA is originally developed for simplified situations for linear total revenue and total production cost curves to facilitate short-term decision making under certainty (Weiser, 1969; Stettler, 1962). Since that, BEA has been expanded for curvilinear analysis (Goggans, 1965; Givens, 1966), uncertainty (Jaedicke & Robichek, 1964; Adar, Barnea & Lev, 1977), long-term decision making (Michell, 1969), and many additional extensions. It has also extended to take account of income taxes (Morse & Posey, 1969).

The purpose of this study is to extend the original version of BEA to take account of characteristics which are important for financial distress prediction, especially in small firm samples. The importance of small firms and their financial performance is very high in most countries. However, scientific research on small business distress prediction is scarce that is partly due to the poor quality of financial statement information impairing the prediction ability of traditional ratios (Balcaen & Ooghe, 2006). Thus, the purpose is to develop especially for small business distresses a new financial predictor performing better than traditional ratios and even non-financial (qualitative) variables (Keasey & Watson, 1987; Balcaen & Ooghe, 2006). In distress prediction models, indebtedness and liquidity ratios often play the main role (Karels & Prakash, 1987) whereas the role of profitability is minor in importance (Ohlson, 1980; Zavgren, 1985; Zavgren & Friedman, 1988). In its original form, BEA is concentrated on the risk associated with profitability. In order to be useful in distress prediction, it should be extended to pay attention also to financial risk. The purpose of this study is to introduce such an extension.

This study is a part of a project (Grant No 126630) financed by Academy of Finland.

The main innovation of the present approach is to solve the point of profit at which the equity of the firm declines to the critical level as measured by the critical value of the equity ratio. The difference between this critical point (measured as net sales and as cost of debt) and the current point (current state) will form a margin of safety (MOS). The percentage MOS is applied as a predictor of payment default and tested against traditional financial ratios. In addition, eight non-financial predictors are used for control purposes. The first research hypothesis assumes that MOSs based on the extended BEA will be efficient predictors of financial distress when compared with traditional financial ratios, especially in a small business sample. The second hypothesis assumes that MOS based on zero-profit target is outperformed by that based on the critical equity. Thirdly, the last hypothesis proposes that MOS measured as cost of debt will be in default prediction less efficient than MOS measured as net sales. These three hypotheses will be tested by the logistic regression analysis (LRA) in a sample of Finnish default and non-default firms. The estimation sample includes 328 payment default firms and 1358 non-default firms. The results are validated in a holdout sample.

The organization of the paper is as follows. The motivation and purpose of the study were outlined in the first introductory section. Section 1 presents the extended BEA and derives MOS for both the net sales and the cost of debt. In this section, the suitability of the extended BEA for financial distress analysis is discussed and the research hypotheses are drawn. Section 2 briefly presents the data and statistical methods of the study while section 3 shows the empirical results. In this section, it will be shown that MOS is an efficient predictor in default prediction. It is also efficient when non-financial predictors are included in the model. These results support the research hypotheses. They suggest that MOS based on an extended BEA is an important innovation in financial statement analysis of financially distressed firms. This result holds especially for very small firms. The last section summarizes the study, discusses its limitations and outlines trends for future research.

1. Extension of BEA for distress analysis

1.1. Traditional BEA. The benefits of BEA are originated in its obvious simplicity and relevance. It is based on an assumption that costs can be classified as fixed and variable. Variable costs are directly related to sales volume so that the unit variable cost is assumed constant. Fixed costs (capacity costs) are dependent on the capacity and thus independent of sales volume. The profit of the firm is calculated as follows:

$$\textit{Selling price} \times \textit{Sales volume}\ (\textit{Net sales}) -$$

$$- \textit{Variable unit cost} \times \textit{Sales Volume}\ (\textit{Variable costs}) =$$

$$= \textit{Sales margin}\ (\textit{Contribution margin}) -$$

$$- \textit{Fixed costs} = \textit{Profit}\ (\textit{before interest and taxes}).$$

If the target profit is set equal to zero (break-even), it is attained by the selling volume that is resulted when dividing fixed costs by the unit contribution margin (difference between selling price and unit variable cost). The firm has to sell this critical volume in order to cover fixed costs by its contribution margin.

The difference between the current sales volume and the critical sales volume is called MOS. MOS can be expressed in absolute selling volume, in money (net sales), or in percent. The less the firm has fixed costs and the higher the contribution margin is, the less is critical sales volume, and the larger is MOS. MOS reflects the degree of freedom (slack) for the firm, its safety buffer for losses. If MOS is large, moderate fluctuations in sales volume (demand) do not expose the profitability of the firm. The firm can reduce the profitability risk (enlarge MOS) by increasing contribution margin or lowering fixed costs.

1.2. Extension of BEA. The traditional BEA is concentrated on profitability analysis. In financial distress prediction, profitability may play a minor role while solvency is of importance (Ohlson, 1980; Zavgren, 1985; Zavgren & Friedman, 1988). However, BEA can be easily extended to take account of solvency firstly by including cost of debt in the fixed costs. This means that the critical sales volume is the volume that leads to the contribution margin which covers both traditional fixed costs and the cost of debt (interest cost). In this version of BEA, it is not necessary to pay attention to taxes, since income taxes can be assumed zero because the taxable (target) profit after cost of debt is set equal to zero. This assumption simplifies BEA remarkably. However, if it is the purpose to apply BEA for distress analysis, solvency must be taken into account more carefully. It may mean that also income taxes have relevance for BEA (Morse & Posey, 1969).

The further extension of BEA will take account of solvency in an explicit way. In this extension, the target (critical) profit is set equal to the net profit (loss) that will diminish the equity of the firm to a critical level. This critical level of equity can, for example be zero or calculated on the basis of the critical value of the equity ratio (say, 10%). If the solvency is good, the firm is able to suffer from even a large loss without exposing its ability to pay financial obligations as they mature. It is the level of solvency that mainly determines the ability of the firm to get additional outside finance in financial distress. If the solvency is critical, the firm usually cannot get additional finance. If the current solvency is already below the critical level, the target profit must be (positive and) large enough to increase it back to this level. In this situation, taxes are relevant for the extension. In addition, it should take account of dividends paid to shareholders.

Table 1 shows the statement that begins from net sales and ends with retained earnings (after dividends) which contribute to the equity. The statement makes it possible to calculate the critical sales volume which makes the net profit zero (extension 1) or equal to a value (a loss) that leads the equity to a critical level (extension 2). Table 2 shows the mathematical solutions for the critical sales volume in these extensions. If the target net profit is set equal to zero (extension 1), the solution is independent of income taxes and dividends. However, when the target profit is set on the basis of the critical equity (extension 2), the solution depends on dividends and may be affected by taxes. Thus, in the latter extension the solution is more complicated.

Table 1. Concepts of income statement

Net sales = Selling price × Sales volume	$p \cdot q$
– Variable cost	$- v \cdot q$
= Sales margin (Contribution margin)	$= (p - v) \cdot q$
– Fixed cost	$- F$
– Depreciations	$- P$
= Profit before interest and taxes	$= (p - v)\, q - F - P$
– Interest cost	$- i \cdot DEBT$
– Income taxes	$- f \cdot [(p - v) \cdot q - F - P - i \cdot DEBT]$
= Net profit	$= [(p - v) \cdot q - F - P - i \cdot DEBT] \cdot (1 - f)$
– Dividends	$- d \cdot EQUITY$
= Retained earnings	$= [(p - v)\, q - F - P - i \cdot DEBT] \cdot (1 - f) - d \cdot EQUITY$

Table 2. Margins of safety (MOSs) for sales volume

Extension 1: Net profit = 0 (target)	*Critical sales volume* = $q^* = \dfrac{[F + P + i \cdot VPO]}{(p - v)}$
Extension 2: Equity ratio = b (target)	*Critical equity* = $EQUITY^* = \dfrac{DEBTb}{(1 - b)}$ $q^* = \dfrac{[F + P + i \cdot DEBT] \cdot (1 - f) + d \cdot EQUITY - (EQUITY - EQUITY^*)}{(p - v) \cdot (1 - f)} = \dfrac{[F + P + i \cdot DEBT]}{(p - v)} - $ $- \dfrac{(1 - d) \cdot EQUITY - EQUITY^*}{(p - v) \cdot (1 - f)}$ *Margin of safety (MOS) in percent* = $M(q) = \dfrac{100 \cdot [q - q^*]}{q}$

The extensions of BEA can be used to analyze distressed firms, since they reflect the ability of the firm to safely react to negative fluctuations in sales volume (demand). If MOS is high, the profitability risk (that the net profit is zero) or the solidity risk (that the solidity in terms of the equity ratio falls to the critical level) associated with firm is low. The extensions of BEA are expected to be useful as predictors of financial distress, because they behave in different way as compared with traditional financial ratios. Therefore, they are expected to be efficient predictors and bring incremental information over those ratios (Hypothesis 1). It is expected that extension 1 is useful especially when predicting profitability crisis. However, it is extension 2 that is expected to be more efficient in predicting financial distress as in this study (Hypothesis 2).

Hypothesis 2 can be theoretically justified, since extension 2 (based on the critical equity) is closely related to the probabilistic theory of bankruptcy (Scott, 1981). Wilcox (1971; 1973; 1976) and Santomero & Vinso (1977) developed a bankruptcy theory based on the gambler's ruin model. Scott (1981) developed the theory further and showed that the probability of failure is an explicit function of the expected value and the standard deviation of the change in retained earnings (net income minus dividends), and the current value of equity, all divided by total assets. Thus, this kind of approach suggests that the profitability together with its volatility and the equity ratio are important predictors of bankruptcy. Extension 2 assumes that the volatility of profitability is directly related to fluctuations in sales volume. Thus, MOS is related to the probability of bankruptcy.

1.3. Critical cost of capital. The traditional BEA is based on the solution of the critical sales volume. However, the sales volume is only one factor that affects the riskiness of the firm. The statements in Table 1 make it possible to solve (in addition to sales volume) critical values, for example, for variable unit cost, price level, or cost of debt which all can be assumed probabilistic (Jaedicke & Robichek, 1964; Adar, Barnea & Lev, 1977). The choice of the factor depends on the expectation how critical the factor is for the financial risk of the firm and how probable it is that it will fluctuate in a relevant way to cause a crisis. In this study BEA is extended to analyze the critical cost of debt. This version will show how safe the profitability or the solvency of the firm is against potential fluctuations (increase) in interest rate. In this analysis, the critical cost of debt is calculated keeping all other factors constant.

Table 3 shows the solutions for the critical cost of debt for extensions 1 and 2. When the target net profit is zero (extension 1), the critical cost is got when the profit (before cost of debt) is divided by the debt. This extension is again independent of taxes and dividends. When the target profit is the value (usually loss) that leads the equity to the critical level (extension 2), the solution is more complicated, because dividends and potentially also taxes must be taken into account. In the extensions for the critical cost of debt, MOS can simply be calculated as the difference between the current cost and the critical cost. Thus, it directly shows how much the cost of debt can increase before exposing the profitability or solvency of the firm.

It is however assumed that these extensions are not as powerful in financial distress prediction as those based on the sales volume (Hypothesis 3). This hypothesis is justified because volume for financial crisis fluctuations in cost of debt may not be as critical as those in sales volume. High cost of debt can be one of the reasons for financial distress but it is the main or the only reason when fluctuations (increases) are exceptionally high. The fluctuations in cost of debt are typically moderately slow and small whereas fluctuations in demand can be very quick and large. Therefore, it is assumed that in short-term financial distress prediction MOS of sales volume (net sales) is more efficient than MOS of cost of debt. It is expected that the research hypotheses hold especially for small firms. The traditional BEA may be too simplified for large firms where the relationships are often curvilinear (Goggans, 1965; Givens, 1966).

Table 3. MOSs for cost of debt

Extension 1: Net profit = 0 (target)	*Critical cost of debt* = $i^* = \dfrac{\left[(p-v)q - F - P\right]}{DEBT}$
Extension 2: Equity ratio = *b* (target)	$i^* = \dfrac{\left[(p-v)q - F - P\right]\cdot(1-f) - d\cdot EQUITY + (EQUITY - EQUITY^*)}{DEBT\cdot(1-f)} = \dfrac{\left[(p-v)q - F - P\right]}{DEBT} + $ $+\dfrac{(1-d)\cdot EQUITY - EQUITY^*}{DEBT\cdot(1-f)}$ *Margin of safety (MOS) in percent* = $M(i) = i^* - i$

2. Data and statistical methods

2.1. Data of the study. The hypotheses on the prediction ability of MOSs derived for the extended BEA will be tested in a sample of payment default and non-default firms. In this sample, financial distress is reflected by officially registered payment defaults. The data include a random sample of Finnish firms which have published annual financial statements in accounting years 2000-2003. These data are obtained from Suomen Asiakastieto Oy (http://www.asiakastieto.fi) for research purposes. The prediction models will be estimated using the last financial statements published before the default date. The payment default to be predicted has emerged after the end of 2003 but before April 30, 2005 (event period of 16 months). The original data include about 400 default and 1700 non-default firms. For validation purposes, 80% of the sample was included in the estimation sample while 20% was left in the holdout sample. In addition, the firm-year observations of the original sample which are not used in estimation are included in the holdout sample to get a more general insight of the prediction ability. Thus, the estimation sample includes 328 default and 1358 non-default firms while the holdout sample respectively includes 635 and 3412 firm-year observations. The median period from the date of annual closing of accounts to the default is (in the estimation sample) 409 days while the upper quartile is 599 days.

In Finland, the most common types of payment default are private-judicial draft protest published or unpublished, unaccounted tax withholdings and value-added tax installments published by the tax authorities, insolvency or other impediment stated in connection of execution proceedings, and judgment by default on demand for payments. More than 40% of the payment defaults in Finland are private-judicial draft protests. Thus, the types of default are heterogeneous and often not as serious as bankruptcies. Therefore, the heterogeneity of defaults makes them difficult to predict for as long prediction horizon as in this study. However, also in many prior studies the concept of distress has been arbitrary and heterogeneous (Karels & Prakash, 1987; Keasey & Watson, 1991; Balcaen & Ooghe, 2006). The traditional concept used in previous studies is a juridical definition of bankruptcy (Altman, 1968). However, models have been applied to predict payment delays referring to a mild form of distress (Wilson et al., 2000). Many studies apply many criteria for distress in the same sample as in this study. Agarwal & Taffler (2008) define the failure as entry into administration, receivership, or voluntary liquidation procedures while Beaver (1966) regarded a firm as failed when any of the following events have occurred: bankruptcy, bond default, an overdrawn bank account, or non-payment of a preferred stock dividend.

The present estimation sample includes a variety of industries. The largest industrial group is consisted of retail and wholesale firms (24.7%). There are also a large number of service firms (21.9%) (real estate, renting, and business activities) and manufacturing firms (18.0%). The most significant differences in the industry between default and non-default firms are in the percent of service and manufacturing firms. The percent of service firms in non-default firms is 22.9% but only 17.7% in default firms. However, the proportion of manufacturing firms is only 17.2% for the non-default firms but 21.0% for the default firms. The sample firms represent statistically the population in Finland and are mainly very small businesses. In Finland, the number of micro firms is about 300000 making 94.5% of all firms (Statistics Finland). For the non-default firms in the sample, the median of net sales is 379 thousands of euros and for the default firms it is only 277 thousands of euros. The sample however also includes a couple of large firms making the size distribution skew.

2.2. Statistical methods and variables. In the present study, the logistic regression analysis (LRA) will be applied to estimate the prediction models for payment default. For this estimation, the dependent variable Y = 1 when the firm has experienced a payment default during the event period and Y = 0 otherwise (non-default). In general, LRA can be used to predict a dependent variable on the basis of continuous or categorical independent variables (Hosmer & Lemeshow, 1989). LRA creates a score (logit) L for every firm. L is used to determine the conditional probability to be a default firm as:

$$p(i,X) = \frac{1}{1+e^{-L}} = \frac{1}{1+e^{-(b_0+b_1x_1..+b_nx_n)}}, \quad (1)$$

where b_j (j = 0,…, n) are coefficients and n is the number of independent variables x_i (j = 1,…, n). The LR models are estimated by the maximum likelihood method in Statistical Package for the Social Sciences (SPSS). The statistical significance of the coefficients is tested by the Wald test. The strength of association is assessed by the Nagelkerke R square. The classification accuracy of the models is evaluated by the percent of Type I and II errors both in the estimation and holdout sample.

The independent financial variables in the preliminary analyses include four traditional financial ratios and four different MOS variables. Because the size distribution of the sample firms is very skew, also the logarithm of total assets is included in the analyses to take account of the size effect. In addition, critical cost-of-debt variables for two extensions are used as independent variables in the preliminary analysis. The traditional financial ratios are selected on the basis of their importance in default prediction recognized in prior studies (Balcaen & Ooghe, 2006; Lensberg et al., 2006): return on investment ratio, quick ratio, traditional cash flow to net sales ratio, and equity ratio. In Finnish financial distress studies, the equity ratio has clearly played the dominant role in default prediction models (Laitinen, 2009). The selected financial ratios measure respectively profitability, traditional liquidity, cash flow, and solidity.

The application of BEA requires that costs are classified as fixed and variable. In this sample, it is assumed that contribution margin is equal to gross profit (the difference between revenue and the cost of producing goods or services sold). The dividends are approximated as the difference between the calculated and actual retained earnings. The cost of debt is calculated as the relation between interest expenses and total debt. The effective tax rate is got when the taxes paid are divided by the calculated taxable income. MOS for the critical equity is estimated assuming that the critical equity ratio equals 10%. It is the critical value implied by Finnish failure research (Laitinen & Laitinen, 2009). In addition, it is the median value for the default firms in the present sample. If MOS was based on a positive taxable profit, a tax rate of 26% is applied.

Because of the nature of data, several additional analyses were carried out to assess the performance of MOSs. First, the LR model was estimated to predict separately bankruptcy and other payment defaults to evaluate the effect of default seriousness and heterogeneity. The arbitrary definition of distress may have serious consequences for the resulting failure prediction model (Balcaen & Ooghe, 2006, pp. 72-73). Second, the model was estimated separately for two size classes based on the median of total assets to assess the size effect. It is expected that the extension of BEA performs well especially for the smaller size class. Because of the dominance of very small firms in the sample, also a LR model including non-financial predictors was estimated for control purposes. These kinds of non-financial predictors are particularly appropriate when studying small firms which often lack reliable annual accounting information (Balcaen & Ooghe, 2006, p. 83). In this test, eight non-financial control variables were chosen on the basis of prior studies: modification of audit report (0 = non-modified report; 1 otherwise), logarithm of firm age, age of financial statements, number of board member personal defaults, number of board members, number of resigned board members (during the last 12 months), default propensity of industry, and non-corporation dummy (0 = limited company; 1 otherwise) (Keasey & Watson, 1987; Laitinen, 1999; Back, 2005; Balcaen & Ooghe, 2006).

3. The results of the study

3.1. Descriptive statistics. Table 4 presents descriptive statistics for the factors of BEA, different MOS variables, and financial ratios. The differences in the variables between default and non-default firms are tested by a median test based on Chi^2 test statistics. The non-default firms are to some degree larger than the default firms as measured by net sales. However, the differences in the balance sheet equity are statistically more significant reflecting the high leverage of default firms. The difference in the distribution of the contribution margin between the groups is not significant. The groups however differ significantly from each other with respect to cost of debt, rate of dividend, and effective tax rate. Default firms pay more for debt as interest expense but less for equity as dividends to shareholders. In fact, only about 25% of them pay dividends while more than 50% of them do not pay taxes at all.

Table 4. Descriptive statistics for the sample

	Non-default firms (n = 1358): Quartiles			Default firms (n = 328): Quartiles				
Factor	25	50	75	25	50	75	*Chi*^{2}	*p*-value
1. Factors of BEA								
Net sales (euro)	120042	379322	1253783	100267	276813	738270	7,00	0,00800
Change in net sales (%)	-0,1046	0,0273	0,1807	-0,2135	-0,0448	0,2056	15,02	0,00000
Contribution margin (%)	0,4035	0,6310	0,9019	0,4150	0,6073	0,8189	1,09	0,29600
Fixed cost (euro)	52368	153997	467307	58160	139429	376390	0,64	0,42400
Depreciations (euro)	2609	8728	34612	2538	8072	22739	0,46	0,49900
Debt (euro)	25743	104234	368234	48418	121326	285108	2,37	0,12400
Equity (euro)	24784	94210	317277	-2129	14514	55525	132,36	0,00000
Cost of debt (%)	0,0012	0,0172	0,0388	0,0230	0,0417	0,0586	110,71	0,00000
Rate of dividend (%)	0,0000	0,0754	0,1584	0,0000	0,0000	0,0011	155,98	0,00000
Effective rate of tax (%)	0,0000	0,2878	0,3036	0,0000	0,0000	0,2669	81,79	0,00000
Critical equity (*b* = 10%) (euro)	2860	11582	40915	5380	13481	31679	2,37	0,12400
2. MOS concepts								
Critical net sales (profit = 0)	102049	315194	1061514	111546	319804	858031	0,00	0,95100
MOS (profit = 0)	-31	28329	143878	-57273	-2026	21616	84,03	0,00000
MOS percent (profit = 0)	-0,0697	10,2056	24,1732	-20,4695	-1,5126	6,3142	121,28	0,00000
Critical net sales (equity = critical)	-650	126911	542056	100978	338432	750581	44,97	0,00000
MOS (equity = critical)	21272	129878	582898	-119783	-8241	51574	149,90	0,00000
MOS percent (equity = critical)	12,9700	40,9208	100,8073	-45,5325	-4,1131	17,5564	205,49	0,00000
Critical cost of debt (profit = 0)	0,0278	0,2132	0,6464	-0,2064	0,0283	0,1363	126,80	0,00000
MOS (profit = 0)	-0,0030	0,1851	0,6377	-0,2568	-0,0179	0,0831	138,13	0,00000
Critical cost of debt (equity = critical)	0,2383	0,9517	2,7203	-0,4394	0,0107	0,2915	181,59	0,00000
MOS (equity = critical)	0,2117	0,9222	2,7136	-0,4655	-0,0427	0,2539	188,29	0,00000
3. Traditional financial ratios								
Logarithm of total assets	10,9828	11,8672	12,8378	11,2860	12,3773	13,5711	24,23	0,00000
Return on investment ratio	3,8000	16,8000	33,4750	-14,3250	5,4500	19,6750	55,06	0,00000

Table 4 (cont.). Descriptive statistics for the sample

	Non-default firms (n = 1358): Quartiles			Default firms (n = 328): Quartiles				
Factor	25	50	75	25	50	75	*Chi*^{2}	*p*-value
Traditional cash flow to net sales ratio	2,9530	9,7129	20,9814	-6,5639	2,1145	8,1357	100,34	0,00000
Quick ratio	0,6000	1,2000	2,6000	0,3000	0,6000	1,0000	125,36	0,00000
Equity ratio	24,8750	48,5000	73,7000	-4,4250	10,2000	27,9750	191,62	0,00000

The table shows that the medians of the four percentage MOS variables in default firms are negative referring to a critical situation. However, the upper quartiles of the variables show a moderately satisfactory level. It is found that MOS variables based on the critical profit (0) differ significantly between the groups. However, the differences in MOS variables reflecting the critical equity (equity ratio = 10%) are more significant supporting Hypothesis 2 in the univariate analysis. In addition, MOS (critical equity) for the net sales shows more significant differences than that for the cost of debt. This result gives support to Hypothesis 3. When comparing financial ratios, the equity ratio shows (as expected) the most significant differences between the groups. However, the statistical significance of the ratio (Wald = 191,62) is not as high as for MOS (critical equity) for the net sales (Wald = 205,49). Thus, on the basis of the univariate analysis the extended MOS variables are expected to be efficient predictors in financial distress analysis supporting Hypothesis 1.

3.2. Logistic regression analysis. Stepwise analysis in Table 5 shows the results for the preliminary analysis carried out by the stepwise LRA. Panel A shows the score test for the six MOS variables and for the five traditional financial variables in the first step (step 0) of the stepwise LRA when the model only includes the constant. This test is used to predict whether or not an independent variable would be significant in the LR model. In each step, the variable with the largest score statistic (whose significance value is less than a specified value) is added to the model. This test shows that MOS (critical equity) for the net sales is very efficient in default prediction. Its test statistic is 279.4 when the statistic for the best traditional financial ratio, the equity ratio, is only 134.6. This MOS measure is thus clearly more significant than the traditional financial ratios supporting Hypothesis 1. The MOS (zero-profit) for the net sales is also an efficient predictor but has a test statistic (107.8) below those of MOS (critical equity) and of the equity ratio. This result supports Hypothesis 2. The MOS variables for the cost of debt do not show any high efficiency in predictive ability supporting Hypothesis 3.

Table 5. The results of the stepwise LRA

Panel A. The score test in the first step of the stepwise LRA		
Variable	Score test	*p*-value
MOS percent (profit = 0) for net sales	107,841	0,00000
MOS percent (equity = critical) for net sales	279,396	0,00000
Critical cost of debt (profit = 0)	3,873	0,04900
MOS (profit = 0) for cost of debt	4,007	0,04500
Critical cost of debt (equity = critical)	7,256	0,00700
MOS (equity = critical) for cost of debt	7,294	0,00700
Logarithm of total assets	28,862	0,00000
Return on investment ratio	68,659	0,00000
Traditional cash flow to net sales ratio	6,620	0,01000
Quick ratio	12,906	0,00000
Equity ratio	134,594	0,00000

Panel B. The last step (step 5) of the stepwise LRA				
Variable	Coefficient	Wald test	*p*-value	Exp(*B*)
MOS percent (equity = critical) for net sales	-0.0140	61.5700	0.00000	0.986
Logarithm of total assets	-0.0830	4.3510	0.03700	0.920
Return on investment ratio	-0.0050	4.5790	0.03200	0.995
Traditional cash flow to net sales ratio	-0.0030	4.6490	0.03100	0.997
Equity ratio	-0.0070	9.0540	0.00300	0.993
Constant	0.1170	0.0580	0.81000	1.124
Nagelkerke R^2	0.2680			

Table 5 (cont.). The results of the stepwise LRA

Panel C. Classification accuracy of the final stepwise LRM				
	Estimation sample Predicted class		Holdout sample Predicted class	
Actual class	Non-default	Default	Non-default	Default
Non-default	72.92	27.08	74.43	25.57
Default	23.78	76.22	28.50	71.50
Total		73.56		73.97

The stepwise LRA stops when the score test indicates that the variables outside the model are not significant. In this analysis, LRA runs five steps so that the final model includes the best MOS variable and four of the five traditional financial ratios: MOS (critical equity) for the net sales, logarithm of total assets, return on investment ratio, traditional cash flow ratio, and equity ratio. Panel B of Table 5 shows the coefficients of this final model. It shows again that MOS is superior in significance when compared with the best traditional financial ratio, the equity ratio. The signs of the five variables are negative so that the risk of payment default is decreasing in MOS, size, profitability, cash flow, and solvency. The final model does not include the quick ratio that reflects liquidity. Panel C shows the classification accuracy of the final stepwise model. For the estimation and holdout sample, the overall classification accuracy is about 74%. This percent is quite high taking into account the heterogeneity of the data and the concept of default.

3.2.1. Fixed models. The final LR models are run in the enter mode (fixed model). In these fixed models, MOS (critical equity) for the net sales (the best MOS measure) and the five traditional financial variables are used as predictors. The quick ratio is included in the analyses although it was not entered in the final stepwise model. However, in the last step of the stepwise model the score test for the quick ratio was 3.68 (p-value is 0.05) being close to enter in the model. This ratio is important in controlling the effect of liquidity. Table 6 presents three different LR models for comparison. Panel A presents the model based on the five financial variables only. In this model, all the variables have a statistically significant coefficient. The equity ratio is as expected the most significant variable (Wald = 42.5) followed by the traditional cash flow to net sales ratio (15.4).

Panel B of Table 6 shows the results for the LR model based on MOS (critical equity) for the net sales alone. The Wald test indicates a very high significance of MOS and the Nagelkerke R^2 is 0.240 that is very close to that of the financial variable model (0.243). Thus, it seems that MOS alone is a good substitute for all the financial variables in default prediction. Panel C shows the results for the combined model including MOS and the five financial variables. In this model, MOS is clearly the most significant variable (Wald = 61.5) followed by the traditional cash flow to net sales ratio (15.0). The equity ratio has not a statistically significant coefficient. The comparison of Panel A and Panel C shows that MOS substitutes the information contained by the equity ratio. In addition, referring to substitution effects it also diminishes the significance of other ratios excluding the traditional cash flow to net sales ratio. The Nagelkerke R^2 for the combined model is 0.29 indicating that MOS brings incremental information in the LR model.

Table 6. The estimated LR models for payment default prediction

Panel A. Traditional financial ratio model				
Variable	Coefficient	Wald test	p-value	Exp(B)
Logarithm of total assets	-0.1210	9.3780	0.00200	0.886
Return on investment ratio	-0.0070	9.5570	0.00200	0.993
Quick ratio	-0.2560	15.3680	0.00000	0.774
Traditional cash flow to net sales ratio	-0.0060	8.5550	0.00300	0.994
Equity ratio	-0.0140	42.5080	0.00000	0.986
Constant	0.9490	3.7390	0.05300	2.583
Nagelkerke R^2	0.2430			
Panel B. Model based on MOS only				
Variable	Coefficient	Wald test	p-value	Exp(B)
MOS percent (equity = critical) for net sales	-0.0200	219.8280	0.00000	0.981
Constant	-1.0710	243.3450	0.00000	0.343
Nagelkerke R^2	0.2400			

Table 6 (cont.). The estimated LR models for payment default prediction

Panel C. Combined model based on traditional financial ratios and MOS				
Variable	Coefficient	Wald test	p-value	Exp(B)
MOS percent (equity = critical) for net sales	-0.0140	61.5130	0.00000	0.986
Logarithm of total assets	-0.1080	7.1520	0.00700	0.897
Return on investment ratio	-0.0050	4.0840	0.04300	0.995
Quick ratio	-0.2470	15.0110	0.00000	0.781
Traditional cash flow to net sales ratio	-0.0030	4.0300	0.04500	0.997
Equity ratio	-0.0030	1.5960	0.20600	0.997
Constant	0.6260	1.5660	0.21100	1.871
Nagelkerke R^2	0.2900			

Table 7 shows the classification accuracy of the three LR models. Panel A shows that the overall accuracy for the financial variable model is about 71% in the estimation and holdout sample. Panel B shows that the MOS model leads about to the same overall accuracy (72%) in the estimation sample, but it gives worse results in the holdout sample (69%). Especially, the MOS model leads to higher inaccuracy in classifying default firms (Type I error). This factually means that some firms have a relatively high MOS but in spite of that have payment defaults. These kinds of default firms can be more efficiently identified by traditional financial ratios. Panel C shows the classification accuracy of the combined model. This panel shows that when combining MOS and the financial variables in the same prediction model, the overall accuracy can be slightly improved. The combined model performs better than the financial variable model in classifying non-default firms but worse in classifying default firms. The percent of classification error types can be altered by the cutoff value for the default probability. In the present analyses, the cutoff value is kept equal to the percent of default firms in the estimation sample.

Table 7. Classification accuracy of the LR models

Panel A. Traditional financial ratio model				
	Estimation sample Predicted class		Holdout sample Predicted class	
Actual class	Non-default	Default	Non-default	Default
Non-default	68.83	31.17	69.78	30.22
Default	20.73	79.27	18.74	81.26
Total		70.86		71.58
Panel B. Model based on MOS only				
	Estimation sample Predicted class		Holdout sample Predicted class	
Actual class	Non-default	Default	Non-default	Default
Non-default	70.84	29.16	69.37	30.63
Default	25.00	75.00	31.50	68.50
Total		71.65		69.23
Panel C. Combined model based on traditional financial ratios and MOS				
	Estimation sample Predicted class		Holdout sample Predicted class	
Actual class	Non-default	Default	Non-default	Default
Non-default	70.82	29.18	71.53	28.47
Default	22.26	77.74	24.09	75.91
Total		72.17		72.22

Note: Estimation sample includes 328 default and 1357 non-default firms. Holdout sample includes 635 default and 3412 non-default firms.

3.2.2. Testing different effects. Table 8 shows the resulted LR models for testing different effects (type of default, size, and non-financial variables). Panel A shows the models estimated separately for bankruptcies (109 events) and other default types (219 events). The results show that for each group MOS is the most significant variable. For bankruptcies, the quick ratio and the traditional cash flow to net sales ratio are the most significant financial variables whereas for other defaults the logarithm of total assets and the quick ratio show the highest significance. The equity ratio is insignificant in each group so that its effect has been substituted by MOS. The panel also shows that the classification accuracy for the bankruptcy group is higher than for the default group. This result is obvious due to the seriousness of bankruptcy as a default event.

Panel B presents the LR models for two size groups. The smaller firm group (204 events) includes firms with total assets less than the median (209389 euros) while the rest of firms belong to the larger firm group (124 events). The MOS variable is clearly the most significant variable in the smaller firm group including very small firms, followed by the quick ratio. However, in the larger firm group the significance of MOS remarkably diminishes in comparison with the financial variables. In this group, the equity ratio is the most significant variable followed by the logarithm of total assets. However, the stepwise LR results in Appendix show that even in the group of larger firms MOS is the most significant univariate predictor (score = 156,8) exceeding that of the equity ratio (128,7). Thus, MOS is an efficient predictor of default especially in very small firms. The larger the firms, the more significant are traditional financial variables when compared with MOS. The classification accuracy for larger firms is remarkably higher than for smaller firms.

Panel C of Table 8 shows the LR model based on MOS, five financial variables and eight non-financial variables. The results show that also in this combined model MOS is the most significant variable followed by the quick ratio and a number of non-financial variables. The inclusion of non-financial variables does not remarkably diminish the significance of MOS or financial ratios, with an exception for the logarithm of total assets. The equity ratio is not significant in this model. The non-financial variables make a remarkable effect on the performance of the model. The Nagelkerke R^2 is 0.37 when it was only 0.29 for the combined model of MOS and financial variables. In the same way, the overall classification accuracy is 77.3% being only 72.2% before including non-financial variables. Thus, in small business samples non-financial variables bring important incremental information over financial variables in default prediction.

Table 8. The parameter estimates of LR models for different samples

	Panel A. Default type				Panel B. Size of firm				Panel C. Non-financial variables	
	Bankruptcy		Other default		Smaller firms		Larger firms		All firms	
Variable	Coefficient	Wald	Coefficient	Wald	Coefficient	Wald	Coefficient	Wald	Coefficient	Wald
MOS percent (equity = critical) for net sales	-0,015	42,965	-0,014	47,453	-0,012	26,318	-0,010	8,159	-0,012	46,150
Logarithm of total assets	-0,004	0,005	-0,146	9,701	0,238	5,971	-0,339	9,829	-0,008	0,025
Return on investment ratio	-0,008	4,661	-0,004	2,289	-0,004	2,011	-0,006	2,192	-0,004	3,303
Quick ratio	-0,373	11,917	-0,214	9,549	-0,214	10,492	-0,138	0,796	-0,242	14,141
Traditional cash flow to net sales ratio	-0,004	6,481	-0,001	0,057	0,000	0,355	-0,015	6,458	-0,003	5,062
Equity ratio	0,000	0,045	-0,002	1,219	-0,003	2,339	-0,025	10,200	-0,001	0,186
Constant	-1,677	5,247	0,624	1,185	-3,172	8,629	4,119	7,881	0,672	0,694
Modification of audit report									-0,644	13,618
Logarithm of firm age (in months)									-0,292	5,497
Age of financial statements (in months)									0,048	10,890
Number of board member personal defaults									0,582	15,863
Number of board members									-0,347	17,664
Number of resigned board members									0,098	0,221
Default propensity of industry									0,123	17,212
Non-corporation dummy									-1,146	0,930
Number of firms (default & non-default)	109	1357	219	1357	204	639	124	718	328	1357
Nagelkerke R^2	0,247		0,243		0,257		0,382		0,371	
Classification accuracy (estimation & holdout)	73,4	72,9	70,7	70,9	71,6	69,4	76,6	74,8	77,3	

Note: Holdout sample for bankrupt firms includes only non-default firms. Size classes are defined as below and above the median size for total assets (209389 euros). There is no holdout sample for the non-financial model.

Conclusion

The purpose of this study was to introduce an extension of the traditional BEA to be applied in financial distress analysis. The traditional BEA is a widely adopted management tool that is concentrated on the analysis of profitability. Financial distress is however largely characterized by difficulties in solvency. Therefore the focus of analysis was moved to the analysis of solvency taking account, firstly, of the cost of debt (extension 1) and, secondly, of the critical equity (extension 2) as a target for profit when calculating MOS. The extended BEA is factually a combination of profitability and solvency analysis. For this extended BEA, both critical net sales and critical cost of debt (rate of interest) were drawn resulting in MOS in terms of net sales and cost of debt. The MOS concepts associated with this analysis measure the solvency buffer against fluctuations respectively in demand (sales) or cost of debt (interest rate). It was expected that these kinds of MOSs would be efficient predictors of financial distress including incremental information over traditional financial ratios (Hypothesis 1). Furthermore, it was expected that extension 1 is outperformed by extension 2 (Hypothesis 2) and that MOS drawn for the cost of debt is outperformed by that for the net sales (Hypothesis 3). In addition, it was expected that MOS concepts are efficient especially in small business samples.

The hypotheses were assessed in a sample of Finnish small default and non-default firms. The financial distress was reflected by officially registered payment defaults as a target event. The statistical analyses based on LRA showed that MOS (critical equity) for the net sales is very efficient as a predictor of default. In Finland, the equity ratio has been traditionally the most powerful predictor of financial distress. However, in this study the equity ratio was clearly outperformed by the new MOS concept. The effect of the equity ratio was in the models entirely substituted by that of MOS. In fact, including traditional financial variables in the model does not remarkably increase the overall classification accuracy of MOS. However, it is remarkable that MOS makes more Type I errors than models based on traditional financial variables do. This means that for a part of default firms MOS can exceed the critical level but still they have payment defaults.

The statistical results showed that MOS is very efficient in predicting both bankruptcies and milder payment defaults. It was also shown that it is effective in predicting payment defaults of very small firms. However, the larger the firms in the sample, the less effective is MOS in prediction as compared with traditional financial variables. This result may mean that the assumptions of BEA and its extended version are too simplified for larger firms. The structure of these firms is complicated and the sales and cost curves may be curvilinear instead of being linear. The results also showed that MOS and financial variables do not lose their significance in prediction when non-financial variables are included in the model. Controversially, MOS, traditional financial variables, and non-financial variables form together an efficient model including incremental information over models based on MOS and financial variables only.

In summary, the statistical analyses supported all research hypotheses. It was shown that the extended BEA can be a very powerful new technique for financial analysts. It is as simple and as relevant as the traditional BEA. However, it is oriented towards solvency instead of profitability alone. MOS is a combination measure that seems to be very useful in financial distress analysis. It can provide for example auditors with an efficient technique to help making the going concern decision. It is also a very promising variable to be included in different failure prediction models along traditional financial ratios and qualitative non-financial variables. MOS is efficient especially in analyzing very small firms which are difficult to assess with traditional financial statement analysis. The present study has however limitations which should be relaxed in further studies. For example, the applicability of MOS in analyzing larger firms should be carefully reconsidered. In this context, the effect of the linearity assumption could be assessed by incorporating curvilinear relationships in the model. In addition, the effect of cutoff value in classification should be analyzed to diminish Type I errors.

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Appendix

Table 1A. The score test in the first step of the stepwise LRA for the size classes

Panel A. Smaller firms (total assets less than 209389 euros)		
Variable	Score test	p-value
MOS percent (equity = critical) for net sales	115,322	0,00000
Logarithm of total assets	0,031	0,86100
Return on investment ratio	25,984	0,00000
Quick ratio	8,612	0,00300
Traditional cash flow to net sales ratio	3,796	0,05100
Equity ratio	60,082	0,00000
Overall statistics	137,423	0,00000

Table 1A (cont.). The score test in the first step of the stepwise LRA for the size classes

Panel B. Larger firms (total assets greater than 209389 euros)		
Variable	Score test	p-value
MOS percent (equity = critical) for net sales	156,750	0,00000
Logarithm of total assets	12,682	0,00000
Return on investment ratio	42,957	0,00000
Quick ratio	4,650	0,03100
Traditional cash flow to net sales ratio	49,199	0,00000
Equity ratio	128,684	0,00000
Overall statistics	193,730	0,00000