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Managing liquidity buffer through core liquidity portfolio

Abstract

Generally, the common practice by banks in liquidity management is that they hold a liquidity portfolio as a unitary portfolio and not segmenting it into sub-portfolios taking into consideration bank specific needs. There is wide agreement that insufficient liquidity buffers are the root causes of the 2007 to 2009 liquidity crisis. The authors construct a core liquidity portfolio to specifically ensure liquidity buffer is met to avoid liquidity problem. A two stage stochastic programming model with recourse is constructed taking into account liquid asset return to maximize liquidity buffer. The authors propose a liquid asset return scenario generation model. The data used to calibrate the proposed model is from International Monetary Fund from January 2000 to April 2014. The researchers found that the behavior of the deterministic liquidity buffer relative to stochastic reveals some general properties of the underlying problem and help predict how a stochastic will perform. By taking into account liquid asset return randomness, the bank self-insure against adverse asset price movements. The authors strongly recommend that banks should design a core liquidity portfolio which is a sub-portfolio of the overall liquidity portfolio, which will specifically focus on meeting liquidity requirements using stochastic programming. However, bank liquidity buffer is restricted by interest rate on liquid assets and required reserves defined by the central bank.

Keywords: core liquidity portfolio, liquid asset return, liquidity buffer, scenario generation, stochastic programming.

JEL Classification: G11, G21.

Introduction

Liquidity risk for a bank includes both the risk of being unable to fund its financing commitments and the risk of being unable to meet the demand for withdrawals. The shortage of liquidity is said to be an assassin of banks and liquidity surplus is considered a drag on competitiveness. Banks face two central issues regarding liquidity. The main purpose of banks is to manage liquidity creation and also to manage liquidity risk. By creating liquidity in the market, the banking industry serves an important role in the country's economy.

According to Ali (2012), liquidity risk emanates from the nature of banking business, from the macro factors that are exogenous to the bank, from financing and operational policies that are internal to the banking firm. The sources of liquidity risk include the maturity mismatch of assets and liabilities, inability of the bank to convert its assets into cash without loss and unanticipated recall of deposits.

Until recently, liquidity was not the main focus of banking regulators. A number of liquidity-related management models have been developed; see Abraham (2011), Bolton et al. (2008), De Alcantara (2008), Ratwowski (2013). Abraham (2011) proposed a method that can be used to calculate liquidity buffer. In his model, Abraham (2011) defined the following variables; the wholesale, retail, off-balance sheet, intra-day and downgrade requirements. The liquidity demand model was considered by Bolton et al. (2008), and computed a liquidity demand arising from a possible maturity mismatch between asset revenues and consumption.

De Alcantara (2008) developed an integrated model for optimal asset allocation in commercial banks that incorporates uncertain liquidity constraints that are ignored in risk-adjusted return on capital and economic value added models. A model of bank liquidity risk driven by solvency was proposed by Ratwowski (2013). In his model, he defined the interaction between liquidity requirements, access to refinancing and liquidity risk.

Previous and most recent liquidity management models developed look at liquidity as a unitary portfolio. The Asian Development Bank (2013) proposed that the bank should establish a sub-portfolio that will specifically focus on preserving liquidity buffer. There is wide agreement that insufficient liquidity buffers are the root causes of the 2007 to 2009 liquidity crisis. Banks are failing to adequately maintain liquidity buffers to meet liquidity needs. Defining the appropriate assets and core liquidity management strategies are a challenge to most banks. Thus the main aim of the research is to design an appropriate core liquidity portfolio management technique to ensure that the liquidity buffer is met. We construct a two-stage stochastic programming model with recourse to manage the core liquidity portfolio. To meet and maintain the buffer requirement, we maximize the liquidity buffer. The paper is organized as follows; Section 1 develops a scenario generation method and then a two-stage stochastic programming model. Section 2 is devoted to methods used to collect data and its statistical characteristics. The stochastic solutions are presented in Section 3. Final section gives meaning to the presented empirical results in conjunction with data characteristics and conclusion.

1. Model formulations

The management needs to decide on the size of the initial liquidity buffer to hold so as to caution itself against stochastic liquidity demand. At each beginning of instant time, the bank decides on how much to invest in high quality liquid assets to satisfy the liquid asset reserve requirement. In between instant times, the bank is exposed to exogenous liquidity demand and the liquidity buffer is depleted. The bank needs to make recourse decisions to maintain the liquidity buffer. We start by defining the decision variables and parameters that we used as follows:

Definition of sets:

- $t \in T$ Set of instant time;
- $i \in I$ Set horizon bands;
- $s_n \in S$ Set of scenarios, for $n \in N$;
- $\omega_j^1 \in \Omega_1$ Set of event space for the unknown liquid asset return, where $j \in J$;
- $\omega_k^2 \in \Omega_2$ Set of event space for the unknown cost of liquid asset, where $k \in K$;
- $P_{\omega_j^1} \in P_1$ Set of probabilities of event space for the unknown liquid asset return where $j \in J$;
- $P_{\omega_k^2} \in P_2$ Set of probabilities of event space for the unknown cost of liquid asset, where $k \in K$;
- $p_{s_n} \in P$ Set of scenario probabilities, for $s_n \in S$;

Deterministic data:

- F_0 Initial liquidity buffer;
- q_0^{AB} Liquid assets held at stage 0 under uncertain liquid asset returns;
- \mathcal{Q}_{ba} Business-as-usual absolute risk aversion coefficient;
- \mathcal{Q}_{cont} Contractual maturity absolute risk aversion coefficient;
- $\beta_{b_i^A}$ Percentage increase in business-as-usual assets takes place upon expanding bank operations, for $i \in I$;
- $\beta_{b_i^L}$ Percentage increase in business-as-usual liabilities takes place upon increase in assets, for $i \in I$;
- Ψ The weight of additional needed funds;
- λ The coefficient of business-as-usual net funding gap;

Random data:

- $b_{its_n}^A$ Business-as-usual assets, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^L$ Business-as-usual liabilities, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Az}$ Business-as-usual asset advance, for $i \in I, t \in T$ and $s_n \in S$;

- $b_{its_n}^{Ad}$ Business-as-usual asset trading, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Aa}$ Business-as-usual other asset, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Le}$ Business-as-usual stable deposits, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Lv}$ Business-as-usual volatile deposits, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Lh}$ Business-as-usual trading and hedging instruments, for $i \in I, t \in T$ and $s_n \in S$;
- $b_{its_n}^{Ll}$ Business-as-usual other liabilities, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^A$ Contractual maturity of assets, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^L$ Contractual maturity of liabilities, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Az}$ Contractual maturity of assets advance, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Ad}$ Contractual maturity of assets trading, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Aa}$ Contractual maturity of other assets, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Le}$ Contractual maturity of stable deposits, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Lv}$ Contractual maturity of volatile deposits, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Lh}$ Contractual maturity of trading and hedging instruments, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^{Ll}$ Contractual maturity of other liabilities, for $i \in I, t \in T$ and $s_n \in S$;
- $c_{its_n}^M$ Net contractual maturity, for $i \in I, t \in T$ and $s_n \in S$;
- r_{its_n} Liquid asset returns, for $i \in I, t \in T$ and $s_n \in S$.

Decision variables:

- $b_{its_n}^G$ The business-as-usual net funding gap, for $i \in I, t \in T$ and $s_n \in S$;
- $f_{its_n}^D$ Planned additional funds, for $i \in I, t \in T$ and $s_n \in S$;
- $q_{its_n}^{AE}$ Liquid asset bought to maintain liquidity buffer, for $i \in I, t \in T$ and $s_n \in S$;
- $q_{its_n}^{AB}$ Liquid assets held at stage t under uncertain liquid asset returns, for $i \in I, t \in T$ and $s_n \in S$.

We define business-as-usual as the normal execution of standard functional operations within the bank. Contractual maturity of assets and liabilities are the inflows and outflows liquidity from on- and off-balance sheet items, mapped to defined time bands based on respective maturities, respectively.

1.1. Scenario tree. We need to construct a scenario for random variable liquid asset returns. Now consider a case where the random vector s , has a discrete and finite distribution, with n scenario set $S = \{s_1, s_2, \dots, s_N\}$. Denote p_{s_n} as the probability of realization of the n^{th} scenario s_n . We assume that $p_{s_n} \geq 0$ for all $s_n \in S$ and that $\sum_{n \in N} p_{s_n} = 1$. A scenario is defined as a path from the root of the tree to one of the leaves. The nodes visited by each path correspond to the values assumed by random

parameters in the model. Now we assume liquid assets return can be computed as shown in equation (1).

$$r_t = \frac{(1+r_a)(q_t^{AB} - C_{CB}) + (1+r_{CB})C_{CB} - q_t^{AB}}{q_t^{AB}}, \quad (1)$$

where r_t is the liquid assets returns at time t . The r_a and r_{CB} are the interest rate on interest earning liquid assets and interest rate on cash reserves deposited with central bank, respectively. We assume that $r_a \geq r_{CB}$. The cash reserve deposited with central bank is C_{CB} . Now, we can define a scenario generation model by equation (2) below:

$$r_{i(t+1)s_n} = \mu_{q_t^{AB}} + \gamma_{CI} \delta_{q_t^{AB}} \varepsilon_{t+1}, \quad (2)$$

where we assume ε_{t+1} is the random error with mean 1 and γ_{CI} is the standard normal value at confidence interval level CI . Now we can construct the scenario tree as shown in Figure 1.

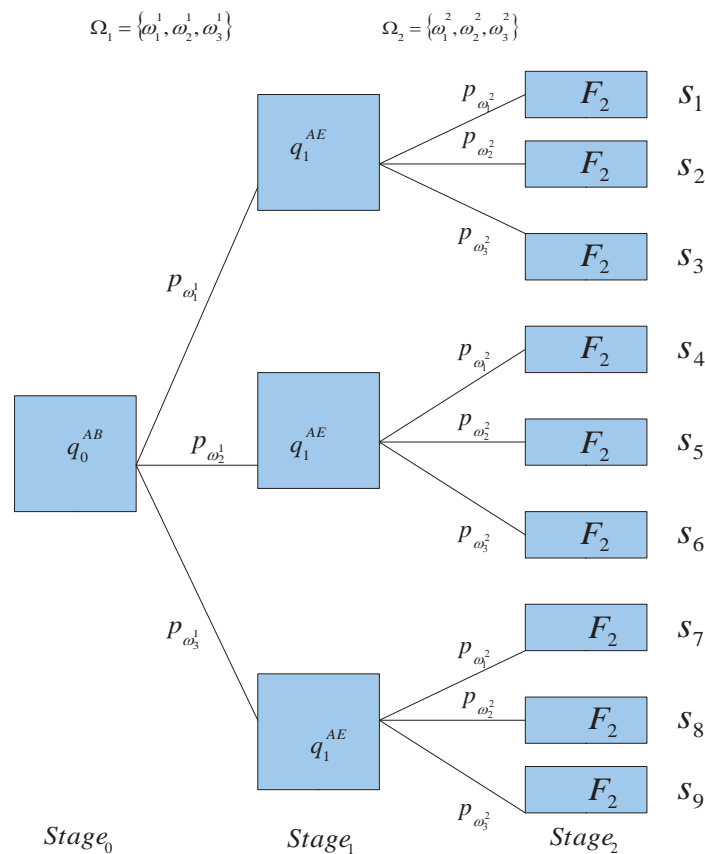


Fig. 1. Liquidity buffer level scenario tree

In the scenario tree at stage 0, the q_0^{AB} represents the initial liquid assets to be decided under event space having three possibilities $\Omega_1 = \{\omega_1^1, \omega_2^1, \omega_3^1\}$, with probabilities $\{p_{\omega_1^1}, p_{\omega_2^1}, p_{\omega_3^1}\}$, respectively. The event space Ω_1 represents uncertain liquid asset returns. At the beginning of stage 1, the liquid asset returns are revealed. A recourse decision to buy additional liquid assets q_1^{AE} is made at the end of stage

1 under event space $\Omega_2 = \{\omega_1^2, \omega_2^2, \omega_3^2\}$ with probabilities $\{p_{\omega_1^2}, p_{\omega_2^2}, p_{\omega_3^2}\}$, respectively. At the beginning of stage 2, the money market announces the cost of liquid assets. We need to note that the event space Ω_2 is the uncertain costs of liquid assets. The liquid asset return and cost of liquid asset bought event spaces probability must satisfy the following conditions; $\sum_{j \in J} p_{\omega_j^1} = 1$ and $\sum_{k \in K} p_{\omega_k^2} = 1$, respectively.

Once the cost of liquid assets event space $\omega_k^2 \in \Omega_2$ in stage 2 has been observed, the liquidity buffer is computed by the model as the final decision bank liquidity buffer.

Each root-leaf path from the scenario tree defines scenario s_n , induced by a sequence of all random events. There is a one-to-one correspondence between the scenarios and the leaf-nodes in our tree. The unconditional probability of a node, in this case is given by $P_{s_n} = P_{\omega_j^1} P_{\omega_k^2}$, which starts from the root and terminates at that node.

1.2. Stochastic programming. The advantage of SP models is that they provide a generic framework to model uncertainties and enables decisions that will perform well in the general case. In this model, we need to maximize liquidity buffer requirement. According to Committee of European Banking Supervisors (2009), we define liquidity buffer as a subset of counterbalancing capacity. The counterbalancing capacity is partitioned into business-as-usual view, planned stress view and protracted stress view. So in this case, liquidity buffer is planned stress view. Therefore, the two-stage stochastic programming model with recourse is as follows:

$$\text{Max } F_0 + \{E[\lambda b_{its_n}^G + \psi f_{its_n}^D]\}, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{3}$$

$$\text{s.t } F_0 \leq q_0^{AB} \tag{4}$$

$$\mathcal{G}_{bau} b_{its_n}^A - (1 - \mathcal{G}_{bau}) b_{its_n}^L \geq b_{its_n}^G, \quad \mathcal{G}_{bau} \in (0,1), \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{5}$$

$$b_{its_n}^z + b_{its_n}^d + b_{its_n}^a \geq b_{its_n}^A, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{6}$$

$$b_{its_n}^e + b_{its_n}^v + b_{its_n}^h + b_{its_n}^l \geq b_{its_n}^L, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{7}$$

$$\mathcal{G}_{cont} c_{its_n}^A - (1 - \mathcal{G}_{cont}) c_{its_n}^L \leq c_{its_n}^M, \quad \mathcal{G}_{cont} \in (0,1), \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{8}$$

$$c_{its_n}^z + c_{its_n}^d + c_{its_n}^a \leq c_{its_n}^A, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{9}$$

$$c_{its_n}^e + c_{its_n}^v + c_{its_n}^h + c_{its_n}^l \geq c_{its_n}^L, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{10}$$

$$(1 + \beta_{b_i^A}) b_{its_n}^A - (1 + \beta_{b_i^L}) b_{its_n}^L - c_{its_n}^M \geq f_{its_n}^D, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{11}$$

$$\sum_{i \in I} q_{its_n}^{AE} \leq Q,$$

$$\forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{12}$$

$$(1 + r_{its_n}) q_{its_n}^{AB} + q_{its_n}^{AE} - \psi f_{its_n}^D \geq b_{its_n}^G, \quad \forall i \geq 1, \forall t \geq 1, \forall n \geq 1 \tag{13}$$

$$F_0 \geq 0, b_{its_n}^A \geq 0, b_{its_n}^L \geq 0, b_{its_n}^z \geq 0, b_{its_n}^d \geq 0, b_{its_n}^a \geq 0, b_{its_n}^G \geq 0, f_{its_n}^D \geq 0 \tag{14}$$

$$b_{its_n}^e \geq 0, b_{its_n}^v \geq 0, b_{its_n}^h \geq 0, b_{its_n}^l \geq 0,$$

$$c_{its_n}^A \geq 0, c_{its_n}^L \geq 0, c_{its_n}^M \geq 0, c_{its_n}^e \geq 0,$$

$$c_{its_n}^v \geq 0, c_{its_n}^h \geq 0, c_{its_n}^l \geq 0, c_{its_n}^z \geq 0,$$

$$c_{its_n}^d \geq 0, c_{its_n}^a \geq 0, q_{its_n}^{AE} \geq 0, q_{its_n}^{AB} \geq 0.$$

The objective (3) is to maximize liquidity buffer and is computed based on the clean market to market value of the securities. We need to note that $q_0^{AB} = r_0 + x_0 + y_0$ since the initial liquidity buffer includes eligible securities especially designated money market funds and central bank reserves and deposits, which are liquid assets. The first stage constraint is denoted by constraint (4), where the initial liquid assets held is the initial buffer. The constraint (5) is the business-as-usual net funding gap as the difference between the business-as-usual assets and liabilities. The constraints (6) and (7) are business-as-usual assets and liabilities, respectively.

The net contractual maturity is denoted by constraint (8). The contractual maturity of assets and liabilities are denoted by constraints (9) and (10), respectively. The constraint (11) denotes the planned additional funds model which is defined as the difference between an anticipated increase in business-as-usual assets $(1 + \beta_{b_i^A}) b_{its_n}^A$ and the anticipated increase in business-as-usual liabilities $(1 + \beta_{b_i^L}) b_{its_n}^L$ and deducting the net contractual maturity. We need to note that $\beta_{b_i^A} \in (0,1)$ and $\beta_{b_i^L} \in (0,1)$. The constraint in (12) is the total value of liquid assets the bank is able to buy to meet the liquidity shortfall. The rebalancing constraint is constraint (13) that shows the relationship between the additional funds needed and the business-as-usual net funding gap. Finally, the constraint (14) denotes the non-negativity constraints.

2. Methodology

The choice of stochastic programmes has been made for several reasons. In this research, SP can accommodate general distributions and dynamic aspects by means of a scenario tree. We do not explicitly assume a specific stochastic process for the liquid asset return but viewed as an independent

and identically distributed random variable; and we rely on empirical data distributions. The Lingo 14 software was used to solve the constructed two-stage stochastic programming model with recourse.

2.1. Sources of data. The constructed two-stage stochastic model is assessed for its viability and performance using South African Reserve Bank. The data are actual liquid assets held, and cash

reserves requirement readily available on South African Reserve Bank website. The considered data is from year January 2000 to year April 2014.

2.2. Statistical characteristics of historical data.

We start by computing the statistical characteristics of historical data. The Lingo 14 software was used to analyze the data. The characteristics of data used are as shown in the following Table 1 below.

Table 1. Statistical characteristics of historical data

Statistical measure	Liquid assets held	Cash reserves	Liquid asset return
Average	1.39788×10^5	3.5344×10^4	4.01228×10^{-2}
Standard deviation	9.7145×10^4	1.9785×10^4	2.842×10^{-3}
Kurtosis	-1.0294046	-1.411302186	-1.269907032
Skewness	$6.87005958 \times 10^{-1}$	$1.41679602 \times 10^{-1}$	$-4.0141792 \times 10^{-2}$
Minimum	3.6634×10^4	6.311×10^3	-1.269907032
Maximum	3.27016×10^5	7.0161×10^4	4.6264502×10^{-2}

The historical liquid assets and cash reserve requirement data exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. In addition, the random variable liquid asset returns exhibit kurtosis, implying heavier tails than the normal distribution. Thus, we need to generate scenarios for liquid asset returns that comply with historical observations, without relying on the normality assumption. We rely solely on the observed actual liquid asset held, cash reserve requirement and liquid asset returns in the market. We estimate the first four moments and we generate scenarios based on these statistical characteristics. The scenarios are generated so that the first four

marginal moments of the random variables match their historical values.

3. Empirical analysis

In this section, we present and discuss the performance of our model. We start by analyzing the SP solution status by observing the global optimal status. In this case, solution status is globally optimal implying that the results are valid. Now we turn to Table 2 where results of different components are shown. Of prime importance, are the stochastic objective values, which are the expected objective values over all different cases and scenarios.

Table 2. Stochastic programming when $\mathcal{G}_{cont} = 15\%$ and $\mathcal{G}_{cont} = 20\%$

Value	Obj	Inf	EV	EWS	EVPI	EM	EVMU
$\beta_1 = 0.2$ $\beta_2 = 0.2$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	5.9605×10^{-8}	3.6061×10^8	1.192×10^{-7}
$\beta_1 = 0.3$ $\beta_2 = 0.2$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	5.9605×10^{-8}	3.6061×10^8	5.9605×10^{-8}
$\beta_1 = 0.5$ $\beta_2 = 0.4$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	0	3.6061×10^8	5.9605×10^{-8}
$\beta_1 = 0.5$ $\beta_2 = 0.5$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	0	3.6061×10^8	5.9605×10^{-8}
$\beta_1 = 0.5$ $\beta_2 = 0.6$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	0	3.6061×10^8	5.9605×10^{-8}
$\beta_1 = 0.6$ $\beta_2 = 0.4$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	0	3.6061×10^8	5.9605×10^{-8}
$\beta_1 = 0.6$ $\beta_2 = 0.5$	3.6061×10^8	0	3.6061×10^8	3.6061×10^8	0	3.6061×10^8	5.9605×10^{-8}

We assume that as β_1 changes regardless whether β_2 is changing and when all other parameters are constant, the overall objective should at least change. However from Table 2, this is not the case the liquidity buffer is constant throughout. Similarly the expected values of liquidity buffer (hereinafter EV), expected value of wait-and-see value of liquidity buffer (hereinafter, EWS) and expected value of policy based on mean outcome (hereinafter, EM) liquidity buffer are constant regardless β_1 and β_2 changes. The basic facts

and notations of stochastic programming are explained in Appendix.

The advantage of using a SP solution is that the bank is well-hedged against worst-case scenarios. The infeasibilities value is zero in all cases implying that no constraints were violated in the deterministic equivalent model. The EV of liquidity buffer is the same as the reported optimal liquidity buffer for the model, further giving weight to the results. The

EWS liquidity buffer reports the expected value of the liquidity buffer if we could wait-and-see the outcomes of all liquid asset returns and uncertain costs of liquid assets borrowed to cover the liquidity gap before making decisions. However, in reality it is not possible to wait. Comparing the values of EWS and EV, we expect $EWS \geq EV$ since we are maximizing liquidity buffer.

The expected value of perfect information (hereinafter, EVPI) is the expected increase in liquidity buffer if we know the future outcome in advance. Thus EVPI is used to compare EWS and EV and measures the maximum amount the decision maker should be willing to pay in return for complete and accurate information regarding the future outcomes of the underlying asset returns. It can be computed as follows; $EVPI = |EV - EWS|$. From Table 2, when $\beta_1 = 20\%$ and $\beta_2 = 20\%$, and $\beta_1 = 20\%$ and $\beta_2 = 30\%$ the EVPI is non-zero. However for all other values of β_1 and β_2 , the EVPI is zero. Therefore in all cases, the ratio of total EVPI to total optimal liquidity buffer value is very small close to zero implying that the decision maker is willing to pay nothing per given period. According to Kall and Wallace (2003), small EVPI shows that randomness plays a minor role in the model. The EM is slightly different from the expected true liquidity buffer as evidenced by expected value of modeling uncertainty (hereinafter EVMU). The EVMU also known as the value of stochastic solution (VSS), is the expected decrease

in liquidity buffer if we replaced each stochastic parameter by a single estimate and act as if this value is certain. The $EVMU = EV - EM$ is non-zero in all cases but very small indicating that we gained little by taking into account uncertainty in our model analysis.

In other words, EVMU measures how good or bad a decision obtained by EV. In this case, $EV \geq EM$, implying that the model is good since there is a very small difference between EV and EM, and there is no potential for improvement. Therefore, we can observe that $EV \leq EM \leq EWS$, $EVPI \geq 0$ and $EM \geq 0$ which are consistent with Barbaro and Bagajewicz (2004) and Birge (1997) conditions. Therefore, at this level, we can infer that the scenarios generated are reliable and valid. We need to note that in some circumstances, there are explicit tradeoffs based on processing all relevant information. We do have situations which are not consistent with Birge (2013) conditions that is when $\beta_1 = 20\%$ and $\beta_2 = 20\%$, and $\beta_1 = 20\%$ and $\beta_2 = 30\%$, the $EVPI > 0$ and $EVMU > 0$ although very small. But for all other β values the $EVPI = 0$ and $EVMU \neq 0$, consistent with Birge (2013) conditions. The condition implies that we had a situation with multiple liquidity buffers and there exist an optimal liquidity buffer. Turning to Table 3, when $\beta_1 = 20\%$ and $\beta_2 = 20\%$, the bank has to buy liquid assets in all bands except band 2 meet liquidity buffer. However, when β_1 and β_2 take any other value, the bank has to buy liquid assets in bands 1, 2 and 4.

Table 3. Liquid asset bought – stage 0 solution

Bands	$\beta_1 = 0.2$ $\beta_2 = 0.2$	$\beta_1 = 0.3$ $\beta_2 = 0.2$	$\beta_1 = 0.5$ $\beta_2 = 0.4$	$\beta_1 = 0.6$ $\beta_2 = 0.5$
1	2.9565×10^7	3.5763×10^7	1.1495×10^8	9.9442×10^7
2	0	1.2093×10^8	1.0726×10^7	1.0726×10^7
3	2.4534×10^7	0	0	0
4	1.9890×10^7	2.0128×10^8	2.3230×10^8	2.4781×10^8
5	1.5420×10^7	0	0	0
6	2.6779×10^7	0	0	0
7	4.4377×10^7	0	0	0
8	1.9741×10^8	0	0	0

The change in risk aversion coefficient and other variables remain the same; will only change the stage 0 solution. Here, the value of recourse decision changes as risk coefficients change. However, the stochastic programming solution such as objective value, EV, WS, EVPI, EM, EVMU and random distribution will remain the same in all cases. The Table 4 shows the liquid assets bought under different values of absolute risk aversion

coefficient. When ($\mathcal{G}_{bau} = 4\%$ and $\mathcal{G}_{cont} = 9\%$), and $\mathcal{G}_{bau} = 10\%$ and $\mathcal{G}_{cont} = 10\%$), the bank should be prepared to purchase liquid assets in all bands. However, when $\mathcal{G}_{bau} = 40\%$ and $\mathcal{G}_{cont} = 20\%$ the bank should buy liquid assets in bands 2 and 5 only. However, we need to note that our assumption is that the absolute risk aversion coefficient does not change with time.

Table 4. Changes in absolute risk aversion coefficient – liquid asset bought

Band	$\mathcal{G}_{bau} = 0.04$ $\mathcal{G}_{cont} = 0.09$	$\mathcal{G}_{bau} = 0.1$ $\mathcal{G}_{cont} = 0.1$	$\mathcal{G}_{bau} = 0.4$ $\mathcal{G}_{cont} = 0.2$
1	7.6437×10^6	1.9601×10^7	0
2	1.3825×10^8	7.0254×10^7	2.9155×10^7
3	6.3011×10^6	1.6246×10^7	6.5974×10^7
4	5.0557×10^6	1.3147×10^7	6.9941×10^7
5	3.8677×10^6	1.0169×10^7	1.9291×10^8
6	6.8998×10^6	1.7743×10^7	0
7	1.1597×10^7	2.9477×10^7	0
8	1.7835×10^8	1.8134×10^8	0

Table 5. Liquid asset return distribution report

Band	Mean	Standard Deviation
1	4.01562×10^{-2}	1.5040×10^{-5}
2	4.01457×10^{-2}	1.81231×10^{-5}
3	4.0162×10^{-2}	1.2942×10^{-5}
4	4.0140×10^{-2}	1.51683×10^{-5}
5	4.0143×10^{-2}	1.5890×10^{-5}
6	4.01463×10^{-2}	1.5189×10^{-5}
7	4.01478×10^{-2}	1.70903×10^{-5}
8	4.01515×10^{-2}	1.56129×10^{-5}

With reference to Table 5, the random distribution of liquid asset return is constant regardless of the values of $(\beta_1$ and $\beta_2)$, and $(\mathcal{G}_{bau}$ and $\mathcal{G}_{cont})$. Applying scenario generation method in equation (3), we obtain scenarios for the liquid asset return. The resulting scenarios are equi-probable in our tests, but generally they do not have to be.

Conclusions

The general practice of the banking industry is to hold a unitary liquidity portfolio and not necessarily segmented to focus on bank specific liquidity needs. A two-stage stochastic programming model with recourse to maximize liquidity buffer was constructed. A scenario generation model to generate liquid asset return scenarios is developed. The liquid asset return is stochastic and therefore, we represent it on a scenario tree.

From Lingo solution output, we observed that the model is feasible. We further analyzed whether the scenarios are reliable and valid. After assessing the value of stochastic solution, we inferred that the scenarios generated and results computed are reliable and valid. In addition, the expected value of modeling uncertainty is non-zero in all cases. This imply that we really benefited by taking into account the liquid asset return randomness. We need to note that stochastic programming is decision making under risk.

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The models assist us in finding the relationship between the optimal liquidity buffer to stochastic models and deterministic models when a situation is random. In designing the portfolio, liquid asset returns are uncertain, and to avoid liquidity problems in the near future, we need to include randomness on asset return. Taking into consideration liquid asset return, the bank is preparing for the worst-case scenarios. Understanding the behavior of the deterministic liquidity buffer relative to stochastic reveals some general properties of the underlying problem and help to predict how a stochastic model will perform. Ignoring uncertainty when making decisions may have serious consequences to the bank. By taking into account randomness, the bank self-insure against adverse asset price movements.

We conclude that the greater liquidity buffer implies more liquidity to the bank, and less liquidity buffer means less liquid. We strongly recommend that banks should design a core liquidity portfolio which is a sub-portfolio of the overall liquidity portfolio, which will specifically focus on meeting liquidity requirements. The stochastic programming is the appropriate technique that can be used to properly manage the portfolio since it includes uncertainty. However, bank liquidity buffer is restricted by interest rate on liquid assets and required reserves defined by the central bank. Further research can be done on other techniques that can be used to construct the core liquidity portfolio, including scenario generation model.

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Appendix

We define the basic facts and notations according to Maggioni and Wallace (2012).

$$\max_{x \in X} E[z(x, \zeta)] = \max_{x \in X} \{f_1(x) + E_{\zeta}[h_2(x, \zeta)]\}, \quad (A1)$$

where, x is First-stage decision variable restricted to the set $X \in \mathfrak{R}^n$, E_{ζ} is the expectation with respect to a random vector ζ defined on some probability space (Ω, F, p) and given probability distribution p on the algebra F , h_2 is the recourse function, the value of another optimization problem, $E_{\zeta}[h_2(x, \zeta)]$ is the expected recourse function.

The recourse function is:

$$h_2(x, \zeta) = \max_{y \in Y(x, \zeta)} f_2(y, x, \zeta). \quad (A2)$$

The function reflects the costs associated with adopting to information revealed through a realization ζ of the random vector ζ . The solution x^* obtained by solving A1 is the here-and-now solution and

$$EV = E_{\zeta} [z(x^*, \zeta)] \quad (A3)$$

is the optimal value of the associated objective function.

Replace all random variables by their expected value and solve the deterministic program – the expected value problem (EV):

$$EV = \max_{x \in X} z(x, \bar{\zeta}), \quad (A4)$$

where $\bar{\zeta} = E[\zeta]$. Let $\bar{x}(\bar{\zeta})$ be an optimal solution to A4, called the expected value solution and let EEV be the expected cost when using the solution $\bar{x}(\bar{\zeta})$:

$$EEV = E_{\zeta} [z(\bar{x}(\bar{\zeta}), \zeta)]. \quad (A5)$$

The value of the stochastic solution (VSS) is then defined as

$$EVMU = EV - EEV, \quad (A6)$$

and measures the expected increase in value from solving the stochastic version of a model rather than the simpler deterministic one.