Olha Kozmenko (Ukraine), Olha Merenkova (Ukraine)

The use of the Karno and Stackelberg models for the study of operational and marketing strategies in reinsurance companies' behavior

Abstract

The article studies the modeling and optimization of the main parameters in the functioning of reinsurance companies based on the analysis of operational and marketing strategies of their behavior on the reinsurance market. It offers to maximize the function of a reinsurance company's profitability by determining the optimal quantity of reinsurances of a certain risk as a result of the Karno and Stackelberg models' adaptation to the reinsurance market. This approach studies models with and without consideration of the competitors' behavior on the market as well as their different combinations.

Keywords: reinsurance companies, reinsurance market, operational and marketing strategies.

Introduction

Under the influence of globalization processes on both the national insurance market and reinsurance market the requirements to the efficiency and profitability of insurance companies are constantly growing. Moreover, due to the increased cooperation between the insuring and reinsuring companies researchers are paying more attention to the development of operational and marketing strategies considering the main areas of competitors' activities and activities of insurance companies involved in risks reinsurance. This, in turn, contributes to the formation of competitive environment and favors competition on the reinsurance market. The modeling and optimization of the insurance companies' functioning based on the analysis of strategies of the main market participants are in the need of constant improvement.

Analysis of the latest publications. The contemporary economic literature actively studies the issue of raising the efficiency of reinsurance companies by developing and analyzing the operational and marketing strategies and their behavior. The most popular optimization models are the analysis models of Karno and Stackelberg.

Unsolved elements of the general problem. Along with the above-mentioned research areas the development of adequate models and strategies for the improvement of reinsurance companies' activities is very important.

Goal of research. The goal of this paper is to develop approaches to the optimization of the number of reinsurances in order to increase the profitability of reinsurance companies based on the analysis of operational and marketing strategies of their behavior on the reinsurance market.

1. Main results of the research

The data analysis of operational and marketing research of the reinsurance market makes it possible to form a complex characteristic of this market's players, areas of their activity and prospects for further development as well as obtain objective data on the reinsurance operations and their volumes on the market. The obtained data is used to plan the activities of reinsurance companies by using the situational analysis which provides a survey of competitive environment and business climate on the reinsurance market, re-insurers' positions on the strategic markets, the analysis of strengths and weaknesses, possibilities and threats.

The efficiency of reinsurance companies depends on the strategies of their activities in accordance with the current demand on the reinsurance market. The function of demand for reinsurance services determines the profitability of financial flows of a reinsurance company as opposite to its expenditures. The ratio between these flows is an indicator of the profitability of their activity and the increasing competition on the reinsurance market.

One of the factors for optimization of reinsurance companies' activity, which allow to raise their profitability, is the volume of risks' transfer. It is offered to determine the number of reinsurances and make some assumptions. We study the insurance companies according to the volumes of their assets. Accordingly, they can reinsure a bigger or smaller share of the market in proportion to others. We will try to optimize the quantity of reinsurances by modeling the profitability of reinsurance companies.

The determination of the optimal number of reinsurances of a certain type of risk will be carried out in the following stages.

1. The formation of demand for risk transferring services. The function of demand on the reinsurance market characterizes the dependency

of the value of operations (p) to the sum of insurance (S_i) on the number of reinsurances of a certain type of risk ($\sum_i q_i$) in the form of linear regression:

$$p = (a - b\sum_{i=1}^{N} S_i (1-\alpha)^{\sum_i q_i}). \quad (1)$$

2. The formation of the function of cost of the reinsurance company (C_i) of both the constant (organizational and managerial) costs (d_i) and variable (liquidation, acquisition, collection) expenses (c_i) depending on the number of reinsurances (q_i) and the volumes of insurance reimbursements (Z_i).

$$C_i = c_i q_i + d_i + Z_i. \quad (2)$$

3. The optimization of reinsurance companies' functioning (profitability of the reinsurance company (π_i)) on the basis of the earlier determined profitable and spent parts of their financial flows.

$$\pi_i = (a - b\sum_{i=1}^{N} S_i (1-\alpha)^{\sum_i q_i}) q_i - c_i q_i - d_i - Z_i. \quad (3)$$

4. The calculation on the basis of statistical data (regarding the functioning of reinsurance companies on the market) of the demand function parameters $a > 0, b > 0$. The result is the cost of reinsurance operations of reinsurance company *i*, with consideration of the assets on the market ($\sum_i A_i$), the company's own share (α), the size of mathematical expectation of insurance sums ($\sum_i S_i$), as well as the number of reinsurances of a certain risk

$$1 - \sqrt[\sum_i q_i]{\frac{\alpha \sum_i S_i}{\sum_i A_i}}. \quad (4)$$

The factor signs of the market demand are the values $\sum_{i=1}^{N} q_i$ of the number of reinsurances of all companies, which take part in the re-insuring of a certain risk. Therefore, the demand function is the following:

$$1 - \sqrt[\sum_i q_i]{\frac{\alpha \sum_i S_i}{\sum_i A_i}} = a - b\sum_{i=1}^{N} S_i (1-\alpha)^{\sum_i q_i}, a > 0, b > 0. \quad (5)$$

Using the method of the least squares we determine the constant values – the parameters $a > 0, b > 0$. The parameter a characterizes the size of the tariff rate for the reinsurance operations when the insurance company accepts only those risks that correspond to the volume of its assets. The quantitative value of the parameter b shows how the cost of reinsurance services decreases if the number of reinsurance stages grows by one.

5. The determination of the function parameters of reinsurance companies by using the method of the least squares.

6. The formation of the corridor of acceptable margins of an insurance company profitability indicator as a result of the following operations:

- the calculation of the minimal $\min\{\pi_i\}$ and the maximal $\max\{\pi_i\}$ values of profitability;
- the calculation of the mean value of profitability (in the range from the lowest to the highest levels);
- the calculation of the mean value of profitability (in the range from the lowest to the average), which looks like $\frac{\max\{\pi_i\} - 3\min\{\pi_i\}}{2}$ and characterizes the lowest acceptable profitability margin of reinsurance companies;
- the calculation of the mean level of profitability values (in the range from the middle to the highest level), which looks like $\frac{\max\{\pi_i\} + \min\{\pi_i\}}{2}$ and characterizes the highest acceptable profitability margin of reinsurance companies.

So, the acceptable corridor of the function for the optimization of reinsurance companies looks like this:

$$\frac{\max\{\pi_i\} - 3\min\{\pi_i\}}{2} \le \pi_i \le \frac{\max\{\pi_i\} + \min\{\pi_i\}}{2}. \quad (6)$$

The optimization of the main parameters of reinsurance companies' functioning based on the analysis of operational and marketing strategies of their behavior on the reinsurance market is carried out on the basis of the following model:

$$\pi_i = (a - b\sum_{i=1}^{N} S_i (1-\alpha)^{\sum_i q_i}) q_i - c_i q_i - d_i - Z_i,$$

$$1 - \sqrt[\sum_i q_i]{\frac{\alpha \sum_i S_i}{\sum_i A_i}} = a - b\sum_{i=1}^{N} S_i (1-\alpha)^{\sum_i q_i}, a > 0, b > 0 \quad (7)$$

$$\frac{\max\{\pi_i\} - 3\min\{\pi_i\}}{2} \le \pi_i \le \frac{\max\{\pi_i\} + \min\{\pi_i\}}{2}$$

where π_i is an indicator of profitability of the reinsurance; a, b are the constant values, the parameters of the function of demand on the reinsurance market in case when this function is determined depending on the number of reinsurances of a certain risk; S_i – is the mathematical expectation of the insurance sums of the i insurance company; α – a share deducted by the insurance company; q_i – number of reinsurances of a certain risk taken by the i insurance company; c_i – variable costs of the i reinsurance company (liquidation, acquisition, collection); d_i – constant costs of the i reinsurance company (organizational, managerial); Z_i – insurance compensations; $1-\frac{\sum_i q_i}{\sqrt{\frac{\gamma\sum_i S_i}{\sum_i A_i}}}$ – size of the tariff rate of carrying out reinsurance operations to the sum of insurance.

One of the practical problems in determining the parameters of the demand function of reinsurance services is the necessity to transfer it into a linear form in order to use the method of the least squares. Therefore, we offer to perform the linearization in the following manner:

$$\ln\left[1-\frac{\sum_i q_i}{\sqrt{\frac{\alpha\sum_i S_i}{\sum_i A_i}}}\right]=\ln a-\sum_i q_i \ln\left[b\sum_{i=1}^{N} S_i(1-\alpha)\right]. \quad (8)$$

As a result of these transformations, the dependency of the cost of reinsurance services on the number of reinsurance stages is as follows:

$$p^*=a^*-b^*\cdot\sum_i q_i,$$

$$p^*=\ln\left[1-\frac{\sum_i q_i}{\sqrt{\frac{\alpha\sum_i S_i}{\sum_i A_i}}}\right], \quad (9)$$

$$a^*=\ln a, b^*=\ln\left[b\sum_{i=1}^{N} S_i(1-\alpha)\right].$$

The following operations are needed for a simpler demand function on the reinsurance market (as compared to model 7):

$$a=e^{a^*}, b=\frac{e^{b^*}}{\sum_{i=1}^{N} S_i(1-\alpha)}. \quad (10)$$

Let's make this model of optimization of the main parameters in the reinsurance companies' functioning in the form of analysis models of operational and marketing behavior strategies according to Karno and Stackelberg:

$$\begin{cases} \pi_i=(a^*-b^*\cdot\sum_i q_i)q_i-c_i q_i-d_i-Z_i, \\ p^*=a^*-b^*\cdot\sum_i q_i,\ a>0, b>0 \\ \frac{\max\{\pi_i\}-3\min\{\pi_i\}}{2}\le\pi_i\le\frac{\max\{\pi_i\}+\min\{\pi_i\}}{2} \end{cases}, \quad (11)$$

$$p^*=\ln\left[1-\frac{\sum_i q_i}{\sqrt{\frac{\alpha\sum_i S_i}{\sum_i A_i}}}\right],$$

$$a^*=\ln a, b^*=\ln\left[b\sum_{i=1}^{N} S_i(1-\alpha)\right].$$

The conditions of minimization and maximization of the profitability parameter for the formation of the efficiency margins in the reinsurance company's functioning can be written in the form of the following system:

$$\begin{cases} \frac{\partial\pi_1}{\partial q_1}=a^*-b^*\sum_{i=1}^{N} q_i-b^* q_1\left(1+\frac{\partial q_2}{\partial q_1}+\ldots+\frac{\partial q_N}{\partial q_1}\right)-c_1=0, \\ \frac{\partial\pi_2}{\partial q_2}=a^*-b^*\sum_{i=1}^{N} q_i-b^* q_2\left(\frac{\partial q_1}{\partial q_2}+1+\ldots+\frac{\partial q_N}{\partial q_2}\right)-c_2=0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \frac{\partial\pi_N}{\partial q_N}=a^*-b^*\sum_{i=1}^{N} q_i-b^* q_N\left(\frac{\partial q_1}{\partial q_N}+\ldots+\frac{\partial q_{N-1}}{\partial q_N}+1\right)-c_N=0. \end{cases} \quad (12)$$

According to the Karno and Stackelberg's models, to solve the system (12) it is offered to make assumptions, which allow to study different competitive positions of reinsurance companies and their strategies of market behavior.

Assumption 1. If every insurance company functioning on the reinsurance market makes decision regarding the terms of providing reinsurance services without consideration of the competitors' behavior $\partial q_i/\partial q_j=0, i\ne j, i=\overline{1,N}, j=\overline{1,N}$.

Mathematically, this condition is defined as follows:

$$\frac{\partial\pi_i}{\partial q_j}\,|\,\partial q_j/\partial q_i=0, i\ne j, j=\overline{1,N}, i=\overline{1,N}. \quad (13)$$

According to the model of Karno in relation to the analysis of operational and marketing strategies of behavior, the number of reinsurances of a certain type of risk and the cost of reinsurance services can be determined by using the following ratio:

$$q^* = \sum_{i=1}^{N} q_i^* = \frac{Na^* - \sum_{i=1}^{N} c_i}{(N+1)b^*}, p^* = a^* - b^* \sum_{i=1}^{N} q_i^* = \frac{a^* + \sum_{i=1}^{N} c_i}{N+1}, \quad (14)$$
$$a^* = \ln a, b^* = \ln\left[b \sum_{i=1}^{N} S_i (1-\alpha) \right].$$

Assumption 2. If every participant of the reinsurance market makes decisions taking into account the behavior strategies of competitors (Stackelberg's assumption). In other words, to simplify the system (12) the initial values according to the number of reinsurance stages look like this: $\frac{\partial q_2}{\partial q_1} = \frac{\partial q_3}{\partial q_1} = \frac{\partial q_N}{\partial q_1} = -\frac{1}{N}$.

With this assumption the following two situations are possible:

- one of the reinsurance companies uses the Stackelberg's reaction and believes that other reinsurance companies choose the Karno reaction, i.e. in planning the behavior strategy this company considers the competitors' activities while other companies use the opposite tactics. The optimal quantity of reinsurances for reinsurance companies, which are used to maximize the profitability figure, are as follows:

$$q_1^* = \frac{a^* - c}{2b^*}, q_2^* = \ldots = q_N^* = \frac{a^* - c}{2Nb^*}, \quad (15)$$
$$a^* = \ln a, b^* = \ln\left[b \sum_{i=1}^{N} S_i (1-\alpha) \right];$$

- all reinsurance companies plan their activities according to the Stackelberg's reaction. In other words, they consider that all other participants of the reinsurance market do not take into account the behavior of competitors. In this case, the optimal number of reinsurance of significant risks has the following form:

$$q_1^* = \ldots = q_N^* = \frac{N(a^* - c)}{(N^2+1)b^*}, \quad (16)$$
$$a^* = \ln a, b^* = \ln\left[b \sum_{i=1}^{N} S_i (1-\alpha) \right].$$

The use of the proposed approach of Karno and Stackelberg relating to the analysis of operational and marketing strategies of re-insurers' behavior with its further adaptation to the optimization of reinsurance companies' activity has a number of advantages and makes it possible:

- to receive an objective characteristic of identification of a reinsurer's functioning efficiency;
- to determine the optimal number of stages for reinsuring significant risks;
- to formalize the acceptable margins of insurance companies' profitable activity;
- to plan strategies of a further functioning and development of reinsurance companies.

Conclusions

The use of the Karno and Stackelberg models for the analysis of operational and marketing strategies of companies with their further adaptation to the reinsurance market makes it possible to carry out the modeling of the major parameters of reinsurance companies' functioning and optimize the number of reinsurance stages according to the degree of risks. The advantage of the proposed method of raising the reinsurance companies' efficiency is the consideration of different combinations of the behavior of reinsurance companies and their competitors on the market.