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## Using simulation to support the reinsurance decision of a medical stop-loss provider<sup>1</sup>

### Abstract

This article illustrates how simulation modeling can be employed to support the reinsurance decision of a medical insurer. We do this in the context of a simplified but realistic example, where a medical insurer is evaluating a request for proposal to provide stop-loss coverage for a trust, which provides comprehensive medical coverage to employees of a major conglomerate. We model claims frequency and individual loss severity under the assumption that the distribution of trended claims in the most recent five-year period is a good approximation to the distribution of claims in the rating period. Then we demonstrate how simulation can be used to evaluate alternative reinsurance options for the stop-loss provider. We incorporate uncertainty about the true loss distribution through the use of alternative distributions to model total claims.

**Keywords:** simulation, health reinsurance, medical stop-loss.

### Introduction

Early Thursday morning, on the 5<sup>th</sup> of January 2006, Elle Belmont, Vice President of Delta Health and Life, received a phone call from Cindy Philips, Director of Victory Trust, advising Belmont that the Trust is requesting Delta for a proposal to provide stop-loss coverage. She is quite pleased with the news, which couldn't have come at a better time: she is facing enormous pressure from the Board to deliver significant premium growth. And so, Elle Belmont decides to make a preliminary assessment of the risk and return associated with the opportunity given alternative reinsurance options.

The risk, which Victory Trust wishes to transfer is a type of risk, which Delta has assumed in the past, but at a smaller scale. Operating with a capital base of \$15 million, Delta's current portfolio consists of insurance policies of small sizes and low maximum exposure. Together, its portfolio of group term life, group dental and group health policies generates total annual gross premium revenue of \$25 million. Victory Trust potentially represents Delta's first "jumbo" policy, a policy covering more than 5,000 lives. From a medical underwriting standpoint, a very large group, like Victory Trust, is more attractive than small groups because small groups have a higher risk of having a disproportionate number of people in poor health<sup>2</sup>. However, the unusually large

size of the Victory stop-loss business raises concerns that the Victory stop-loss business opportunity is riskier than Delta's current portfolio. This is of particular concern at this time, given the upcoming rating agency annual review.

The request is for a proposal ("RFP") to provide stop-loss coverage with an attachment of \$200,000 and an \$800,000 maximum exposure per life per coverage period. The RFP states that only policies with a maximum rate of \$13.50 per individual per month<sup>3</sup> and a 6 months run-out period<sup>4</sup> will be given serious consideration. At \$13.50 PPPM and 20,000 exposures per month, the Victory Trust stop-loss opportunity represents \$3,240,000 increase in annual premium revenue. The question is whether, at Victory Trust's offered rate of \$13.50, the stop-loss opportunity exposes Delta's capital position to tolerable downside risk while providing attractive enough upside risk. We proceed first by analyzing what the average performance might look like. Next, we consider possible deviations from the average scenario, using Monte Carlo simulation. Finally, we assess alternative reinsurance strategies, which Delta can adopt to manage the downside risk. We incorporate uncertainty about the true loss distribution through the use of alternative distributions to model total claims.

### 1. Expected profitability of the stop-loss business

Victory Trust is the funding vehicle for the self-insurance plan of a major manufacturing conglomerate with thousands of employees in the Midwest<sup>5</sup>. The first graph, reproduced in Figure 1 below,

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<sup>1</sup> The authors are grateful to anonymous referees and also to participants at the RMIR Forum for helpful comments. Domingo Castelo Joaquin is grateful to Illinois State University for providing URG support for this research.

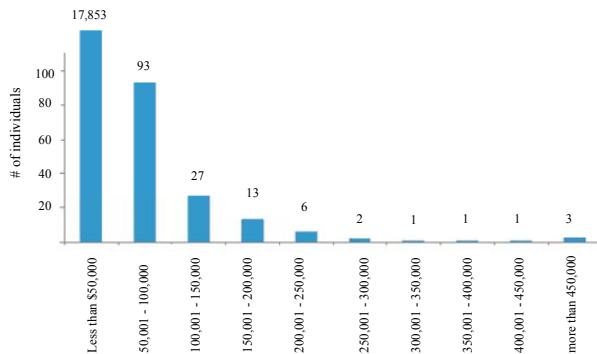
<sup>2</sup> A major reason for medical underwriting is to avoid situations, where persons apply for health insurance only when they are already sick. Such adverse selection problem is minimized for a very large group plan, like Victory Trust, where participation is triggered by employment, and not elected by those who already have some affliction – very large groups do not have disproportionately number of people in poor health. Accordingly, medical underwriting for large groups focuses on group characteristics like claims history, age and gender composition, geographic location, and employment conditions, as opposed to the health conditions of individual members of the group. In this setting we can invoke on the operation of the Law of Large Numbers.

<sup>3</sup> This is based on an expected total number of exposures during the rating period of 20,000 per month.

<sup>4</sup> This means claims must be paid within the policy period plus 6 months after the end of the policy period to be eligible under the stop-loss policy.

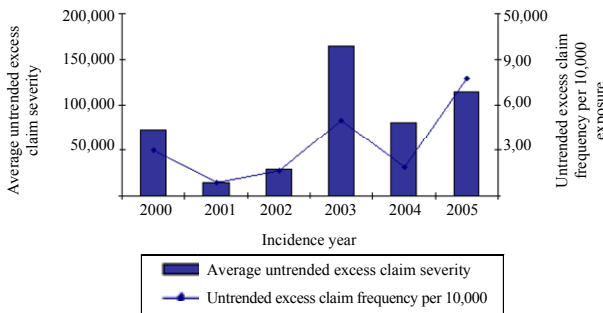
<sup>5</sup> The Trust is self-administered through its wholly owned third party administrator. The Trust collects and pays benefits from employer and employee contributions.

displays the frequency distribution of claims in 2005<sup>1</sup>. The general shape of the distribution, with a high frequency of small claims and a low frequency of large claims, is typical of Victory Trust's experience. The desire to stabilize cash flow and manage funding requirements led the Trust Director, Cindy Philips, to recommend that the Trust buy a per individual stop-loss insurance policy. Trading off insurance premium cost with expense fluctuation, the Victory's Management Committee agreed that the per individual deductible at \$200,000 would give the Trust enough stability.



**Fig. 1. Individual annual medical expense**

The next graph, reproduced in Figure 2 below, summarizes the risk that would have been transferred with the stop-loss insurance over the past five years. There is significant fluctuation in both the excess claim frequency and average excess claim severity. There is also a general upward trend on the excess claims frequency and excess claim severity<sup>1</sup>.



**Fig. 2. Untrended excess claim severity**

Given the expected premium revenue, the expected profitability of the stop-loss opportunity would depend on the expected stop-loss claims. For larger groups, expected claims are traditionally projected from prior claim experience<sup>2</sup>. The medical experience data provided by Victory Trust covers claims of at least \$50,000 each from year 2000 to year

2005<sup>3</sup>. To make the claims in different years comparable, in Table 1 we index all the claims to year 2005 by multiplying the actual claims during the incidence year by the corresponding cumulative trend factors<sup>4</sup>. We model claims frequency and individual loss severity under the assumption that the distribution of trended claims in the experience period of 2000-2005 is a good approximation to the distribution of claims in the rating period 2006<sup>5</sup>.

We are interested in claims, which pierce through the \$200,000 attachment point for the stop-loss policy. For example, a ground-up claim with Victory Trust of \$114,775 in 2000 would be multiplied by 1.78 to produce the trended value of \$204,063 for 2005 and stop-loss or excess claim of \$4,063. A ground-up claim of \$120,064 in 2003 would be multiplied by 1.27 to produce the trended value of \$151,953 for 2005 and not generate any excess claim. The resulting trended excess claim cost on a per person per month (PPPM) basis is graphically illustrated in Figure 3, together with the offered premium rate of \$13.50 PPPM. The opportunity looks promising, but warrants closer scrutiny. We need to find out if the margin between the offered premium and the trended experience, as illustrated in Figure 3, is adequate to cover: (1) the expected claims in 2006; (2) the known potential claims<sup>6</sup>; (3) administrative expenses; (4) a reasonable range of claim cost fluctuation considering that risk controls rest largely on the Trust<sup>7</sup>; and (5) return on capital.

<sup>3</sup> Delta examined and concluded that all historical claims over \$50,000 were paid before the run-out period expired so there is no need to truncate any claims to reflect the run-out period. Victory Trust certified that claims over \$100,000 are complete in the 2000-2005 historical experience period and there are no other potential claims that will penetrate the \$200,000 level during that period. Since the claims are complete, there is no adjustment to account for unknown claim development in the 2000-2005 experience period.

<sup>4</sup> Annual trend factors are based on trend rates published by actuarial and employee consulting firms including Segal, Towers Perrin, Aon and Milliman.

<sup>5</sup> In reality, the period from 2000 to 2005 may not be homogeneous from year to year. The age and gender distribution of covered persons can change. If so, among others, this could have an effect on the risk of pregnancy and heart problems. The employer being served by the Trust could have expanded to different geographic regions, which may exhibit differences in availability of health care facilities. The employer may change its activities, dropping previous ones and engaging in new businesses, thereby altering the type of work-related health risks to which its employees are exposed. There could also be a change in medical technology or a change in attitude toward the adoption on medical technology. For example, low birth weight (high claims) has become more of an issue in recent years partly due to the practice of in-vitro fertilization. The number of transplants has also increased tremendously. Both of these developments are influenced by many factors, including income levels and age distribution.

<sup>6</sup> As part of the medical underwriting process, the insurer has identified four covered persons, who have known medical conditions that could result in medical expenses above the \$200,000 deductible level.

<sup>7</sup> The Trust determines the eligibility of who is covered in the trust, handles all the administration including claim adjudications and obtains stop loss indemnity from the insurer on claims in excess of the self-funded amount for each covered life. This exacerbates Delta's risk at front end at the acceptance stage, as well as, the backend at the claims payment stage because the controls largely rest on the Trust, not Delta's.

<sup>1</sup> This is largely due to a general trend in medical costs for the population covered by the Victory Trust.

<sup>2</sup> Due to the short-tail nature of the business, it is customary to apply full credibility to prior claim experience data for health benefits for very large groups. See Lundberg (2003) for a discussion of large group medical underwriting.

Table 1. Historical trend factors

Year	Annual trend	Cumulative trend factor to 2005
2000	11%	1.78
2001	12%	1.60
2002	13%	1.43
2003	13%	1.27
2004	12%	1.12
2005		1.00

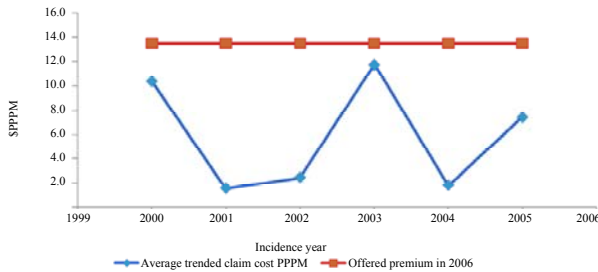


Fig. 3. Trended excess claim cost PPPM

## 2. Simulating the stop-loss opportunity

While an opportunity may appear profitable on the average, realized results will likely deviate from expected values. From a capital employment and capital position perspective, the Board is interested in the Victory Trust’s new business opportunity. On the other hand, it is also concerned about the possibility that the new business could deplete Delta’s capital position in the event of underwriting loss. The Board has some idea about the probability that Delta’s capital position would be depleted by at least five percent<sup>1</sup>. But it also is interested in the probability that the capital position will improve. We employ simulation modeling to estimate these probabilities.

Base-case ending capital position
EC = Ending capital <sup>2</sup> = S + NI
S = Starting capital = \$15,000,000
NI = Net income after tax = NIBT × (1 - T%)
NIBT = Net income before tax = P - C - A
T% = Assumed tax rate = 35%
P = Premium revenue = \$3,240,000 = \$13.50 PPPM × 12 months × 20,000 exposures
C = Stop loss claims = Simulated stop loss claims
A = Administrative expenses = \$278,200 = \$100,000 + 5.5% × P

Equation 1. Base-case ending capital position

Delta’s capital position at the end of the year is equal to its starting capital plus net income generated during the year. To focus on the potential impact of the Victory Trust business opportunity on Delta’s ending capital position, equation 1 defines incremental net income before tax as premium revenue from Victory Trust minus stop-loss claims, and minus administrative cost. The uncer-

tain variable here is the size of the stop-loss claims. In the Appendix, we show how to use experience data and trend factors to develop three models of total excess claims. We refer to these as Model 1, Model 2, and Model 3<sup>3</sup>.

Table 2 displays the results from running 10,000 iterations of the simulation models. The Model 1 and Model 2 paint a rosier picture than the Model 3. Under the first two models, the mean ending capital position is, respectively, \$15,096,990 and \$15,200,800. In other words, the capital position is expected to add about \$100,000 to \$200,000 to the starting capital of \$15,000,000. The probability of losing five percent of starting capital ranges from around 2% to 5%, but the probability of improving Delta’s capital position is between 60% and 70%. This is very encouraging. However, the Model 3 paints a bleak outlook, putting a probability of 40% that Delta will lose at least five percent of its starting capital and only a probability of 20% that Delta will fare better than breaking even. Those who believe that Model 1 and Model 2 are closer to the true model of total excess claims would likely support the Victory Trust stop-loss business. Those who believe that Model 3 is closer to the truth would be cautious and worried about the downside risk. Organizations are likely to have their share of optimists and skeptics. Neither group can ignore the other’s beliefs. Since, given the scale of the stop-loss policy, the support of both groups is necessary, as it stands the Victory Trust business opportunity is a no-go. It is necessary to reduce the downside risk to get the skeptics on board. Given the limited time Delta has to respond to Victory Trust. An efficient way of reducing downside risk is to transfer part of the risk to another insurer. This is what reinsurance does.

Table 2. Ending capital position: base case

	Model 1	Model 2	Model 3
Minimum	13,016,180	13,239,700	10,684,770
Maximum	6,308,390	16,330,050	16,197,110
Mean	15,096,990	15,200,800	14,384,980
Standard deviation	476,066	411,814	696,552

<sup>3</sup> All three models use the same Poisson distribution to model claims frequency, the same generalized beta distribution to model severity of claims from known sources, and the same generalized beta distribution to model the distribution of 2006 trend factors. A simulation model of total claims is referred to Model 1, Model 2, or Model 3, respectively if the severity of loss from unknown conditions is modeled using log-normal, inverse Gauss, or log logistic distribution, respectively. We incorporate uncertainty about the true-loss distribution through the use of all three alternative distributions to model total claims. These three distributions fit the trended claims data best out of more than ten continuous parametric distributions with positive support. Details are provided in the Appendix.

<sup>1</sup> Delta’s Management Committee fears that five percent depletion in capital position will attract unwanted attention from industry watchdogs.

<sup>2</sup> Investment income on capital is excluded. Investment income on the incremental cash flow from the stop-loss policy is marginal due to the short tail nature of the policy.

Table 2 (cont.). Ending capital position: base case

	Model 1	Model 2	Model 3
Variance	226,638,700,000	169,590,500,000	485,184,300,000
Skewness	(0.54)	(0.50)	(0.43)
Kurtosis	3.33	3.42	3.21
Number of errors	-	-	-
Mode	15,261,760	15,359,460	14,246,240
5.0%	14,236,050	14,468,640	13,160,680
10.0%	14,457,140	14,657,730	13,450,260
15.0%	4,607,680	14,777,410	13,663,460
20.0%	14,707,870	14,864,240	13,821,670
25.0%	14,803,430	14,949,700	13,953,090
30.0%	14,886,180	15,017,570	14,056,990
35.0%	14,956,940	15,072,650	14,163,760
40.0%	15,019,420	15,126,920	14,253,170
45.0%	15,083,460	15,181,040	14,348,060
50.0%	15,141,330	15,235,360	14,435,680
55.0%	15,196,480	15,284,080	14,526,420
60.0%	15,256,880	15,334,450	14,609,330
65.0%	15,316,120	15,385,610	14,691,060
70.0%	15,377,970	15,441,650	14,780,080
75.0%	15,436,370	15,491,520	14,880,000
80.0%	15,507,210	15,549,600	14,984,700
85.0%	15,581,310	15,616,050	15,093,710
90.0%	15,673,920	15,697,850	15,242,110
95.0%	15,794,830	15,821,650	15,450,260
Pr (end cap > start cap)	61.6%	71.4%	19.4%
Pr (lose at least 5%)	5.2%	1.9%	39.8%

### 3. Managing downside risk with reinsurance

Reinsurance is insurance provided by a reinsurer on policies originally issued by a primary insurer<sup>1</sup>. In exchange for a reinsurance premium, a reinsurance contract transfers uncertain financial consequences of covered loss exposures from the insurer to the reinsurer<sup>2</sup>. Reinsurance can serve different purposes. It can enable the primary insurer to write a large volume of similar policies, or a large amount of insurance on a single policy, or a policy in a new

<sup>1</sup> See Chapter 14 of Outreville (1998) for an introduction to reinsurance. See also Tiller and Tiller (1995) for a comprehensive introduction to life, health and annuity reinsurance.

<sup>2</sup> Reinsurance is basically a method of financing risk. On the average, the reinsurance premium is supposed to cover expected loss payments, expenses, and the cost of capital. If expected loss payments are high then the reinsurance premium is high as well. Reinsurance, as the term is used in the insurance industry and in this case study, should be distinguished from government-sponsored reinsurance. For example, Bovbjerg (1992) and Swartz (2003) make the case for the government to act as reinsurer to individual and small group health insurers and assume the risk of extremely high-cost people. In this case, the premium charged would be less than the present value of expected loss payments. The discount is essentially a subsidy, and government-sponsored reinsurance is a mechanism for providing subsidy to individual and small group health insurers. The hope is that the cost-sharing arrangement underlying the government-supported reinsurance program would make health insurance more accessible and affordable. See Blumberg and Holahan (2004) for an examination of potential savings to the private sector and potential costs to the government of alternative cost-sharing arrangements.

line of business. Reinsurance can make it possible for the primary insurer to survive the impact of catastrophic loss. It can also reduce earnings and cash flow volatility by putting a cap on the potential liabilities of the primary insurer. Finally, reinsurance can reduce the required unearned premium reserve, thereby providing surplus relief, which may be necessary to permit further premium growth.

Property & Casualty (P&C) losses tend to be very volatile and more prone to catastrophes. Consequently, P&C insurers tend to depend more on reinsurance for stabilizing cash flows and catastrophe protection. Very large health insurers with written premiums of upwards of \$1 billion a year tend to have claims that are predictable with high degree of accuracy. As a result, they rely less on reinsurance. Among health insurers, reinsurance would most likely be needed by small and medium sized insurers, who are particularly vulnerable to claims volatility or may need surplus relief. Since the health care market is dominated by jumbo health carriers, the demand for reinsurance by health carriers tends to be rather low<sup>3</sup>.

There are two types of reinsurance: treaty and facultative. Treaty reinsurance requires the primary insurer to cede all policies and requires the reinsurer to accept all cessions that satisfy the terms and conditions of the treaty. With facultative reinsurance, the primary insurer and the reinsurer negotiate each policy, which the primary insurer wishes to reinsure. Facultative reinsurance is usually employed in the reinsurance of catastrophic or unusual risk. A reinsurance contract can be a pro-rata or an excess of loss reinsurance contract. With a pro-rata contract, premiums, claims, and applicable expenses are shared proportionally. With an excess of loss contract, in exchange for a reinsurance premium, the reinsurer indemnifies the insurer when the latter's loss exceeds an agreed retention limit. Pro-rata reinsurance can be effective in enhancing an insurer's ability to write a large volume of similar policies or a large amount of insurance on a single policy. It also provides surplus relief by reducing the required unearned premium reserves. Excess of loss reinsurance can be effective in stabilizing loss experience and in providing catastrophe protection. Delta is considering two alternative risk management strategies: (1) a 50% quota share reinsurance; and (2) an aggregate excess reinsurance combined with a 50% quota share reinsurance.

<sup>3</sup> According to 2007 A.M. Best data, among health insurers with written premiums in the year 2007 of at least \$10 million, the top 50 firms account for 80 percent of all premiums written. Each top 50 health insurer is a jumbo carrier with written premium of at least \$1 billion in 2007.

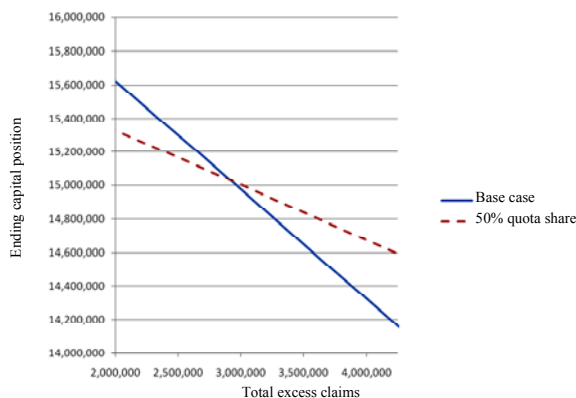
#### 4. Capital position with 50% quota share reinsurance

Under the 50% quota share contract, Delta shares with its reinsurer a fixed proportion (50%) of the premium, the claims, and applicable insurer’s expenses. In exchange, the insurer receives a ceding commission (10% of ceded premium revenue) from the reinsurer. Delta’s capital position at the end of the year is equal to its starting capital plus net income generated during the year. In equation 2, incremental net income before tax is defined as premium revenue from Victory Trust net of ceded premium, minus stop-loss claims net of quota share reinsurance claim recovery, minus administrative cost net of ceding commission.

Ending capital position with 50% quota share reinsurance	
EC = Ending capital = S + NI	
S = Starting capital = \$15,000,000	
NI = Net income after tax = NIBT × (1 – T%)	
NIBT = Net income before tax = P – C – A – (RP–RC–CC) =	
= (P–RP) – (C–RC) – (A–CC)	
T% = Assumed tax rate = 35%	
P = Premium revenue = \$3,240,000	
C = Stop-loss claims = Simulated stop loss claims	
A = Administrative expenses = \$278,200	
RP = Ceded quota share reinsurance premium revenue = 50% × P = \$1,620,000	
CC = Quota share reinsurance ceding commission = 10% × RP = \$162,000	
RC = Quota share reinsurance claim recovery = 50% × stop-loss claim	

**Equation 2. Ending capital position with 50% quota share reinsurance**

The downside risk reduction potential offered by 50% quota share reinsurance is illustrated in Figure 4. It plots ending capital position against total excess claims. Without reinsurance, Delta keeps the full premium received from Victory Trust, but suffers the full impact of an increase in excess claims. With 50% quota share reinsurance, Delta shares with the reinsurer not only the premium but also the impact of an increase in excess claims. Thus, the 50% quota share case corresponds to the flatter line. If total excess claims are very low, then Delta would be better off without reinsurance so it does not have to share its income. If total excess claims very high, then Delta is better off with reinsurance so it can share the loss with the reinsurer – the downside risk is reduced.



**Fig. 4. Risk reduction with quota share**

When total excess claims are \$2,916,000 in Figure 4, the ending capital position is at \$15,029,770 for the base case and also with 50% quota share. This corresponds to the point at which the two capital position lines cross. At the crossover level of total excess claims, it does not matter whether Delta reinsures or not. But since total excess claims are uncertain, realized excess claims may turn out to be higher or lower than the crossover level.

**Table 3. Ending capital position with 50% quota share reinsurance**

	Model 1	Model 2	Model 3
Minimum	4,022,970	14,134,740	12,857,270
Maximum	15,669,080	15,679,910	15,613,440
Mean	15,063,380	15,115,280	14,707,380
Standard deviation	238,033	205,907	348,276
Variance	56,659,680.00 0	42,397,630.00 0	121,296,100.00 0
Skewness	(0.54)	(0.50)	(0.43)
Kurtosis	3.33	3.42	3.21
Number of errors	-	-	-
Mode	15,145,760	15,194,610	14,638,000
5.0%	14,632,910	14,749,210	14,095,220
10.0%	14,743,460	14,843,750	14,240,020
15.0%	14,818,720	14,903,590	14,346,610
20.0%	14,868,820	14,947,000	14,425,720
25.0%	14,916,600	14,989,740	14,491,430
30.0%	14,957,970	15,023,670	14,543,380
35.0%	14,993,360	15,051,210	14,596,760
40.0%	15,024,600	15,078,350	14,641,470
45.0%	15,056,620	15,105,400	14,688,910
50.0%	15,085,550	15,132,560	14,732,720
55.0%	15,113,130	15,156,930	14,778,100
60.0%	15,143,320	15,182,110	14,819,550
65.0%	15,172,950	15,207,690	14,860,410
70.0%	15,203,870	15,235,710	14,904,930
75.0%	15,233,070	15,260,640	14,954,890
80.0%	15,268,490	15,289,690	15,007,240
85.0%	15,305,540	15,322,910	15,061,740
90.0%	15,351,840	15,363,810	15,135,940
95.0%	15,412,300	15,425,710	15,240,020
Pr (End cap > start cap)	64.0%	73.6%	20.6%
Pr (Lose at least 5%)	1.1%	0.1%	10.4%

Table 3 displays the results from running 10,000 iterations of the simulation models. With 50% quota share reinsurance, Model 1 and Model 2 still paint a rosier picture than Model 3. Under the first two models, the capital position is expected to improve by about \$60,000 to \$115,000. This is less than the \$100,000 to \$200,000 expected improvement in capital position in the absence of reinsurance. But the probability of improving Delta’s capital position now ranges from 64% to 73.6%. This is slightly better than the 60% to 70% range without reinsurance. However, although there is more opportunity for improving capital position with 50% quota

share, the profit opportunities are for the most part, less rewarding. As Figure 4 illustrates, when total excess claims are low enough, then the payoff without reinsurance is higher than with it. This is confirmed by the fact that the upper percentiles (50<sup>th</sup> through the 95<sup>th</sup> percentile) for ending capital position are lower with 50% quota share (Table 3) than without reinsurance (Table 2). The real contribution of reinsurance is in the reduction of downside risk. Under Model 1 and Model 2, the probability of losing at least five percent of starting capital is now down to between 0.1% and 1.1% compared to 1.9% and 5.2% without reinsurance. The reduction in probability is even more dramatic under Model 3: the probability of losing at least five percent of starting capital drops from 39.8% to 10.4%. Those believing that the appropriate model is closer to the Model 1 or Model 2, may be satisfied that the 50% quota share reinsurance lowers the downside risk level to quite tolerable level. However, those who find Model 3 to be very plausible may still be uncomfortable, that there is still a one in ten chance, that Delta will end up depleting its capital position by at least five percent. Delta might want to consider supplementing the 50% quota share reinsurance with an aggregate excess reinsurance.

**5. Capital position with 50% quota share and aggregate excess reinsurance**

Equation 3 calculates the ending capital position when the 50% quota share reinsurance is supplemented with aggregate excess reinsurance contract, which obligates the reinsurer to indemnify Delta up to a maximum amount of \$1,000,000, when Delta's aggregate retained loss for the year exceeds the retention limit of \$1,950,000. In exchange, Delta pays a reinsurance premium of \$100,000 to cover expected excess claims with allowance for fluctuations, management expenses, and to earn a reasonable return on invested capital.

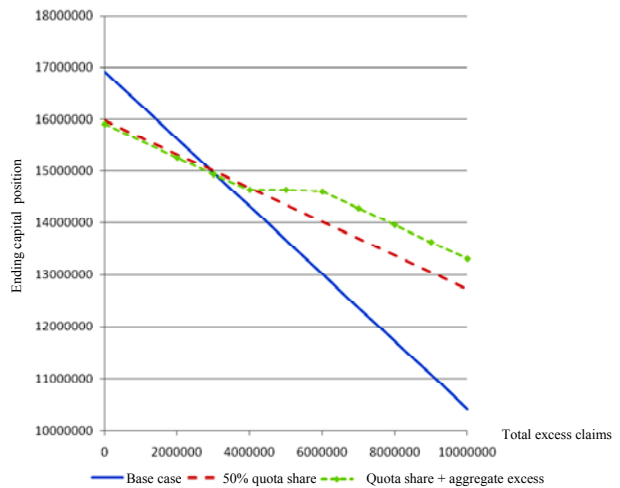
The aggregate excess cover is analogous to the stop-loss insurance, which Delta is being invited to provide to Victory Trust. The aggregate excess reinsurance kicks in when the total excess claims that remains, after the 50% quota share claim recovery, pierces through the \$1,950,000 attachment point. The additional downside risk reduction potential offered by aggregate excess reinsurance is illustrated by the flat segment in Figure 5. This segment reflects the immunization of ending capital position from further erosion with an increase in total excess claims, when excess claims are between \$3,900,000 and \$5,900,000<sup>1</sup>. When total excess claims are less

than \$3,900,000 then the addition of an aggregate excess cover reduces ending capital position by an amount equal to the after-tax aggregate excess reinsurance premium (\$65,000 = \$100,000 × (1-35%)).

Ending capital position with 50% quota share plus aggregate excess	
EC = Ending capital = S + NI	
S = Starting capital = \$15,000,000	
NI = Net income after tax = NIBT × (1 - T%)	
NIBT = Net income before tax = (P - C - A) - (RP - RC - CC) - (ARP - ARC) = (P - RP - ARP) - (C - CC - ARC) - (A - CC)	
T% = Assumed tax rate = 35%	
P = Premium revenue = \$3,240,000	
C = Stop-loss claims = Simulated stop-loss claims	
A = Administrative expenses = \$278,200	
RP = Ceded quota share reinsurance premium revenue = 50% × P = \$1,620,000	
CC = Quota share reinsurance ceding commission = 10% × RP = \$162,000	
RC = Quota share reinsurance claim recovery = 50% × Stop-loss claim	
ARP = Aggregate excess reinsurance premium = \$100,000	
ARA = Aggregate excess attachment point = \$1,950,000	
ARM = Aggregate excess maximum coverage = \$1,000,000	
ARC = Aggregate excess reinsurance recovery = MIN(MAX[(C - RC) - ARA, 0], ARM)	

**Equation 3. Ending capital position with 50% quota share plus aggregate excess**

When total excess claims are more than \$5,900,000 then the addition of an aggregate excess cover to the 50% quota share reinsurance increases the ending capital position by the difference between the maximum coverage amount after-taxes (\$650,000 = \$1,000,000 × (1-35%)) and the after-tax aggregate excess reinsurance premium.



**Fig. 5. Risk reduction with quota share and aggregate excess reinsurance**

Table 4 displays the results from running 10,000 iterations of the simulation models. With the addition of an aggregate excess reinsurance, the probability of losing at least five percent of starting capital is practically nil under the Model 1 and Model 2, and reduced to 0.5% under the Model 3. Concerns about downside risk would be put to rest with the addition of an aggregate excess cover. But, after

<sup>1</sup> Fifty percent of \$3,900,000 equals the attachment point of \$1,950,000. Fifty percent of \$5,900,000 equals \$2,950,000, which, in turn, is equal to the \$1,950,000 attachment point plus the \$1,000,000 maximum coverage.

paying the aggregate excess reinsurance premium, is there enough upside left to make the new business still attractive? Under Model 1 and Model 2, capital position is expected to improve by only \$9,880 to \$54,320. This is much less than the \$63,380 to \$115,280 expected improvement in capital position when only 50% quota share reinsurance is in place.

Table 4. Ending capital position with 50% quota share + aggregate excess reinsurance to \$54,320.

(This is much less than the \$63,380 to \$115,280 expected improvement in capital position when only 50% quota share reinsurance is in place).

	Model 1	Model 2	Model 3
Minimum	14,607,970	14,644,970	13,442,270
Maximum	15,604,080	15,614,910	15,548,440
Mean	15,009,880	15,054,320	14,774,610
Standard deviation	213,237	195,532	196,780
Variance	45,470,180,000	38,232,680,000	38,722,300,000
Skewness	(0.04)	(0.20)	0.85
Kurtosis	2.23	2.52	4.30
Number of errors	-	-	-
Mode	14,644,970	14,644,970	14,644,970
5.0%	14,644,970	14,684,210	14,644,970
10.0%	14,678,460	14,778,750	14,644,970
15.0%	14,753,720	14,838,590	14,644,970
20.0%	14,803,820	14,882,000	14,644,970
25.0%	14,851,600	14,924,740	14,644,970
30.0%	14,892,970	14,958,670	14,644,970
35.0%	14,928,360	14,986,210	14,644,970
40.0%	14,959,600	15,013,350	14,644,970
45.0%	14,991,620	15,040,400	14,644,970
50.0%	15,020,550	15,067,560	14,667,720
55.0%	15,048,130	15,091,930	14,713,100
60.0%	15,078,320	15,117,110	14,754,550
65.0%	15,107,950	15,142,690	14,795,410
70.0%	15,138,870	15,170,710	14,839,930
75.0%	15,168,070	15,195,640	14,889,890
80.0%	15,203,490	15,224,690	14,942,240
85.0%	15,240,540	15,257,910	14,996,740
90.0%	15,286,840	15,298,810	15,070,940
95.0%	15,347,300	15,360,710	15,175,020
Pr (End cap > start cap)	53.5%	62.5%	14.8%
Pr (Lose at least 5%)	0%	0%	0.5%

## Concluding remarks

Of the three models we considered, the first two models paint a much rosier picture than the last. Those who believe that Model 1 and Model 2 are closer to the true model of total excess claims would likely support the Victory Trust stop-loss business. Those who believe that Model 3 is a distinct possibility understandably would be cautious and worried about the downside risk of the proposed business. To achieve a consensus about the decision to offer stop-loss coverage to Victory Trust, it may be necessary to reduce the

downside risk. Delta can transfer part of the downside risk through reinsurance. From the simulation results, we found that a 50% quota share combined with an aggregate excess cover would reduce the probability (under Model 3) of losing 5% of initial capital from 29.8% to 0.5%. In addition, Delta can stipulate in its initial offer that the stop-loss policy would be renewable at Delta's option. Thus, one can view the current Victory Trust business as creating the option for Delta to provide a series of stop loss coverage to Victory Trust in the future<sup>1</sup>. If at the end of the policy period the majority of the Board agrees that the appropriate claims distribution is closer to the Model 1 or Model 2, Delta can offer to renew stop-loss coverage to Victory Trust. This time they may decide to drop the aggregate excess cover. If the consensus after one year is that the appropriate distribution is closer to Model 3, then they can modify their offer accordingly.

Aside from generating probability estimates, a simulation also provides a ranking of input variables in terms of their influence on an output variable<sup>2</sup>. Table 5 displays such ranking in terms of standardized betas, obtained from multivariate stepwise regression<sup>3</sup>. The precise ranking can vary somewhat across simulations. However, the general pattern would be the same and is more significant than the precise ranking in a specific simulation. In Table 5, the rankings of input variables are about the same across all risk management policy regimes<sup>4</sup>. The number of claims filed with the trust is the most influential variable, followed by severity of claims (in 2005 values) from the two, more costly, known conditions. The severity of claims (in 2005 values) from unknown conditions and the 2006 trend factor also have significant and practically the same degree

<sup>1</sup> This type of option is referred to in the finance literature as a real option. Real options differ from financial options in that the underlying asset for a real option is not a traded financial asset. In the current example, the underlying asset is a series of stop loss contracts in the future. See Culp (2001) for a discussion of real options in the context of risk management.

<sup>2</sup> In our example, the output variable of interest is the ending capital position, and the input variables are the number of claims filed with the Trust, the severity of claims (in 2005 values) from each of the four known conditions, the severity of claims (in 2005 values) from unknown conditions, and the 2006 trend factor.

<sup>3</sup> The stepwise regression technique regresses input variables against the output variable and excludes all variables that provide an insignificant contribution to the explanatory power of the model. The standardized betas are obtained by multiplying the coefficients from the final regression by the ratio of the standard deviation of the specific input variable to the standard deviation of the output variable. Thus, a standardized beta of 2 indicates that if  $x$  is increased by one standard deviation, then  $y$  is expected to increase by 2 standard deviations.

<sup>4</sup> The standardized betas are identical in the case of no reinsurance and 50% quota share reinsurance. This is to be expected since the 50% quota share does not change the relationship between marginal ending capital position and marginal claim costs. On the other hand, the introduction of an aggregate stop-loss would alter this relationship by introducing a discontinuity at the attachment point. And so, the coefficients for the last case are slightly different.

of influence on the ending capital position. From Table 5, we see that lowering the intensity rate for the claims distribution, which reduces claims by 1 standard deviation would improve ending capital position by 0.17 standard deviations. On the other hand, reducing severity by 1 standard deviation would improve ending capital position by only around 0.08 standard deviations.

The sensitivity analysis in Table 5 is suggestive of avenues for achieving a better distribution of the ending capital position. To be effective, such efforts should address weaknesses in the contractual relationship between Victory Trust and Delta Life and Health: Delta relies on the self-funded, self-administered Trust to perform accurate and unbiased administration consistent with guidelines. This, by itself, is a significant risk. Reinsurance would not be effective in managing this type of risk. What can help address this problem would be risk management procedures like:

- a) Clear administration and stop-loss agreements with strong contractual ramifications.
- b) Frequent underwriting, claim and transactional audits on the Trust's administration monitoring such as on-going claim experience reports, reports on individuals hitting catastrophic claim diagnosis, and reports on individuals reaching 50% of deductible.
- c) Claims and medical management support such as health care provider reimbursement negotiations, contracts with tertiary care and specialty provider network networks.
- d) Close working relationship with the Trust as an "involved" partner informed of underwriting and administration development.

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<sup>1</sup> See Lam (2002) for a comprehensive introduction to enterprise risk management.

Table 5. Sensitivity analysis

	Standardized beta / without reinsurance	Standardized beta / 50% quota share	Standardized beta / with aggregate excess
Number of claims filed with trust	(0.170)	(0.170)	(0.170)
Claim severity/3rd known condition	(0.087)	(0.087)	(0.088)
Claim severity/4th known condition	(0.080)	(0.080)	(0.082)
Claim severity/unknown condition	(0.068) to (0.082)	(0.068) to (0.082)	(0.065) to (0.077)
2006 trend factor	(0.075)	(0.075)	(0.075)

The optimal combination and configuration of risk management techniques is best chosen at the enterprise level, after analyzing, individually and collectively, all the risks to which Delta is exposed, not just the risks associated with the Victory Trust stop-loss business opportunity in isolation from other businesses of Delta Insurance. The enterprise risk management (ERM) approach can exploit natural hedges created by different businesses in the insurer's total portfolio and reveal opportunities for more efficient management of risk<sup>1</sup>. For example, Delta's current portfolio consists of insurance policies of small sizes and low maximum exposure. Its current portfolio is therefore likely to have very low, possibly zero, correlation with Victory Trust's stop-loss policy, which has an attachment point of \$200,000. Thus, the introduction of Victory Trust stop-loss policy would create diversification benefits, which could be assessed by supplementing our simulation model with an enterprise-level simulation model, which includes not only the Victory Trust stop-loss business opportunity, but also Delta's current portfolio of group term life, group dental, and group health policies.



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## Appendix.

### 1. Simulating total excess claims

Total excess claims consist of excess claims from known conditions and excess claims from unknown conditions. There are four known conditions covered by the policy. Severity of claims from known conditions is random. The number of claims from unknown conditions is also random, so the severity of individual claims is from unknown conditions. Loss severity in 2005 values is trended using a stochastic trend factor. The combination of all these random variables underlies the simulation model of total excess claims<sup>1</sup>. We model claims frequency and individual loss severity under the assumption that the distribution of trended claims in the experience period of 2000-2005 is a good approximation to the distribution of claims in the rating period of 2006<sup>2</sup>.

### 2. Simulating future costs: claims frequency<sup>3</sup>

We distinguish between claims filed with Victory Trust and stop-loss or excess claims filed with Delta. Based on the trended claims data, summarized in Table A1 below, 895 claims were filed out of a total number of 81,000 exposures. If the past trend continues, that would put the probability of a claim being made at  $p = 1.10\%$ . Given the expected total number of  $n = 20,000$  exposures in the rating period of 2006, this would translate to an expected number of claims of  $np = 220.99$ . Assuming that the loss events are independent, the variance of the number of claims would be  $np(1-p) = 218.55$ .

Table A1. Claim frequency based on trended data

Incidence year	Frequency	Exposure	Relative frequency
2000	96	10,000	0.009600
2001	142	11,000	0.012909
2002	123	12,000	0.010250
2003	225	14,000	0.016071
2004	159	16,000	0.009938
2005	150	18,000	0.008333
Total/Composite	895	81,000	0.011049

Since the mean is practically equal to the variance, we choose the Poisson distribution with a mean of 221 to represent the frequency of reported claims<sup>4</sup>. This distribution is illustrated in Figure A1 below<sup>5</sup>. We complete our model of claims frequency by incorporating information on potential prospective claims, identified during the standard stop-loss underwriting process. Delta established that known medical conditions during the experience period are expected to lead to four claims during the rating period that could penetrate through the stop-loss deductible. The calculation of the aggregate number and severity of claims are calculated and summarized in Equation A1.

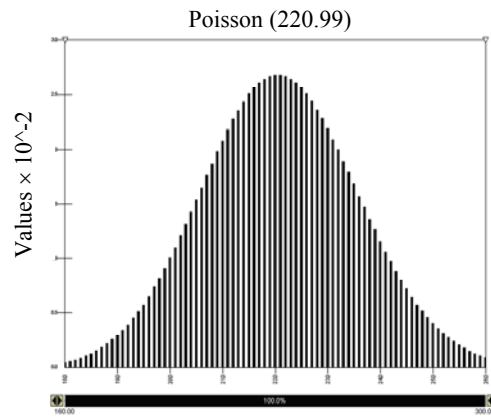
<sup>1</sup> We use @Risk, an Excel add-in, to run the simulation model. See Joaquin (2007) for a step-by-step, teach by example introduction to @Risk.

<sup>2</sup> In reality, the period from 2000 to 2005 may not be homogeneous from year to year. In addition to statistical fluctuations, certain factors might or might not be present in each year. If they are, the degree and influence could be different. For example, low birth weight (high claims) has become more of an issue in recent years partly due to the practice of in-vitro fertilization. The number of transplants has also increased tremendously. Both of these developments are influenced by many factors, including income levels and age distribution. If an underwriter knowingly or unknowingly put bad risks on the book, the book will perform poorly. It is unlikely, that we have the same underwriter for all the years. Even when we do, many things could influence the underwriter's practice - s/he could become more experience in risk selection, s/he could be influenced by how s/he is compensated (by volume or profitability), etc.

<sup>3</sup> Claims here mean medical expenses exceeding \$50,000 filed with Victory Trust. They might or might not become stop-loss claims.

<sup>4</sup> If we compute the average frequency and variance per 20,000 for each year, it turns out that our assumption about the mean and the variance is equivalent to assuming that the expected number of claims is equal to the average of the annual average frequency and the variance is equal to the average of the annual variances. We note further that the minimum value of the number of insured claims distribution is less than 5 (the trended 2004 claim frequency per 20,000 covered lives) so the distribution is inclusive of the historical outcome.

<sup>5</sup> This distribution assumes an intensity rate equal to the average intensity for the experience period (the past six years). From Table A1, we note that this is higher than the relative frequency for the most recent two years. A higher relative frequency is expected with a stop-loss contract in which Victory Trust, not Delta Health, determines the eligibility of who is covered in the trust, handles all the administration, including claim adjudications and obtains stop-loss indemnity from Delta Health on claims in excess of the self-funded amount for each covered life.



**Fig. A1. Distribution of claim frequency**

$$C = \text{total stop loss claims} = C(U) + C(K)$$

$$C(U) = \text{Total claims from unknown conditions}$$

$$C(U) = \sum_{i=1}^N C(U_i)$$

$$C(K) = \text{Total claims from known conditions}$$

$$C(K) = \sum_{j=1}^4 C(K_j)$$

$$N = \text{Number of covered claims from unknown conditions}$$

$$N \sim \text{Poisson}(\lambda = 221)$$

$$\text{Number of claims from known conditions} = 4$$

**Equation A1. Total simulated claims**

### 3. Simulating future costs: claim severity

In the previous section, we trended claims during the experience period of 2000-2005 using historical trend factors. What we want to do next is to find the distribution that reflects the population in the rating period of 2006. We do this by finding the distribution that fits the trended data base in terms of the Anderson-Darling statistic<sup>1</sup>. The Anderson-Darling statistic takes the whole distribution into account by taking the probability-weighted sum of the squared vertical distance between the cumulative distribution function of the fitted distribution and that of the data at all data points. Thus, if the required data is available, the Anderson-Darling goodness of fit criterion would be ideal for loss modeling applications<sup>2</sup>. Based on this criterion, the distributions which fit the trended claims data base are the lognormal distribution, the inverse Gauss distribution, and the log logistic distribution<sup>3</sup>. These distributions are shifted versions of standard distributions. For example, the fitted lognormal distribution is obtained by shifting the location of a lognormal distribution (with a mean of \$61,591 and a standard deviation of \$76,935) by \$26,018<sup>4</sup>. Thus, the mean of the shifted distribution is equal to \$87,610 compared to the observed average of \$87,343. The shift variable only affects location,

<sup>1</sup> The selection was made from continuous parametric distributions with positive support. The candidate distributions included the generalized beta, exponential, gamma, inverse Gauss, log logistic, lognormal, pareto, Pearson, triangular, uniform, and the Weibull distribution.

<sup>2</sup> The two other common measures of goodness of fit are the Chi-square statistic and the Kolmogorov-Smirnov statistic. The Chi-square statistic measures how well the expected frequency, given the fitted distribution, compares with the observed frequency from a histogram of the observed data. It is most useful for fitting distributions to nominal data. It is also suited for group data. But an issue arises as to how the data is split to construct the histogram. The Kolmogorov-Smirnov statistic avoids the problem of specifying the number of intervals to split the data into by focusing only on the maximum vertical distance between the cumulative distribution function of the fitted distribution and that of the data. Its main weakness is that it ignores the lack of fit over the rest of the distribution. In particular, it will generally be insensitive to lack of fit at the tails of the distribution. See Chapter 13 of Klugman, Panjer, and Willmot (2004) for a detailed discussion of goodness of fit criteria and the critical values for the Anderson-Darling test.

<sup>3</sup> All three distributions are plausible candidate distributions at the 10%, 5%, and 1% significance levels. Appendix A of Klugman, Panjer, and Willmot (2004), contains a summary of the statistical properties of the three distributions. The notation in Table 3 is adopted from this Appendix.

<sup>4</sup> The lognormal distribution (with different parameters) also is the best fitting distribution to the 2006 statistics on health care spending in the United States. See Kaiser Family Foundation (2007) for 2006 statistics on health care spending.

but not the shape of the distribution. And so, the standard deviation of the shifted distribution is the same as the unshifted value, i.e., \$76,935. A comparison of the fitted distributions with the empirical distribution given in Table A2 below reveals that, consistent with the experience data, all three fitted distributions put the probability of a loss piercing through the \$200,000 attachment point at slightly more than 5%.

The next step is to trend samples from this fitted distribution to the rating period of 2006, using a stochastic trend factor. Based on industry forecasts available as of the end of year 2005, we model this trend factor using Pert distribution with a minimum value of 10%, a maximum value of 18% and a most likely value of 12%<sup>1</sup>. The associated density curve is given in Figure A2. The stop-loss policy under consideration has a \$200,000 deductible and the maximum exposure per life per coverage period is \$800,000. The calculation of severity of individual claims from unknown conditions is summarized in equation A2.

Table A2. Fitted loss distributions

Parameters	Input	LogNormal	Inverse Gauss	Loglogistic
		$\mu=61591$	$\mu=64061$	$\gamma=1.6999$
		$\sigma=76935$	$\theta=55311$	$\theta=36716$
		Shift=26018	Shift=23281	Shift=27475
Distribution statistics				
Minimum	27983	26019	23282	27476
Maximum	861004	+Infinity	+Infinity	+Infinity
Mean	87343	87610	87343	98027
Mode	52443	41053	40402	44070
Median	62766	64511	64513	64193
Std. deviation	73565	76935	68942	+Infinity
Skewness	3.9871	5.6964	3.2286	+Infinity
Kurtosis	27.9014	93.2036	20.3728	+Infinity
Percentiles				
5%	33849	33830	33873	33971
10%	37826	37129	37071	37557
90%	156691	159379	163915	161201
95%	203435	215694	218758	235020
A-D statistic		0.359	0.4951	0.864
Cr. value @ 0.10		1.933	1.933	1.933
Cr. value @ 0.05		2.492	2.492	2.492
Cr. value @ 0.01		3.875	3.875	3.875

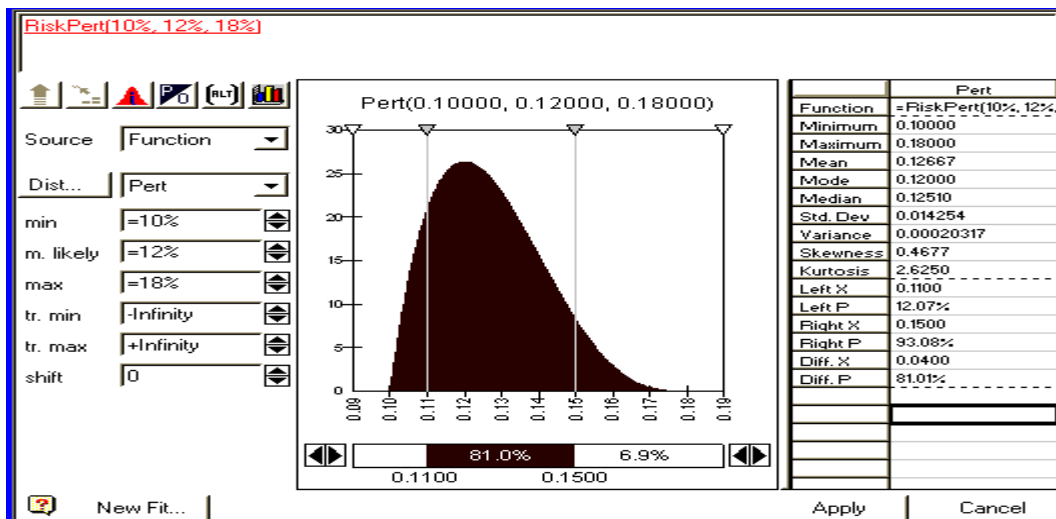


Fig. A2. Distribution of 2006 trend factor

<sup>1</sup> Mathematically, the Pert distribution is generated as a special case of the generalized beta distribution by imposing the constraint that the mean of the generalized beta be equal to (minimum + 4\*most likely + maximum)/6. This constraint allows the four parameters of the generalized beta distribution to be determined from three input values: the minimum, most likely and maximum values. The distribution can be made symmetric by choosing the mode to be midway between the maximum and the minimum. It can be made skewed by choosing the mode to be closer to one of the end-points. See Appendix A of Klugman, Panjer, and Willmot (2004) for a summary of the statistical properties of the generalized beta distribution. See also, Vose (2000) for further discussion of the Pert distribution.

$T = 2006 \text{ trend factor}$

$T \sim \text{Pert}(\text{min} = 8\%, \text{most likely} = 12\%, \text{max} = 18\%)$

$C(U_i) = \text{MIN}[\text{MAX}((1 + T) \times U_i - 200000, 0), 800000]$

$U_i \sim \text{LOGNORMAL}(\mu = 61591, \sigma = 76935, \text{shift} = 2619)$

**Equation A2. Severity of claims from unknown conditions**

Let us now incorporate the four known claims that could penetrate through the stop-loss deductible. Based on expert analysis of the four known conditions, the severity of these claims in 2005 values is assumed to follow the Pert distribution with parameters given in Table A3 below<sup>1</sup>.

Table A3. Severity of known claims

Known claims	Ground-up claims best estimate	Ground-up claims minimum	Ground-up claims 95th percentile	Best estimate claims trended to 2006	Minimum claims trended to 2006	Ground-up claims 95th percentile trended to 2006	Simulated claims with known condition in excess of 200,000
1	\$250,000	\$225,000	\$350,000	\$280,000	\$252,000	\$392,000	113,048
2	\$250,000	\$225,000	\$350,000	\$280,000	\$252,000	\$392,000	113,048
3	\$450,000	\$400,000	\$600,000	\$504,000	\$448,000	\$672,000	349,844
4	\$450,000	\$400,000	\$600,000	\$504,000	\$448,000	\$672,000	349,844
Total							925,784
Allowance for claims on known conditions							\$96,248
Claims on known conditions in excess of allowance							\$829,536

The attachment point is \$200,000 and the maximum exposure per life per coverage period is \$800,000. Also, there is an allowance for total claims from known conditions equal to \$96,248<sup>2</sup>. This means, for example, that Delta is expected to pay \$829,536 to Victory Trust if the total of excess claims with known conditions is equal to \$925,784. The calculation of severity of individual claims from known conditions is summarized in equation A3.

$C(K_i) = \text{MIN}[\text{MAX}((1 + T) \times K_i - 200000, 0), 800000]$

$K_1 \sim \text{PERT}(\text{Mean} = 250000, \text{Min} = 225000, \text{95th percentile} = 350000)$

$K_2 \sim \text{PERT}(\text{Mean} = 250000, \text{Min} = 225000, \text{95th percentile} = 350000)$

$K_3 \sim \text{PERT}(\text{Mean} = 450000, \text{Min} = 400000, \text{95th percentile} = 600000)$

$K_4 \sim \text{PERT}(\text{Mean} = 450000, \text{Min} = 400000, \text{95th percentile} = 600000)$

**Equation A3. Severity of claims from known conditions**

**4. Three simulation models of stop-loss claims in the rating period**

We run three simulation models of stop-loss claims in the rating period. All three models use the same Poisson distribution to model claims frequency, the same generalized beta distribution to model severity of claims from known sources, and the same generalized beta distribution to model the distribution of 2006 trend factors. The models differ in

<sup>1</sup> An evaluation of the four identified cases allowed us to specify upper and lower bounds, and also the most likely value of loss severity. In the case of known claims, the random severity variable may be more naturally represented by distributions, like the Pert distribution, which are characterized by these parameters (minimum, maximum, and most likely value), rather than by other loss distributions, like the lognormal distribution, which are not bounded above. We use the “Alt Parameters” feature of @Risk to substitute the expected value, the minimum value, and the 95th percentile as parameters of the Pert distribution, instead of the minimum, the maximum and the most likely value.

<sup>2</sup> The allowance equals five percent of the product of the expected number of claims in the rating period and the average claim severity trended to 2005, the final year of the experience period.

terms of the distribution used to model severity of loss from unknown conditions. A simulation model of total claims is referred to Model 1, Model 2, or Model 3, respectively if the severity of loss from unknown conditions is modeled using lognormal, inverse Gauss, or log logistic distribution.

Table A4 displays the results from running 10,000 iterations of the simulation models. Under the first two models, the mean total simulated excess claims are lower than the total premium revenue by about \$500,000<sup>1</sup>. The probability of total excess claims being lower than the total premium revenue is between 70% and 80%. This is in sharp contrast with Model 3, under which it is expected that total stop-loss claims will exceed total premiums by roughly \$500,000; and where there is about 70% probability that total excess claims will exceed total premium revenue. Those who believe that Model 1 and Model 2 are closer to the true model of total excess claims would likely support the Victory Trust stop-loss business. Those who believe that Model 3 is a distinct possibility would be cautious and worried about the downside risk of the proposed business. We incorporate uncertainty about the true-loss distribution through the use of alternative distributions to model total claims.

Table A4. Simulated total excess claims

	Model 1	Model 2	Model 3
Minimum	948,898	915,569	1,120,088
Maximum	6,013,837	5,669,954	9,600,611
Mean	2,812,580	2,652,883	3,907,983
Standard deviation	732,409	633,560	1,071,618
Variance	536,423,000,000	401,397,600,000	1,148,365,000,000
Skewness	0.54	0.50	0.43
Kurtosis	3.33	3.42	3.21
Number of errors	-	-	-
Mode	2,559,096	2,408,792	4,121,438
5.0%	1,738,961	1,697,328	2,267,188
10.0%	1,924,935	1,887,922	2,589,248
15.0%	2,066,738	2,014,023	2,817,437
20.0%	2,181,382	2,116,157	2,985,295
25.0%	2,290,315	2,205,266	3,146,397
30.0%	2,379,917	2,282,294	3,300,110
35.0%	2,475,428	2,368,404	3,436,998
40.0%	2,566,559	2,447,053	3,562,698
45.0%	2,659,335	2,524,606	3,690,176
50.0%	2,744,364	2,599,551	3,829,923
55.0%	2,833,305	2,683,100	3,964,715
60.0%	2,931,725	2,766,490	4,110,658
65.0%	3,026,953	2,849,678	4,247,805
70.0%	3,136,591	2,934,671	4,412,442
75.0%	3,264,174	3,038,759	4,571,656
80.0%	3,410,702	3,170,403	4,774,163
85.0%	3,565,040	3,303,578	5,017,932
90.0%	3,796,933	3,487,834	5,345,570
95.0%	4,136,955	3,778,705	5,790,397
P(C<3,240,000)	74.3%	82.8%	28.0%
P(C<2,916,000)	59.2%	68.9%	17.7%
P(C<3,900,000)	92.0%	96.3%	52.7%
P(C<5,900,000)	100.0%	100.0%	95.9%

<sup>1</sup> Total premium revenue = \$3,240,000 = \$13.50 × 20,000 × 12. Expected total claims are \$2,812,580 and \$2,652,883 under Model 1 and Model 2, respectively.