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The evolution of risk diversification

Abstract

Reducing portfolio risk is a major concern for most investors. Diversification has always been the simplest way to address such a concern. The previous literature shows clearly that the random purchase of a sufficiently large number of stocks can substantially reduce risk and thus achieve diversification. But an important question remains: how many stocks can achieve diversification? The author traces back in details the development of the literature addressing this question and presents some evidence contradicting the current trend that claims that a diversified portfolio requires hundreds of stocks.

Keywords: diversification, risk, portfolio, size.

Introduction

The topic of risk reduction through diversification has grabbed the attention of researchers for over half a century. Researchers' conclusions about diversification have evolved and changed substantially over the years. Whilesome recommend a portfolio as small as ten stocks, others argue that hundreds of stocks are needed to achieve diversification.

The purpose of this paper is twofold. First I review a comprehensive literature about portfolio diversification. Second, I present some new evidence contradicting the conclusions of recent research and supporting earlier research.

This paper is organized as follows. First, a comprehensive literature review is presented. Second, new empirical tests are conducted and the results are presented. Finally, the paper concludes and provides recommendations.

1. Literature review

Ever since the influential work of Markowitz (1952) and Sharpe (1963, 1964) the question of the number of common stocks required to achieve diversification has received considerable attention. To address this question some researchers focus on naïve diversification and others focus on deliberate diversification strategies. But these are not the only issues explored. Important topics such as the cost difference between direct and indirect diversification, under-diversification, and time diversification are also researched. This section explores all these issues and links them to the objective at hand.

This section is divided into five major parts. The first part offers a description of research on the difference between direct and indirect diversification. The second part provides evidence and potential reasons of under-diversification. The third part relates the issue of time diversification to this study. The fourth part reviews some of the work done on passive diversification. Finally, the fifth

part describes some of the work done on active diversification.

1.1. Direct vs. indirect diversification. Investors who seek diversification can have the choice of either doing it directly or indirectly through a mutual fund. Surprisingly few papers attempt to compare both strategies.

Smith and Schreiner (1970) develop a cost comparison model of direct vs. indirect diversification. They assume that securities grow at a constant rate and make the comparison on a cash-to-cash basis by including various cost components. Their model proceeds as follow. Let I represents the initial wealth and R_n the value of the investment after n years. The ratio of final wealth to initial wealth for a direct diversifier is given by:

$$\frac{R_n}{I} = \frac{(G - 2FT)^n(1 - F)}{1 + F},$$

where, G is the growth rate, F is the brokerage transaction cost expressed in percentage of wealth invested, and T is the percentage of the portfolio that is turned over.

For an indirect diversifier, the ratio of final wealth to initial wealth is given by:

$$\frac{R_n}{I} = (1 - L)(G - 2F'T - M)^n,$$

where, L is the load charge, F' is the trading commission, T' is the fund turnover, and M is the management fee.

Smith and Schreiner consider 5 mutual funds, 4 initial wealth levels (\$1,000, \$10,000, \$100,000, and \$1,000,000) to accommodate different investment levels, and 5 investment horizons (1, 2, 4, 8, and 16 years). The growth rate, G , is assumed to be 10 percent. The mutual fund management fee, M , is 0.5 percent. The turnover of securities, T' , in each of the funds varies from 12.9 percent to 77 percent. T is assumed to be equal to T' for each comparison. The buying or selling average commission F' varies from 0.41% to 0.58%. The individual investor's buying or

selling commission F is based on the sixteen largest holdings of each fund. Sixteen is assumed to be the size of a well-diversified portfolio. F varies from 0.42 percent for a large investment to 9.09% for a small investment. Finally, the loading charges for the funds vary across funds and investment levels. For example, one mutual fund charges 8.5 percent loading fee if the initial investment was \$1,000 and 1 percent if the initial investment was \$1,000,000.

The results of their investigation suggest that a small investor would find it more rewarding to diversify through a fund and a large investor would be better off diversifying directly. The paper also concludes that the investment size for which the investor is indifferent as to which alternative he uses to obtain diversification increases with the investment horizon length. The authors call this size the indifference point.

In a subsequent paper Schreiner and Smith (1980) explore the impact of Mayday on diversification costs. On May 1, 1975 the practice of fixed brokerage commission for securities trading was ended and negotiated commissions became the norm. Schreiner and Smith use the same cost model and find that, because small investors do not have bargaining power, this new law makes it harder for them to diversify directly. On the other hand, large investors, who usually have bargaining power, can diversify directly with more ease. Fielitz (1974) finds that direct diversification can be done at a reasonable cost. He accounts for transaction costs and concludes that investors can build portfolios which net performance equates the performance of an index fund.

Smith and Schreiner (1970), Schreiner and Smith (1980), and Fielitz (1974) base their remarks on the previous findings that diversification can be achieved using a small number of stocks. However, as will be discussed later in this chapter, there is rising evidence that this number is actually significantly larger.

Poterba (2001) reports, that 41 percent of U.S. households own mutual funds. The big growth of the mutual funds is to some extent a puzzle, at least from a tax perspective. The Investment Company Act of 1940 specifies how mutual funds are taxed. A direct investor is not taxed on capital gains until he realizes the gains. With a mutual fund, an investor can be taxed on capital gains he did not realize. When a fund sells assets, the gains are passed through to the investors who pay taxes on them. This means that investors can be taxed even if capital gains are reinvested.

An indirect investor could also pay taxes even if no capital gains are realized. Many funds have an overhang of unrealized capital gains. An investor who just purchased the fund could pay taxes on capital gains he didn't earn because the gains are distributed on a pro rata basis to all shareholders in the fund. Dickson, Shoven and Sialm (2000) explore another situation where investors' after-tax return depends on the behavior of other shareholders. Redemption by a large number of investors can force the mutual fund to sell some of its equity to pay the investors who want out. This involuntary sale produces taxable capital gains. Dickson, Shoven and Sialm (2000) show, that this involuntary sale is an important determinant of the after tax performance of equity mutual funds. In sum, an indirect investor does not have the liberty of deferring capital gains.

An additional advantage of direct diversification is the possibility of short selling. Short selling can substantially enhance return and make diversification easier to achieve. Traditionally a long-short strategy consists of buying stocks with positive expected return and short selling stocks with negative expected returns. For example, Kao (2002) finds that hedge funds that use long-short strategies have higher and more consistent alphas than mutual funds that use only long strategies. Jacobs, Levy, and Starer (1999) argue that long-short portfolios benefits outweigh their costs. It must be noted, however, that short selling is a restricted activity and has often been blamed for destabilizing the market. One of the current restrictions is the uptick rule (SEC rule 10a-1 and 10a-2) introduced in 1938. The 10a-1 rule prohibits investors from selling an exchange-listed stock short unless the stock's last trade was at the same price or higher than the previous trade. The 10a-2 rule requires brokerage firms that sell a stock short or allow their customers to sell short to first make sure that the shares can be borrowed or that delivery of the securities can be made to the purchaser by the settlement date.

Short selling is not cost-free. Investors who sell securities short cannot reinvest the proceeds but rather have to keep them in a brokerage account. Therefore they forgo an investment opportunity. Additionally, they have to deposit a margin as if they have a long position. The margin can be cash or any restricted security held long. Other costs include costs due to delayed trading (rule 10a-1) and other trading costs.

Schroeder (2001) reports, that the SEC may propose easing restrictions on short selling of big stocks. The uptick rule is likely to be replaced by a new rule saying that short selling can occur only when the last best bid is an increase from the previous best bid price. This change is likely to affect only the 100

to 150 of the most actively traded stocks because reliable information is widely available about them and are, therefore, the hardest to manipulate. Indeed the SEC fears a practice called short and distort where an unscrupulous trader could short a stock and then spread false negative information to drive down the price of the targeted stock. Schroeder (2001) also reports that a major reason behind the new easing of restrictions on short selling is the introduction of single-stock futures. The SEC is concerned that investors could shift their investments to the futures market because it has no short selling restrictions.

Transactions costs are the key factor when comparing direct versus indirect investments. According to a Securities Industry Association (SIA) survey, online trading will make up 50 percent of stock market trades in 2003, up from 37 percent in 1999. This exponential growth of online trading was accompanied by an equally dramatic decrease in transaction costs. Lower transaction costs means that investors have more incentives to invest directly if they wish to do so. This is especially true for investors whose objective is to diversify.

There are other factors that could influence the balance in favor of direct or indirect diversification. Investing in an index fund, for example, guarantees diversification. On the other hand, investors, as discussed in the next section, can choose to invest directly for psychological reasons. They may want to put themselves in a situation where they can personally control their portfolio.

1.2. Investors and under-diversification. There is overwhelming evidence that investors are not well diversified. The Wharton survey of 1975 reported in Blume and Friend (1978), for example, reports that the median number of stocks held was found to be less than four, and 34 percent of the investors had two or less stocks in their portfolio. Goetzmann and Kumar (2002) examine the portfolios of more than 40,000 equity investment accounts of a discount brokerage firm. They find that an overwhelming majority of investors are under-diversified. Many papers attempt to explain this phenomenon.

Goldsmith (1976) develops a transactions costs approach. His model shows that the fraction of the portfolio invested in risky securities rises with wealth at a declining rate and that the number of holdings increases approximately as the square root of total wealth. This finding is later confirmed by Goetzmann and Kumar (2002) who find a positive relationship between wealth and number of holdings.

Conine and Tamarkan (1981) suggest that investors' preference for positive skewness restricts the num-

ber of securities held. The intuition is that a well diversified portfolio will reduce variance (positive aspect) and decrease positive skewness (negative aspect). The optimal number is obtained when the marginal increase in expected utility from a decrease in variance is equal to the marginal decrease in expected utility from the reduction in skewness.

There are numerous papers that provide explanations for the phenomenon of under-diversification. Merton (1987), for example, argues that search and monitoring costs severely limits portfolio size. DeBondt (1998) contends that investors develop a false perception that they can manage their portfolio through understanding of each security rather than through diversifying. Huberman (2001) finds that investors tend to invest in familiar stocks which create an illusion of control and a dangerous level of overconfidence which in turn leads to under-diversification. Kelly (1995) argues that this overconfidence leads investors to believe that they can beat the market and, therefore, do not need to diversify. Odean (1999) confirms Kelly's findings; he argues that investors who are overconfident tend to adopt an active strategy. Finally, Goetzmann and Kumar (2002) find that young investors and investors in low-income and non-professional categories hold the least diversified portfolios.

More recently Guiso and Jappelli (2009) document the extent of portfolio diversification through a survey of a large sample of Italian clients of an Italian bank. They find quite conclusively that the lack of financial literacy is the main variable explaining the under-diversification phenomena. They also find that the older households, risk averse investors, and low educated people were the most likely to be financially illiterate and thus the most under-diversified.

1.3. Time diversification. Bernstein (1976) argues that time diversification is as important as asset diversification. He bases his argument on a simple historical fact; over time, above-average returns tend to offset below-average returns. Lloyd and Haney (1980) argue that time diversification is more important than asset diversification. They contend that in order for a well-diversified portfolio to realize the market return it has to be held for a sufficiently long period of time. They use a universe of 100 stocks over the period from 1971 to 1976 to show that holding one stock for 6 years, for example, is less risky than holding 100 stocks for one year.

Samuelson (1990) argues that it is misleading to claim that time is on the investor's side. Samuelson adopts a set of assumptions such as investors are risk averse and have a utility function equal to the logarithm of wealth, returns are random, and future wealth depends only on investment results. To make

his point, Samuelson notes that while the dispersion of returns converges toward the expected value with the passage of time, the dispersion of terminal wealth diverges away from its increasing expected value. This means that the decrease in likelihood of loss over time is accompanied by an increase in the magnitude of potential loss.

Despite Samuelson's argument, many scholars attempt to resurrect time diversification. For example, Lee (1990) shows that when returns are predictable, time can reduce risk. On the other hand, if returns follow a random walk the investment horizon becomes irrelevant. Lam and Zou (2000) argue that Samuelson looked only at a restricted model in which the amount invested in risky assets does not vary across time. If this assumption is relaxed risky investments can become more attractive (increased utility) as the investment horizon is extended. Krizman (1994) makes five arguments in favor of time diversification. First, if investors believe that returns are not random, then the standard deviation of terminal wealth increases at a more moderate rate. Second, bad outcomes are extremely unlikely. If they occur, chances are that riskless assets might also default. Third, investors are often willing to accept risk over longer horizons than shorter horizons because they can adjust their consumption habits when they are given time to do so. Therefore the assumption that future wealth depends only on investment performance, if relaxed, can lead investors to favor longer term risky assets. Fourth, investors may have a discontinuous utility function. Finally, investors behave irrationally.

Because this research uses the reduction of terminal wealth standard deviation as a measure of diversification, the issue of time diversification must be acknowledged. Samuelson notes that the dispersion of terminal wealth increases with time. This implies that the number of assets needed to achieve diversification, as measured by the reduction of terminal wealth standard deviation, may increase with time. The time effect is not captured if diversification is measured through reduction of times series standard deviation.

1.4. Passive diversification. Direct diversifiers can engage in either a passive or an active diversification strategy. In a passive diversification strategy investors choose their stocks randomly. Their only concern is portfolio size. In an active diversification strategy investors choose their stock deliberately and/or engage in a weighting scheme.

It appears that passive diversification is approached from three different angles. Some papers such as Evans and Archer (1968) look at the reduction of risk as diversification increases. Others such as Elton and Gruber (1977) look at the possibility that

returns can differ across equally diversified portfolios. Finally, a third but smaller number of papers such as Statman (1987) uses asset pricing theory to evaluate the optimal size of a diversified portfolio.

The oldest and most cited paper is Evans and Archer (1968). Evans and Archer find that a stable relationship exists between portfolio size and the level of portfolio dispersion. This relationship decreases rapidly to an asymptote. The asymptote approximates the level of systematic variation in the market. Following Sharpe (1963), the total variation of a portfolio can be separated into two forms: systematic variation and unsystematic variation. The reduction in variation of a portfolio's return due to increased diversification is entirely due to the reduction of the unsystematic portion of total variation.

The data used in their paper consists of 470 of the securities listed in the Standard and Poor's Index for the year 1958. Observations on each security are taken at semi-annual intervals for the period from January 1958 to July 1967. The statistics employed are the geometric mean of the ex-post returns and the standard deviation of the logarithms of the geometric returns. The hypothesis that portfolio standard deviation decreases to an asymptote as diversification increases is tested using the regression function:

$$Y = \frac{B}{X} + A,$$

where X is the portfolio size, Y are the mean portfolio standard deviations at each level of X , A and B are constants.

This function yields a very good fit ($R^2 = .9863$). These results are confirmed using t-tests and F-tests. The t-tests test for the reduction of successive mean portfolio standard deviation and the F-tests test for the reduction of standard deviations about the mean portfolio standard deviation. Overall, the results of Evans and Archer (1968) raise qualms concerning the justification of increasing portfolio sizes beyond 10 securities since there appears to be no marginal benefit from increasing portfolio size at this level.

Evans and Archer's paper is innovative because for the first time the question of portfolio size is addressed. However, their model addresses only one dimension of a two dimensional problem. The dimension addressed is risk and the dimension ignored is return. Another problem with this paper is that the biggest portfolio investigated is of size 40. This means that the authors implicitly assume that a portfolio of size 40 is idiosyncratic risk-free. This study uses larger portfolios.

Solnik (1974) uses the Evans and Archer approach and adds international data in his investigation. The

author looks at diversification in major European stock markets and the NYSE. The seven countries are the United Kingdom, Germany, France, Switzerland, Italy, Belgium and the Netherlands. A major finding in Solnik's paper is that the American market, because of its larger size and correlation structure, offers more opportunities for diversification than most of the European markets. When the entire data is blended, there is an even greater reduction in risk compared to the most attractive market (NYSE). This proves that the gains from international diversification are far from negligible. For example, Solnik finds that an internationally well-diversified portfolio is half as risky as the U.S. market. This risk reduction is even more dramatic if the European market is compared to an international portfolio. Solnik (1974), in addition to Evans and Archer (1968), is one of the most cited pieces in the investment books. Unfortunately they both ignore return and the portfolios considered are too small.

In a recent paper Campbell, Lettau, Malkiel and Xu (2001) trace the development of volatility of individual stocks, industries and the overall market from 1962 to 1997. Their data consists of the entire NYSE-NASDAQ-AMEX universe. Campbell et al. (2001) come to the following conclusions: the volatility of individual stocks has risen over time, the correlation among stocks returns has fallen over time, the volatility of the market and most of the industries have not changed, and the number of stocks necessary to achieve diversification has increased. The methodology used to confirm the implication that the number of stocks necessary to achieve diversification has increased is identical to Evans and Archer's. But unlike Evans and Archer, Campbell et al. do not seek any statistical significance and present only a visual illustration to their argument. This study tries to extend Evans and Archer, and Campbell et al.'s works by including return in the analysis, by testing for statistical significance, and by including large portfolios in the analysis.

Upton, Jessup and Matsumoto (1975) recognize the findings that unique risk is practically exhausted by holding eight to 16 stocks. However, they emphasize that what should be looked at is the dispersion of portfolio returns around the market's return. They note that while the reduction of times series standard deviation is quickly exhausted, the reduction of dispersion of portfolio returns around the market's return is not quickly exhausted. This means that while it is possible to construct a small portfolio that is as risky as the market, it takes a large portfolio to achieve market return consistently.

Elton and Gruber (1977) provide an analytical solution to Upton, Jessup and Matsumoto's approach.

They develop two models. The first one measures the expected variance of a portfolio of N securities and the second one measures the variance of returns across portfolios of size N . The data used in Elton and Gruber (1977) consists of weekly returns of 3,290 securities selected from the NYSE over the period from June 1971 to June 1974. They conclude that the risk, measured by the first model, is quickly exhausted but the risk, measured by the second model, is not quickly exhausted.

Elton and Gruber (1977) and Upton, Jessup and Matsumoto (1975) account for possible variations in portfolio return, but they do not measure the rate of decrease of both measures of risk as diversification increases and do not provide any specific recommendation, they simply conclude that a diversified portfolio is much larger than what previous papers infer.

In a comprehensive study, Fisher and Lorie (1970) examine the frequency distributions and dispersion of wealth ratios of investments in different-sized portfolios of NYSE stocks from 1926 to 1965. A large number of holding periods and portfolio sizes are examined. Fisher and Lorie report the frequency distributions of wealth ratios in great details. For example, they report the 5th percentile, 10th,...etc., up to 95th percentile, the maximum, minimum, arithmetic mean, measures of absolute dispersion, relative dispersion, skewness, and kurtosis. The part of Fisher and Lorie's paper examines the effect of increasing the number of stocks in a portfolio on the distribution of returns. The authors conclude that portfolios containing eight stocks have similar frequency distributions to those of portfolios containing larger numbers of stocks. To illustrate, 40 percent of reduction is obtained by holding two stocks; 80 percent by holding eight stocks; 90 percent by holding sixteen stocks; 95 percent by holding thirty-two stocks; and 99 percent, by holding 128 stocks. This means that dispersion is rapidly exhausted.

Fisher and Lorie's study is extensive without doubt. It is also appealing because it uses wealth ratios. However, the authors perform their study by eyeballing numbers rather than comparing the significance of changes in wealth ratio distribution as portfolio size increases.

O'Neil (1997) attempts to find out how many funds constitute a diversified mutual fund portfolio. He runs simulations using quarterly mutual funds returns collected from the Morningstar OnDisk database for the period from 1976 to 1994. All the mutual funds are categorized as growth or growth and income. The three variables in his analysis are objective (growth or growth and income), holding period (5, 10, 15 or 19 years), and number of funds (1-8, 10, 12, 14, 16, 18, 20, 25, or 30). The explicit

assumptions are that one specific mutual fund meets the inventor's needs and that every investor has a fixed investment horizon.

On average, a growth fund holds 78 securities. It is not surprising therefore that O'Neil finds that the time series standard deviation, which is the method used by Evans and Archer (1968), Solnik (1974), and Campbell, Lettau, Malkiel and Xu (2001), ceases to decrease after the first fund. However when O'Neil uses standard deviation of terminal wealth he finds a significant decrease of risk after including multiple funds. Although the expected terminal wealth doesn't seem to be impacted by the number of funds, the terminal wealth standard deviation decreases to between 31 percent and 41 percent for growth funds and to between 47 percent and 52 percent for growth and income funds. Longer holding periods requires more funds to achieve diversification because wealth tends to be more dispersed. O'Neil's paper is interesting because it implies that if terminal wealth standard deviation is used rather than time series standard deviation to investigate the size of a diversified portfolio, the outcome could be that more stocks are required to achieve diversification than previously thought. This is one of the issues investigated in this paper.

Newbould and Poon (1993) argue that the recommendations to form a portfolio of size 8 to 20 stocks is faulty, and that the minimum number of stocks needed to achieve diversification is much higher than 20 stocks. They add that the actual number would depend upon the universe of stocks being analyzed and the weighting scheme used to construct portfolios. Newbould and Poon use market weights and notice that risk seems to decrease to an asymptote as diversification is increased. However the asymptote is reached later compared to an equally weighted portfolio.

Statman (1987) criticizes the Evans and Archer findings and provides a new approach. Statman, unlike previous work, tries to answer the question using asset pricing theory. The approach in his paper is to contrast the marginal benefits and marginal costs. Portfolio size can be increased as long as the marginal benefits exceed the marginal costs.

The 500-stock line (capital market line), where all portfolios $P(n)$ lie, is defined as follow:

$$E[R_{P(n)}] = (R_F + \alpha) + \left\{ \frac{E[R_{P(500)}] - (R_F + \alpha)}{\sigma_{P(500)}} \right\} \sigma_{P(n)},$$

where $E[R_{P(n)}]$ is the expected return of portfolio $P(n)$, R_F is the risk-free rate, α is the excess of the borrowing rate over the lending, $E[R_{P(500)}]$ is the expected return of the 500-stock portfolio, $\sigma_{P(n)}$ is the

standard deviation of portfolio $P(n)$, and $\sigma_{P(500)}$ is the standard deviation of the 500-stock portfolio.

To compare the benefits of diversification, a portfolio of n randomly selected stocks, $G(n)$, is compared to a portfolio $P(n)$ that lies on the 500-stock line and has a standard deviation identical to that of portfolio $G(n)$. In general, $E[R_{P(n)}] - E[R_{G(n)}]$ can be interpreted as the benefit from increasing the number of stocks in a portfolio from n to 500. This benefit is then compared to the cost of investing in funds, also known as total expense ratio, that mimic the S&P 500 index. Assuming that no costs are incurred in buying, selling, and holding of portfolios $G(n)$ composed of less than 500 stocks, a leveraged 500-stock portfolio, $P(n)$, is preferable to a portfolio $G(n)$ if the costs of $P(n)$ are lower than the benefits that come with increased diversification. Based on this criterion Statman concludes that the appropriate size is 30 stocks for a borrowing investor and 40 stocks for a lending investor.

Statman incorporates return in his paper but not like in Elton and Gruber (1977), Upson, Jessup and Matsumoto (1975), or Fisher and Lorie (1970). Whereas those three papers acknowledge the fact that portfolios of the same size can be equally risky but have different returns from the market portfolio, Statman compares the forgone return from not being fully diversified to the cost of investing in a fully diversified portfolio. As long as the cost is lower than the forgone return, investors are better off investing in the fully diversified portfolio. To put it another way, Statman compares two portfolios with the same total risk but where one, $G(n)$, has some unsystematic risk and where the other, $P(n)$, has only systematic risk. Because finance theory tells us that only systematic risk is priced, investors would find it more rewarding to invest in $P(n)$ as long as the cost of doing so is not high.

Statman's paper appears to contain some flaws. First, it is not clear how lending is accounted for. Because all the portfolios have standard deviations higher than or equal to the S&P500, they all lie on the borrowing side of the curve¹. That is, none of the portfolios can be compared to a lending portfolio because lending lies on the left side of the curve. What Statman is merely doing is considering two borrowing cases. In the first case the borrowing rate is higher than the risk free rate ($\alpha > 0$), in the second case the borrowing rate is equal to the risk free rate ($\alpha = 0$). So his results could be rephrased as follows: the appropriate size is 40 stocks for investors who

¹ Statman used the standard deviations values calculated in Elton and Gruber (1984). The calculations were made using monthly data for all stocks listed in the NYSE.

can borrow at the risk free rate and 30 stocks for investors who cannot borrow at the risk free rate. Second, Statman makes calculation errors. He assumes that the cost of investing in an index fund is 0.49%. This number would also be the cost of $P(n)$ if and only if the investors puts 100% of his wealth in the index fund. But the investor does not put all his wealth in the index fund, he actually invests more.

Another blow to Statman (1987) comes from Murphy (1991). Murphy argues that the standard deviations Statman used are questionable. He uses instead data from the Kansas City Board of Trade over the period from 1977 to 1981 and finds the median standard deviation for single stocks to be approximately 31 percent whereas Statman's number is 49.236 percent. For a portfolio of ten stocks the average standard deviation was 20.650 percent whereas Statman's number is 23.932 percent. If Murphy's numbers are accurate than the number needed to achieve diversification is substantially less than what Statman predicted, regardless of the calculation errors.

Shanker (1989) compares the marginal return from direct diversification and the marginal cost of direct diversification. The marginal return from diversifying is calculated as in Statman (1987) and the marginal cost of diversifying is calculated as in Smith and Schreiner (1970) and Schreiner and Smith (1980). The calculations of marginal cost include investor's portfolio turnover, commission rate, rate of portfolio growth, investor horizon and investor's initial wealth. In sum, Shanker finds that a borrowing investor needs 20 stocks to be diversified and a lending investor needs 50. It must be noted, however, that while Shanker measures the benefits of diversification similarly to Statman (1987), he measures the costs of diversification differently. While Statman measures the cost of diversification as what an index fund would charge its customers, Shanker views the cost of diversification as the cost of directly holding and maintaining the S&P500 portfolio. More recent papers such as Statman (2004), Benjelloun and Siddiqi (2006), Dale, David and Marie (2007) reach the conclusion that a diversified portfolio contains hundreds of stocks.

1.5. Active diversification. In an active strategy, investors get into some sort of predetermined action. They can either choose specific stocks, criterion, risk levels, recommendations, or weight structures. This section reviews some of the evidence on active strategies.

Bloomfield, Leftwich, and Long (1977) assess different portfolio selection strategies. These strategies vary from a naïve strategy where investments are equally weighted to sophisticated strategies where

investments weights are periodically reevaluated. The first strategy involves monthly rebalancing to equal weights and serves as a benchmark for evaluation of other strategies. The second strategy involves monthly rebalancing to the weights of the tangency portfolio. The tangency portfolio is estimated at the beginning of each five-year evaluation period. The third strategy is similar to the second except that the tangency portfolio is re-estimated monthly. The fourth strategy is similar to the second except that the optimal portfolio is calculated using a different algorithm. The algorithm is based on the assumption that excess return is proportional to the beta for all securities. The last strategy is similar to the third except that the new algorithm is used.

The authors use 3-5 year intervals and monthly return data of over 800 stocks to reach the conclusion that the use of sophisticated strategies does not constitute an improvement over the naïve strategy. It appears from the results that the third and fifth strategies are consistently better than the others. However, both strategies involve much higher implementation costs, which are the monthly cost of reevaluating the tangency portfolio, and therefore may not be more successful than a naïve strategy on an after-cost basis. Another important conclusion of the paper is that regardless of the strategy, risk seems to be reduced as portfolio size increases and no significant decrease was noted after the size reaches 17. This result corroborates the earlier results of Evans and Archer (1968) and Fisher and Lorie (1970).

An important remark about Bloomfield, Leftwich, and Long's paper is that portfolio sizes are nominal not actual. That is, a designed portfolio size may end up being reduced if the solution portfolio assigns zero weights to some stocks. This complication is due to the fact that short selling is not allowed.

Wagner and Lau (1971) compare diversification using stocks with high ratings and stocks with low ratings. The ratings used are taken from the Standard & Poors' Stock Guide and portfolios are formed for each quality class for the five-year period from 1960 to 1965 and the ten-year period of 1960-1970. Four measures for each class of stocks are reported. The measures are standard deviation as a measure of total risk, beta as a measure of systematic risk, R-square as a measure of how much of the variability in the returns of the portfolio is associated with the variability of the market, and portfolio returns. The paper reaches the following conclusions: the higher the quality rating the lower the beta, portfolios' returns increase as quality decreases, standard deviation decreases as the number of holdings increases, the decrease is quicker the higher is the rating, increasing portfolio size does increase return, R-

square increases as the portfolio size increases, and R-square increase is higher the higher is the rating except for the A+ rating. These findings imply that portfolios consisting of large numbers of higher risk securities may be less risky than portfolios consisting of small numbers of low risk stocks, yet earn a substantially higher rate of return. In other words, the investor is better off diversifying through the use of a large number of risky securities. As an illustration of this finding, a portfolio of 5 A+ type of securities performs much worse than a portfolio of 15 B+ type of securities. The two portfolios have similar levels of risk.

Jacob (1974) provides a different type of active strategy. In her model, Jacob considers the decision problem where an individual desires to select a specific portfolio size out of a given universe of stocks. Using the market model, Jacob formulates the decision problem as a mixed-integer quadratic programming problem where the investor desires to minimize total variability given a certain level of return. Given the computational difficulties, Jacob makes constraining assumptions for the model to become manageable by the kind of computers available in 1974.

To estimate the accuracy of the model, a universe of 50 randomly selected securities is created from the Compustat Quarterly Industrial tape. The data spans the period from the second quarter of 1967 to the ending of the first quarter of 1971. To estimate the parameters of the market model, each security is regressed against the Dow-Jones Thirty Industrials. The problem is solved for portfolio sizes ranging from 2 to 20. For example, when the desired level is 5, the program is asked to find the best portfolio of size 5, where the five stocks are drawn from the 50 stock universes. The results seem to suggest an improvement over random selection. With only six securities, an investor using the model might expect a risk slightly lower than the market; with fewer securities the risk is slightly higher.

For the approach to be truly valid, the performance of the portfolios has to be compared to the performance of six mutual funds. The comparison is done over the period ranging from April 1, 1971 through March 31, 1972. Load charges and transaction costs are accounted for in the comparison. The results suggest that growth-oriented mutual funds tend to dominate the solution portfolios. However, the solution portfolios dominate the income oriented funds.

Klemkosky and Martin (1975) use the market model to investigate the process of portfolio diversification. They find a positive relationship between beta and the variance of the error term and assess the impact of their finding on diversification. The au-

thors argue that their finding of a positive relationship between beta and the variance of the error term implies that for portfolios having an equal number of securities, the one with the larger beta has the larger residual variance and therefore more unsystematic risk. The relationship between market and residual risk is tested over the period ranging from July 1963 to June 1973. Three hundred and fifty NYSE listed common stocks are selected. First, the betas and the variance of diversifiable risk are regressed on the residual variances. A significant positive relationship is found. These results are corroborated using the Spearman rank correlation. Next they investigate whether this relationship is also valid for portfolios. To do that, securities are ranked according to their betas and approximately 1000 portfolios are selected from the 350 available stocks. When, for example, the desired portfolio size is 10, 35 portfolios are formed with the first one containing the highest 10 betas and the last one containing the lowest 10 betas. The results suggest that there exists a positive relationship between portfolios' betas and residual portfolio variance for each of the portfolio sizes from 2 to 25.

These results have implications on the process of diversification. Holding size constant, an investor will be better off diversifying with low beta stocks than with high beta stocks because the latter contains more unsystematic risk than the former. These results confirm Wagner and Lau's findings that diversification with risky stocks requires a larger portfolio size than a portfolio with less risky stocks.

In a subsequent paper Martin and Klemkosky (1976) study the impact of industry effects, also known as group effect, on portfolio risk. The group effect arises from the significant presence of covariances between residual terms calculated from the market model. To achieve their objective, Martin and Klemkosky measure risk using both the market model, that assumes zero covariances between residual terms, and a model that incorporates covariances in estimating portfolio risk. The total sample includes 150 stocks over the period from January 1968 to December 1973. It includes 40 growth stocks, 44 cyclical stocks, 44 stable stocks, and 22 oil stocks. Portfolio variances are calculated for ten different randomly chosen portfolios containing two, three, ... ten stocks using both methods. Wilcoxon matched-pairs, signed-ranks test is used to test for statistical significance of differences between the two measures. The results reveal that, for portfolios of size four or higher, the group effect is positive and significant. The strongest group effect is found in the oil group followed by the growth, stable and then cyclical group.

While Evans and Archer find that, for a ten-security portfolio the ratio of unsystematic risk to total risk is

91 percent, Martin and Klemkosky find that this ratio is 75 percent for growth stocks, 77 percent for cyclical stocks, 62 percent for stable stocks, and 47 percent for oil stocks. When all the groups are mixed, the group effect is minimal. These results imply that diversification can be more readily achieved when stocks are chosen across industries.

Tole (1992) argues that most investors do not base their choices on randomness or beta levels but rather on recommendations from brokerage firms or financial research firms. These recommendations are based on fundamental and/or technical analysis. As a consequence, Tole thinks that it is relevant to investigate the diversification levels for investors who construct portfolios from the recommendations of brokerage firms, research services, and other source of investment information. The author collects 1500 recommendations from fifteen brokerage firms. Portfolios are constructed from recommendations made from March 1969 to May 1970. They are held constant from May 1970 to June 1971 (bull market) and the period from July 1971 to March 1974 (long-term neutral market). Each portfolio is classified as a conservative growth portfolio, an income portfolio, or a speculative portfolio. Monthly prices are obtained from the quarterly Compustat tapes. For each portfolio time-weighted rates of return, mean rates of return for the two separate periods, standard deviations of monthly returns from mean returns for both periods, portfolio betas, and R-squared are calculated. To verify the results, the author uses the three models used by Evans and Archer (1968), Wagner and Lau (1971), and Klemkosky and Martin (1975). The three methods of measuring unsystematic risk show that a greater number of securities are required in a portfolio consisting of recommendations than in portfolio of random selections.

Woerheide and Persson (1993) argue that most of the literature does not answer the question of whether a portfolio is actually diversified or not, it only provides an estimation of the minimum level required for diversification. Another problem, according to the authors, is that most of the literature is based on equally-weighted portfolios. The objective of Woerheide and Persson is to present five measures of diversification based on the distribution of weights. These measures are inspired from the industrial organization literature.

One of the diversification measures is based on the familiar Herfindahl index:

$$DI(1) = 1 - HI = 1 - \sum_{i=1}^N W_i^2,$$

where DI is the diversification index, HI is the Herfindahl index, W_i is the proportion of wealth invested in security i , N is the the number of securities in the portfolio.

The methodology is the same as in Evans and Archer (1968) with the exception that the weights are randomly assigned to stocks. In total, 1,740 portfolios are constructed out of 483 companies. Monthly returns are extracted from the CRSP tape and data covers the period from December 1965 to December 1985. The five indices are then compared to the standard deviation of returns of randomly selected portfolios. The authors run 7 regressions: average standard deviation on portfolio size assuming equal weights, standard deviation on portfolio size assuming unequal weights, and portfolio standard deviation on each index assuming unequal weights. The results suggest that three indices are good measures of diversification. However the authors recommend the use of the Herfindahl index because of its high adjusted R-squared, popularity, and simplicity of use. Finally the authors, based on comparison with Evans and Archer (1968) and Fisher and Lorie (1970), conclude that portfolios with index values greater than 0.91 are adequately diversified.

A problem with Woerheide and Persson (1993) is that it accounts only for the weight structure. Two portfolios can have the same size, and the same weight structure but a completely different return and/or risk structures. It is, therefore, insufficient to measure diversification using only the Herfindahl index.

2. Portfolio risk measures, data, and results

This paper uses the following risk measures:

1. Time series standard deviation:

$$TSSD_N^i = \sqrt{\sum_{s=1}^S \frac{(R_s^i - \overline{R}_N^i)^2}{S-1}},$$

where, $TSSD_N^i$ is the time series standard deviation of the N -stock portfolio i ,

$$R_s^i = \sum_{j=1}^N \frac{r_{j,s}^i}{N}$$

$r_{j,s}^i$ is the return on stock j , in portfolio i , at time s ,

$$\text{and } \overline{R}_N^i = \sum_{s=1}^S \frac{R_s^i}{S}$$

is the average time series return, over time, of portfolio i .

The average time series standard deviation of K portfolios, each of size N is given by:

$$\overline{TSSD}_N = \sum_{i=1}^K \frac{TSSD_N^i}{K}. \tag{1}$$

Market-value weights can also be used. The calculations above except that R_s^i is calculated using market value weights not equal weights:

$$R_s^i = \sum_{j=1}^N w_j r_{j,s}^i,$$

where w_j represents market weight of stock j in portfolio i .

2. Standard deviation of terminal wealth. For equally-weighted portfolios, the terminal wealth of a portfolio of size N is the sum of the terminal wealth of all the stocks in the portfolio:

$$TW_N^i = \frac{1}{N} \sum_{j=1}^N TW_j^i,$$

where, $TW_j^i = \prod_{s=1}^S (1+r_{j,s}^i)$.

Average terminal wealth over K portfolios, each of size N , is given by:

$$\overline{TW}_N = \sum_{i=1}^K \frac{TW_N^i}{K}.$$

Terminal wealth standard deviation over K portfolios, each of size N , is given by:

$$TWSD_N = \sqrt{\sum_{i=1}^K \frac{(TW_N^i - \overline{TW}_N)^2}{K - 1}}. \tag{2}$$

For market-weighted portfolios the calculations are similar except for TW_N^i that is calculated using market value weights not equal weights:

$$TW_N^i = \sum_{j=1}^N w_j TW_j^i,$$

where w_j represents market weight of stock j in portfolio i .

The sample in this study consists of all the firms listed in the CRSP tape for the period ranging from 1980 to 2000. The data used consists of monthly returns and market values. Market values are used to determine market weights and are calculated by dividing the market value of the underlying stock by the market value of the portfolio.

The simulations are performed with replacement for every portfolio size. Ten thousand ($K = 10,000$) portfolios for every level of N are generated. N takes the values 1, 10, 20, 30 and so on until 100. For every portfolio size $TSSD$ and $TWSD$ are measured using monthly returns following equations (1) and (2). For every combination of time period, weighting scheme, and measure of portfolio risk the following regression is evaluated:

$$Y = A \frac{1}{N^2} + B,$$

where N is portfolio size as defined earlier and Y is one of the measures of risk described earlier ($TSSD$ or $TWSD$). A is the slope, and B is the intercept. When portfolio size grows large that is when N becomes large Y converges toward B .

A portfolio is diversified when its risk is equal or smaller than B . The smallest portfolio with a risk less or equal to B is said to be diversified and the corresponding size is the size of a well-diversified portfolio. Beyond B risk reduction is considered negligible. This kind of conclusion can only be validated if and only if the coefficient of determination is sufficiently high, that is if the regression equation fits well the outcome of the simulations. The close attention is, therefore, given to R-squares.

Tables 1 through 3 provide the results of the simulations and regressions.

Table 1. Average time series standard deviations of monthly portfolio returns

Portfolio size	Equally weighted			Market weighted		
	1981-1990	1991-2000	1981-2000	1981-1990	1991-2000	1981-2000
1	0.1144	0.1285	0.1056	0.1149	0.1297	0.1049
10	0.0609	0.0581	0.0529	0.0684	0.0617	0.0629
20	0.0554	0.0493	0.0476	0.0621	0.0534	0.0572
30	0.0535	0.0458	0.0456	0.0594	0.0498	0.0548
40	0.0526*	0.0440*	0.0446*	0.0580*	0.0473	0.0534*
50	0.0520	0.0428	0.0440	0.0569	0.0457*	0.0524
60	0.0516	0.0421	0.0436	0.0563	0.0442	0.0516
70	0.0513	0.0415	0.0433	0.0556	0.0432	0.0512
80	0.0510	0.0410	0.0430	0.0554	0.0423	0.0509
90	0.0509	0.0407	0.0429	0.0547	0.0416	0.0505
100	0.0507	0.0404	0.0428	0.0546	0.0410	0.0503
Goodness of fit (R^2)	97.78%	96.30%	97.69%	95.04%	94.69%	94.90%

Table 1 (cont.). Average time series standard deviations of monthly portfolio returns

	Equally weighted			Market weighted		
	1981-1990	1991-2000	1981-2000	1981-1990	1991-2000	1981-2000
Intercept	0.0529	0.0444	0.0449	0.0580	0.0469	0.0534
Slope	0.0616	0.0842	0.0608	0.0570	0.0830	0.0516

Notes: This table provides the results of the simulations. The average standard deviation is calculated using equation (1). The last three rows are the outcome of the following regression: $Y = A \frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below B . The corresponding size is the size of a well-diversified portfolio. This level is marked by an asterisk (*).

Table 2. Average time series standard deviation of monthly portfolio returns

Portfolio size	Equally weighted				Market weighted			
	1981-1985	1986-1990	1991-1995	1996-2000	1981-1985	1986-1990	1991-1995	1996-2000
1	0.1170	0.1314	0.1295	0.1499	0.1166	0.1323	0.1284	0.1508
10	0.0592	0.0695	0.0569	0.0717	0.0659	0.0714	0.0527	0.0804
20	0.0532	0.0625	0.0474	0.0613	0.0589	0.0648	0.0447	0.0692
30	0.0510	0.0603	0.0434	0.0572	0.0562	0.0619	0.0413	0.0642
40	0.0500*	0.0590*	0.0414*	0.0551*	0.0543*	0.0600*	0.0392	0.0609
50	0.0492	0.0582	0.0401	0.0539	0.0533	0.0588	0.0374*	0.0588*
60	0.0487	0.0577	0.0392	0.0529	0.0522	0.0580	0.0364	0.0571
70	0.0485	0.0573	0.0385	0.0524	0.0520	0.0573	0.0353	0.0557
80	0.0481	0.0570	0.0380	0.0519	0.0513	0.0568	0.0347	0.0547
90	0.0479	0.0567	0.0376	0.0516	0.0512	0.0564	0.0341	0.0537
100	0.0477	0.0566	0.0372	0.0511	0.0505	0.0561	0.0336	0.0528
Goodness of fit (R^2)	97.58%	97.37%	95.78%	96.03%	94.93%	96.15%	96.13%	92.24%
Intercept	0.0502	0.0594	0.0418	0.0557	0.0545	0.0600	0.0388	0.0606
Slope	0.0669	0.0721	0.0878	0.0943	0.0622	0.0724	0.0898	0.0904

Notes: This table provides the results of the simulations. The average standard deviation is calculated using equation (1). The last three rows are the outcome of the following regression: $Y = A \frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below B . The corresponding size is the size of a well-diversified portfolio. This level is marked by an asterisk (*).

Table 3. Standard deviation of terminal wealth for monthly returns

Portfolio size	Equally weighted			Market weighted		
	1981-1990	1991-2000	1981-2000	1981-1990	1991-2000	1981-2000
1	4.6976	15.8043	34.4385	4.9945	14.2489	33.3349
10	1.5285	4.6504	11.1564	2.0026	7.0612	13.6860
20	1.0769	3.3819	7.9023	1.5190	3.8534	10.5191
30	0.8775	2.7253	6.2903	1.2609	3.0315	8.7594
40	0.7568	2.4281	5.4014*	1.1155	2.3374*	8.0074
50	0.6696*	2.1755*	4.8184	0.9921*	2.1526	7.3416*
60	0.6113	1.9336	4.3977	0.9078	1.9154	6.6531
70	0.5675	1.7816	3.9948	0.8532	1.8049	6.2764
80	0.5303	1.6602	3.7732	0.7898	1.6597	5.9835
90	0.5046	1.5321	3.5553	0.7404	1.5542	5.6337
100	0.4738	1.4675	3.3704	0.7195	1.4815	5.4388
Goodness of fit (R^2)	93.98%	95.21%	93.87%	90.77%	82.99%	91.23%
Intercept	0.7527	2.3503	5.4150	1.0830	2.6627	7.7839
Slope	3.9529	13.4780	29.0835	3.9213	11.6311	25.6137

Notes: This table provides the results of the simulations. The standard deviation of terminal wealth is calculated using equation (2). The last three rows are the outcome of the following regression: $Y = A \frac{1}{N^2} + B$. An asymptote is reached as soon as the calculated number falls below B . The corresponding size is the size of a well-diversified portfolio. This level is marked by an asterisk (*).

The results show that risk decreases as portfolio size increases. Additionally all R-squared except one are higher than 90 percent, all regressions fit very well the risk pattern.

According to the results all portfolios seem to be diversified. It also appears that diversification is often reached faster (with less stocks) when equal weights are used compared to market weights. On the other hand it looks like minimum risk (B) is higher for market weights, this result is counter intuitive because it implies that big stocks are more volatile than small stocks.

The most important result however is that diversification is reached with 40 to 50 stocks regardless of the risk measure, time period, or weighting scheme. This

is an unexpected result as numerous well established papers, as mentioned earlier, predicted otherwise.

Conclusion

I provide an extensive review of the literature dealing with portfolio diversification. Early research has shown that a portfolio with as small as ten stocks can be well diversified. Recent research however claims that hundreds of stocks are needed to achieve diversification.

Using some new evidence and new methodology I revert back to earlier findings and find that a portfolio of 40 to 50 stocks is all that is needed to achieve diversification. This results questions the appropriateness of using index funds by investors as a vehicle for achieving diversification.

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