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Formalization of the "risk" category during the realization of reinsurance operations on the basis of the economic and mathematical apparatus

Abstract

The article offers to substantiate the peculiarities of formation of the "risk" category during the realization of reinsurance operations by using the instruments of economic and mathematical modeling. The identification of the "risk" category involves the consideration and complex representation of three components: the possibility of insured accident, the measure of variability of obtained results, the degree of deviation from the desired result.

Keywords: risk, reinsurance operations, contingency coefficient, synergy effect.

Introduction

Problem statement. The carrying out of any economic activity is impossible without an efficient management of situations, which are connected with uncertainty. In the face of inevitable choices during the decision-making the formalization of uncertainty in identification of such economic category as "risk" in reinsurance operations grows in importance. First of all, it is conditioned by the fact that only significant in terms of the size and catastrophic consequences risks are subject to reinsurance. The necessity to substantiate the peculiarities of identification of the "risk" category during the carrying out of reinsurance operations is also highlighted by the negative content of the risk category, which manifests itself in the occurrence of insured accidents and the coverage of corresponding claims.

Analysis of the latest research and publications. The analysis of the contemporary literary sources [1, 2, 3, 4, 5, 6, 7, 15] dedicated to the identification and formalization of the "risk" category during the carrying out of any economic activity in general and reinsurance operations in particular shows the lack of the common approach and theoretically substantiated conception. This fact is explained both by the general and specific peculiarities of the use of this category in each particular case.

Earlier unsolved parts of the general problem. The existing approaches to the interpretation of the "risk" category show that scientists give considerable attention to the identification of this concept including highly specialized studies that do not always adequately assess the risk as a complex concept.

Goal of the study. Substantiation of the peculiar features in the formalization of the "risk" category during the realization of reinsurance operations through the accumulation of the existing approaches in the application of instruments of economic and mathematical modeling.

1. Main results of the study

We will consider the general patterns in the formation of the "risk" category according to such aspects as the definition of economic essence, the main causes of emergence, impact on the degree of achieving goals, as well as the existing approaches to the formalization of the concept on the quantitative level.

The results of the conducted generalization of the study's areas are presented in Figure 1.

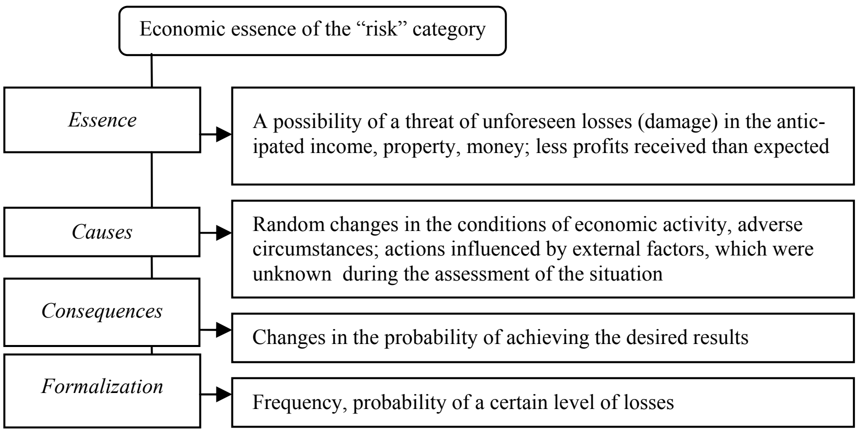

Fig. 1. The essence of the economic "risk" category

Considering the general approaches to the definition of risk, we argue that this concept is identified as one of the following three components: the probability of insured accident, the measure of variability of obtained results, the degree of deviation from the desired result. The use of each of the above interpretations has both advantages and drawbacks, especially during the realization of reinsurance activities.

Focusing on the formalization of risk as a probability of insured accidents during the conclusion of agreements dealing with the reinsurance of a certain part of an insurance company's liabilities, we will consider the advantages given by this interpretation of risk:

- the notion of probability implies the occurrence of insured accident; it is one of the criteria of quantitative characteristics of this accident, which takes into account the fact that risk is possible;
- makes it possible to carry out the precise identification of the risk degree depending on the available quantitative characteristics since it is characterized by a certain interval of possible values, minimum and maximum limits, which correspond to different qualitative interpretations;
- takes into account the random nature of insu-rance accident, anticipating the possibility of its occurrence and nonoccurrence, which is followed by the formation of financial flows of the insurance (reinsurance) company of different volumes.

For the analysis of the next form of the risk category interpretation and its quantitative assessment (the measure of variability of obtained results) we will conduct a comprehensive study of the major positive features of the application of this interpretation:

- as one of the key aspects of the quantitative interpretation of the risk level it considers the requirement, the essence of which is that the realization of risk should not be conditional on the will of the insurer, the insured or any other interested party. This makes it possible to formalize the variability of obtained results, which take into account both favorable and unfavorable accidental events;
- focuses attention on the nature of the insured accident, which can occur, allowing the managers of insurance (reinsurance) companies to make alternative science-based management decisions;
- the notion of degree of variability provides an opportunity to objectively measure and quantify the impact of the risk in case of adverse events.

Parallel to the above-presented approaches to the identification of significant insurance risks an important form of formalization is the definition of risk as a degree of deviation from the desired result, the use of which provides an opportunity to achieve the following advantages in the practical activities of insurance (reinsurance) companies:

- provides an opportunity for operational and strategic planning of losses resulting from the insured accident and the consequent formation of reasonable reserve funds;
- the quantitative measurement of risk takes into account both the degree of achieving the desired result, and the degree of deviation from the predicted values, which allow the managers of insurance (reinsurance) companies to carry out the adjustment of their activities;
- accidental deviations from the desired result correlates to the analysis of certain related objects causing the formation of an adequate risk assessment.

In addition, each of these forms of formal representation of risk as a quantitative criterion does not allow to consider several crucial aspects:

- ability to compare and juxtapose the results of the risk level assessment obtained by using different approaches;
- provision of objective and adequate qualitative characteristics of risk levels depending on different combinations of the highest possible values for each of the quantitative characteristics;
- consideration of different phases of the life cycle of adverse accidental events, which serve as the object of an insurance contract concluded with the corresponding insurance (reinsurance) company;
- possibility to obtain static and dynamic integrated risk assessment that takes into account both current preventive actions of an insurance (reinsurance) company and strategic directions of its detection and overcoming;
- taking into account the need for flexible adjustment of current risk level calculated on the basis of the existing information about its characteristics and in accordance with the intensity of the flow of new information;
- definition and quantitative assessment of the synergy effect of the risk from simultaneous occurrence of factors contributing to the insured accident by multiple quantitative criteria.

In order to overcome the negative trends of a separate use of each of the defined approaches to quantitative assessment of insurance risks and taking into account the fact that risk is a complex multilevel system of interconnected components, the functioning of which provides an opportunity to obtain new characteristics, we will make generalized interpretations of the categories of risk. Therefore, the risk of carrying out reinsurance operations can be identified as a combination of the following three components:

$$R_p = f\big(P_X(H1); SV(X); SSG(X)\big), \quad (1)$$

where R_p is the integral risk level; $P_X\,(H1)$ is the probability of insured accident, which is proposed to define as a conditional probability based on the use of Bayesian approach; $SV(X)$ is a measure of variability of the obtained result, which makes it possible to conduct quantitative analysis of risk as an indicator of semi-variance; $SSG(X)$ is the degree of deviation from the desired result, which takes into account the nature of accidental events that have taken place in terms of their impact on the operational efficiency of an insurance and reinsurance company, and is defined as an indicator of the semiaquare deviation from the geometric average.

Each of these components is a complex system that depends on many factors and influences the formation of other economic indicators. We will analyze the nature of quantitative risk assessment as the probability of insured accident, which can be determined with the following equation (2) [9]:

$$P_B(H1) = \frac{1}{1 + \frac{1-p_t}{p_t} \prod_i \left(\frac{1-b_i}{1-g_i}\right) \left(\frac{g_i(1-b_i)}{b_i(1-g_i)}\right)^{Bi}}, \quad (2)$$

$$p_t = C_T^t \cdot \sum_{k=0}^{t-1} (-1)^k \cdot C_t^k \cdot \left(\frac{t-k}{T}\right)^T,$$

where P_B(H1) is the probability that the analyzed insured accident will happen subject to the availability of information B; p_t is the probability that in the period t of the risk's life cycle the corresponding insured accident will happen (the probability of the necessity to transfer all or part of the risk to reinsurance); $B = (B_1, B_2, \ldots B_n)$ is a set of binary features, where B_i has the value of 1 if the corresponding indicator shows the possibility of insured accident, and 0 – otherwise; b_i is the probability of the situation $B_i = 1$ for the likely insured accidents, and g_i – for the unlikely ones; T is the duration of the project's life cycle; t is the time period of the life cycle of risk, when the assessment of the probability of the insured accident is carried out (it is the time when the decision about the transfer of all or part of the risk to reinsurance is made); $k = 0\,/\,(t-1)$ is the period of the life cycle of risk till the next analysis period; C_T^t, C_t^k is the number of combinations of t elements in the T aggregate (combinations of k elements in the t aggregate).

Another quantitative characteristic of the level of risk is the measure of variability of results, the solution of which is proposed to interpret as an indicator of semi-variance:

- For discrete random variable (equation (3)):

$$SV(X) = \frac{1}{P^-} \sum_{j=1}^{n} \alpha_j p_j \big(x_j - M(X)\big)^2, \quad (3)$$

$$P^- = \sum_{j=1}^{n} \alpha_j p_j .$$

- For continuous random variable (equation (4)):

$$SV(X^-) = \frac{1}{P^-} \int_{M(X^-)}^{+\infty} (x - M(X^-))^2 f(x)dx, \quad (4)$$

$$P^- = \int_{M(X^-)}^{+\infty} f(x)dx ,$$

where X is a random event that characterizes the occurrence of the insured accident; $SV(X)$ is the indicator of semi-variance; P^- is the identifier of probable adverse deviation from the desired (predicted) result, which characterizes the occurrence of insured accident; x_j is the qualitative characteristics of random event in j observation; p_j is the probability of the insured event, which is analyzed in the j observation; α_j is a binary identifier of the adverse deviation from the desired (predicted) result in the j observation;

$$\alpha_j = \begin{cases} 0, & \text{in case of favourable deviation.} \\ 1, & \text{in case of adverse deviation.} \end{cases}$$

If a decision is made to consider the fact of losses as adverse deviation, the binary identifier takes the following form:

$$\alpha_j = \begin{cases} 0, & x_j \le M(X^-) \\ 1, & x_j > M(X^-) \end{cases}; \qquad j = \overline{1,n}.$$

$M(X)$ is the mathematical expectation of a random event that characterizes the fact of the insured accident.

The third, but equally important component of the complex concept of risk in reinsurance activities is the degree of deviation from the desired result, which this study offers to formalize on the basis of deviation from the weighted geometric average:

$$SSG(X) = \sqrt{SG(X)} = \sqrt{\sum_{j=1}^{n} \alpha_j p_j (x_j - G(X))^2}, \quad (5)$$

$$G(X) = a - \varepsilon + \prod_{j=1}^{n} (x_j - a + \varepsilon),$$

where $G(X)$ is the geometric average evaluation of the random variable x in case when the random variable x is discrete;

$$a = \min\{x_1, x_2, ..., x_n\};$$

$$\varepsilon \geq 0.$$

After the formalization of existing approaches to the definition of risk category in the form of specific quantitative criteria and taking into account the peculiarities of the use of this concept in the reinsurance activities, we feel the need to form the approach to the calculation of generalized characteristics.

For this goal we will introduce the algorithm of scientific and methodical approach to the definition of generalized risk assessment (contingency coefficient) in carrying out reinsurance operations as a combination of three components (the possibility of insured accident, the measure of variability of obtained results, the degree of deviation from the desired result) in the form of the following consecutive stages.

Stage 1. The calculation of quantitative assessment of risk's components as a probability of insured accident ($P_X(H1)$), the measure of variability of obtained results ($SV(X)$) and the degree of deviation from the desired result ($SSG(X)$) on the basis of the above-mentioned mathematical ratios (2)-(5). The results of this phase are the information base for further calculations and the basis for the formation of integrated risk assessment and detection of specific features of the risk category (in carrying out reinsurance operations) as a complex multilevel system.

Stage 2. The comparison of three quantitative risk criteria defined in the previous stage by bringing them to the same scale of measurement. The necessity of this stage is explained by the following factors: character of formation, specific character of identification, units of measurement, areas of practical application of risk's components. We conduct the normalization of the parameters $P_X(H1)$, $SV(X)$ and $SSG(X)$ by using the equation (6), because an increase in absolute value of each of these criteria leads to the deterioration of obtained results, which means the growth of generalized levels of risk [1]:

$$q_i = \begin{cases} = 0, x_i = \min(x_i), \\ = (\max(x_i) - x_i)/(\max(x_i) - \min(x_i)), \min(x_i) < x_i < \max(x_i), \\ = 1, x_i = \max(x_i), \end{cases} \quad (6)$$

where x_1 is the value of probability of insured accident $P_X(H1)$; x_2 is the value of the measure of variability of obtained results $SV(X)$; x_3 is the value of the degree of deviation from the desired result $SSG(X)$; $q_1(q_2, q_3)$ are the normalized characteristics $P_X(H1)$ (respectively $SV(X)$ and $SSG(X)$); $\min(x_i)$ are the minimal values of quantitative criteria of risk assessment; $\max(x_i)$ are the maximal values of quantitative criteria of risk assessment.

Stage 3. The definition of the levels of quality characteristics of the components $P_X(H1)$, $SV(X)$, $SSG(X)$ and the definition of normalization intervals for the values of corresponding risk components. The most common approach in the contemporary economic literature [10, 13] dedicated to covering the issues of quality characteristics of risks is the identification of three levels: normal, raised and high. Within the study of minimal and maximum values of interval limits of the normalized values for the respective risk components it is proposed to use the approach, which was formed in the statistical analysis of economic data [14] and which has the following intervals: [0; 0,5) for normal, [0,5; 0,7) for raised and [0,7; 1] for high levels of risk.

Stage 4. Establishment of conformity of normalized characteristics $P_X(H1)$, $SV(X)$ and $SSG(X)$ with the interval limits for the normalized value of risk components. Practical realization of this stage of scientific and methodical approach to the definition of generalized assessment of reinsurance operations is conducted on the basis of the second and third stages. According to this binary indicators are calculated:

$$\begin{matrix} Normal \\ Raiced \\ High \end{matrix} \begin{pmatrix} \overset{P_X(H1)}{b_{11} = \begin{cases} 1, q_1 \in [0;0,5) \\ 0, q_1 \notin [0;0,5) \end{cases}} & \overset{SV(X)}{b_{12} = \begin{cases} 1, q_2 \in [0;0,5) \\ 0, q_2 \notin [0;0,5) \end{cases}} & \overset{SSG(X)}{b_{13} = \begin{cases} 1, q_3 \in [0;0,5) \\ 0, q_3 \notin [0;0,5) \end{cases}} \\ b_{21} = \begin{cases} 1, q_1 \in [0,5;0,7) \\ 0, q_1 \notin [0,5;0,7) \end{cases} & b_{22} = \begin{cases} 1, q_2 \in [0,5;0,7) \\ 0, q_2 \notin [0,5;0,7) \end{cases} & b_{23} = \begin{cases} 1, q_3 \in [0,5;0,7) \\ 0, q_3 \notin [0,5;0,7) \end{cases} \\ b_{31} = \begin{cases} 1, q_1 \in [0,7;1] \\ 0, q_1 \notin [0,7;1] \end{cases} & b_{32} = \begin{cases} 1, q_2 \in [0,7;1] \\ 0, q_2 \notin [0,7;1] \end{cases} & b_{33} = \begin{cases} 1, q_3 \in [0,7;1] \\ 0, q_3 \notin [0,7;1] \end{cases} \end{pmatrix}. \quad (7)$$

Stage 5. The accumulation of results of stages 2, 3 and 4, their presentation in the form of Table 1 and the analysis of the most risky areas of quantitative assessment of risk in reinsurance operations.

Table 1. Conformity of risk components to the interval limits of their qualitative characteristics

Qualitative characteristics of risk	Interval limits of the normalized value of risk components	Risk components		
		Possibility of insured accident	Measure of variability of obtained results	Degree of deviation from the desired result
		$P_X(H1)$	$SV(X)$	$SSG(X)$
Normal	[0;0,5)	b_{11}	b_{12}	b_{13}
Raised	[0,5;0,7)	b_{21}	b_{22}	b_{23}
High	[0,7;1)	b_{31}	b_{32}	b_{33}

Stage 6. Calculation of integrated risk assessment for reinsurance operations as a combination of three components (the possibility of insured accident, the measure of variability of obtained results, the degree of deviation from the desired result) in the form of contingency coefficient. The basis for determining integral characteristics of risk levels are binary parameters obtained in the previous stage. The contingency coefficient (K_k) is calculated in the following way (equation (8)):

$$K_k = \frac{b_{11}b_{22}b_{33} + b_{21}b_{13}b_{32} + b_{31}b_{12}b_{23} - b_{13}b_{22}b_{31} - b_{21}b_{12}b_{33} - b_{11}b_{23}b_{32}}{\sqrt{(b_{11}+b_{12}+b_{13})(b_{21}+b_{22}+b_{23})(b_{31}+b_{32}+b_{33})(b_{11}+b_{21}+b_{31})(b_{12}+b_{22}+b_{32})(b_{13}+b_{23}+b_{33})}}\,. \quad (8)$$

Stage 7. Detection and quantitative assessment of the synergy effect of risk resulting from simultaneous occurrence of factors leading to insured accident according to multiple quantitative criteria $P_X(H1)$, $SV(X)$ and $SSG(X)$. The necessity of this stage is conditioned by the fact that we present the risk as a complex multilevel system that has three interrelated elements, which cause one another and lead to the formation of new features and characteristics of integrated risk assessment not inherent in any of the individual components. The mathematical correlation as the basis for the identification of synergy effects takes the following form (equation (9)):

$$SE = \left[\frac{b_{11}+b_{12}+b_{13}}{3}\right] | \left(b_{11}+b_{12}+b_{13}\right) \ge 2 + \left[\frac{b_{21}+b_{22}+b_{23}}{2}\right] | \left(b_{21}+b_{22}+b_{23}\right) \ge 2 + \\ + \left(b_{31}+b_{32}+b_{33}\right) | \left(b_{31}+b_{32}+b_{33}\right) \ge 2. \quad (9)$$

The essence of the synergy effect of integrated risk assessment formalized by equation (9) is the highlighting of the following aspects:

- it is found within each risk component;
- occurs in a situation if within any level of risk quality characteristics (normal, raised or high) at least two factors of a risk situation are observed or when the sum of binary characteristics is bigger than or equals two: $\begin{bmatrix} \left(b_{11}+b_{12}+b_{13}\right) \ge 2 \\ \left(b_{21}+b_{22}+b_{23}\right) \ge 2 \\ \left(b_{31}+b_{32}+b_{33}\right) \ge 2 \end{bmatrix.$;
- proposed levels of qualitative characteristics of risk (normal, raised or high) have different impact on the formation of integrated risk assessment, assuming the value of sums of binary indicators for the three components with different weight coefficients: $\begin{bmatrix} 1/3(b_{11}+b_{12}+b_{13}) \\ 1/2(b_{21}+b_{22}+b_{23}) \\ 1(b_{31}+b_{32}+b_{33}) \end{bmatrix.$.

Stage 8. Formation of generalized risk characteristics that takes into account the peculiarities of reinsurance activity is a complex function of risk components $f\left(P_X(H1); SV(X); SSG(X)\right)$ and is comprised of two key elements – contingency coefficient and the component formed under the influence of synergy effects from simultaneous occurrence of facts leading to insured accident according by several qualitative criteria. The formalization of generalized risk characteristics on the basis of mathematical algorithms has to be carried out in the following way (equation (10)):

$$R_p = K_k + SE\,. \quad (10)$$

Stage 9. Qualitative characteristics of the objects of study from the scientific and methodical approach to determining the generalized risk assessment for reinsurance operations. The following grouping is proposed [11]:

- if the obtained generalized assessment belongs to the interval from $\min\{R_p\}$ to $\frac{\max\{R_p\} - 3\min\{R_p\}}{2}$, its risk level is normal;
- if it belongs to the interval from $\frac{\max\{R_p\} - 3\min\{R_p\}}{2} \le R_p \le \frac{\max\{R_p\} + \min\{R_p\}}{2}$, its risk level is acceptable;

- if it belongs to the interval $\frac{\max\{R_p\}+\min\{R_p\}}{2} \le R_p \le$ $\le \frac{3\max\{R_p\}+\min\{R_p\}}{4}$, its risk level is high;
- for the interval from $\frac{3\max\{R_p\}+\min\{R_p\}}{4}$ to $\max\{R_p\}$ the risk level is critical.

Stage 10. Systematization of the obtained results and making sound managerial decisions by managers of insurance (reinsurance) companies on the basis of the analysis of quantitative and qualitative characteristics of the risk of insured accidents. In addition, on the basis of the obtained results it becomes possible to form a comprehensive system of corrective measures within the current activities of insurers and to conduct strategic planning of the areas for further development.

Conclusions of the study and recommendations for further research in this field

Within this research and the study of peculiarities of the "risk" category during the carrying out of reinsurance operations we have:

- summarized the existing approaches to the identification of risk categories and insurance risk, in particular;
- identified specific features and requirements, which concern the definition of risk during reinsurance activities and which include the negative content of the "risk" category manifesting itself in the occurrence of the insured accident and making corresponding claims;
- proposed the formalization of risk during the carrying out of reinsurance operations as a combination of three components (the possibility of insured accident; the measure of variability of obtained results; the degree of deviation from the desired result) in the form of an implicit function, which is the contingency coefficient;
- validated the necessity to consider the synergy effect of the risk from simultaneous occurrence of factors contributing to the insured accident by multiple quantitative criteria;
- proposed a scientific and methodical approach to determine the generaled risk assessment in carrying out reinsurance operations as a system for making complex managerial decisions.

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