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Market value of insurance liability and life insurance securitization in a CDO framework

Abstract

This work will study the market value of insurance liability and life insurance securitization in a collateralized debt obligation (CDO) framework. The key concept of CDO technology is the tranching of the liabilities. The senior tranches have the priority over the junior tranches. If we consider the policyholder as senior tranche and the insurance company as the junior tranche, we could study the life insurance company's operation in a CDO framework. Specifically, this paper derives a market value of insurance liability formula, studies the benefit of life insurance securitization. An example is given to illustrate the concept. To simplify, the author assumes the tax rate is zero.

Keywords: tranche, market value of liabilities, life insurance securitization.

Introduction

Fair value accounting measures an insurance company's operation from a value's perspective. It is the new direction of the insurance company financial reporting. It further enhances the financial disclosure of the insurance companies, and provides possibilities of comparison across countries and industries. Briefly speaking, in this accounting framework, assets and liabilities are accounted at their fair value (market value).

It is easy to define the market value of assets. It is harder to define the market value of liability (MVL) for the insurance business. There have been lots of research and proposals on this topic (for some of the discussions, see American Academy of Actuaries, 2002; Babel, 2002; Perrot and Hines, 2002; Wallace, 2002), one of which is the Luke Girard (2000) decomposition formula based on the actuarial appraisal method. Dutta Gupta derives an explicit formula for MVL, which is more direct and provides insights of the components of the MVL. In this work, based on the CDO framework, we explain the components of MVL further, and offer a new look at the benefits of the life insurance securitization.

Securitization of life insurance business has developed rapidly in recent years. It is a mechanism that capital market could participate in liability side of the life insurance business, such as taking a view on the mortality experience. Some people believe that it will change the life insurance industry in next few decades; just like how the invention of the mortgage backed security changed the mortgage industry. For more discussions on the topic, see Cummins (2004).

A simple cash flow CDO issues two classes of notes, a senior note and a junior note (this is just for illustrative purposes. In reality, the structure is much more complicated). The CDO's obligation to pay the senior note always takes the priority to its obligation to the junior note. The junior note is also

called the equity tranche, because it takes the first loss and profit of the CDO's performance. The senior note is also called the senior tranche. The CDO uses the proceeds of the two notes to buy a pool of assets. The cash flow of the assets will be used to service the interest of the two notes. This pool of asset serves as collateral for the two notes.

In this simple case, the senior note holder provides the leverage to the junior note holder. The junior note holder hopes to obtain higher return by taking more risk. Say the notional amount of the senior and junior are the same, and CDO assets return 7%. As the senior note has less risk than any of the CDO assets itself, the note might has a coupon rate at 5%. Then the junior note will have a return of 9%. While junior note has a higher expected return, it also has more risk. If any asset in the CDO asset pool defaults, the junior will suffer the loss first. For example, if, after the defaults, the asset has return of 4%, then the senior note will still have 5% returns, but the junior will have only 3%.

This example only illustrates the concept of tranching. The CDO technology has been developing in rapid pace in recent years. There are synthetic CDOs, CDO of CDOs, ..., and innovation continues.

An insurance operation (operating company) could be considered, in a way, a CDO of two tranches, a policyholder tranche, and a company (holding company), or owner tranche. The policyholder tranche is a senior tranche. The obligation to the policyholder should always be met before any other obligations. Let us assume that cash flow with the policy holder is $L(t)$ at time t , which is the net cash flow to the policyholders, the premiums minus the benefits. For simplicity, we assume that the policies are not participating. There is no dividend paid to the policyholders based on the performance of the business and the company. The owner tranche is the junior tranche. Let us assume that the cost of capital (CC) of the owner is k . The owner takes the first loss and gain of the business. Nevertheless, it expects the

return rate of k . If the return is lower than k , the owner losses economic value. The owner tranche provides capital when required and receives dividend (interest) when available.

There are assets backing the business, the fair value of the business, from the point of view of the owner, is then

Market Value of Assets (MVA) – Market Value of Liability (MVL).

The liability is the obligation to the policyholders. Here it is the obligation to the future cash flow $L(t)$. What is the market value of the liabilities? Is it just discounted liability cash flow? There are other components to the puzzle, such as reserve and capital requirement. The reserve and capital requirement is set by the regulator and rating agency to operate the business and receive a desired rating.

For simplicity, we will not consider the tax impact. That is we assume the tax rate is 0.

1. CDO as pass through without collateral

We first consider the case where there is no reserve and capital requirement for the operation. Generally speaking, the return of the invested assets (of the CDO or insurance operation) has a lower return than the cost of capital of the owner. Then the most efficient way to fund the obligation to the senior note holder, the policyholders, is fund as you go.

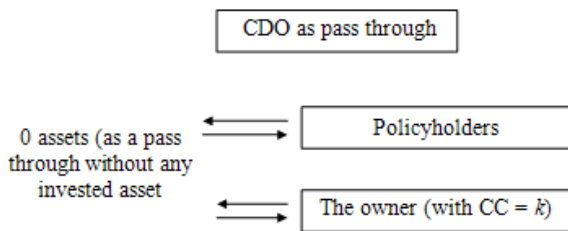


Fig. 1. CDO as pass through

The owner will provide (and receive) the cash flow with the senior note holder whenever it occurs. The owner will provide the $L(t)$ to the policyholder at time t . To value this stream of cash flows, we discount the cash flow at k .

$$MLV1 = \sum_{t=1}^N \frac{L(t)}{(1+k)^t} \tag{1}$$

This value of the liability is a hypothetical value. In fact, it implies that the CDO has no collateralization. The senior tranche are exposed directly to the risk of junior tranche's ability to pay. The positive cash flow from the policy holder is distributed to owner immediately, and the owner will raise capital to pay the policyholder when the cash flow is negative, such as a large amount death benefit is due.

This assumption is clearly not the reality. The policyholder is exposed too much liquidity, credit risk of the owner company. There is no certainty of owner's ability to pay when policy benefit is due. This concern is addressed by the reserve and capital requirement.

To simplify the notations, we define as Cummins (2004), the following

$$NC(i, k, F) = \frac{\sum_{t=1}^N (k - i(t))F(t-1)}{(1+k)^t},$$

where $F = F(t)$, $i = i(t)$. For basic properties of this function, see Cummins (2004). Conceptually, it is the value lost while maintaining a balance of assets at $F(t)$ earning rate of $i(t)$, assuming the owner of the asset has a cost of capital of k .

2. CDO with collateral

The insurance business is highly regulated. There are reserve requirements and capital requirements. In a sense, it is the collateralization for the senior tranche. The level of collateral is set by the regulators. In this CDO framework, it means that the invested asset of the CDO should at least be $SVL(t)$, and $RS(t)$, where $SVL(t)$ is the statutory reserve at time t , and $RS(t)$ is the required surplus at time t . Let us assume that the reserve asset are invested with a more conservative strategy, the rate of return is $i(t)$, in year t . Let us assume the required surplus assets are invested with a relatively aggressive strategy, the rate of return is j .

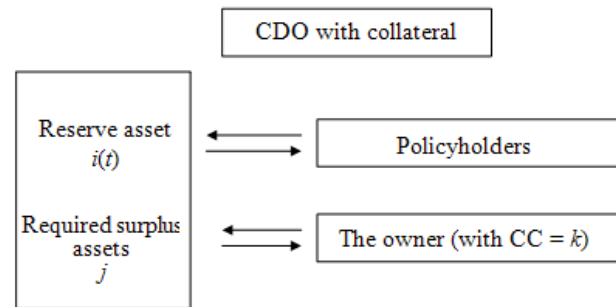


Fig. 2. CDO with collateral

Generally speaking, the rate $i(t)$ and j are less than k . The cost to the owner of maintaining the account for the reserve in year t is the lost interest at time t . It is $(k - i(t))SVL(t - 1)$. The total present value lost over the years is

$$NC(i, k, SVL) = \frac{\sum_{t=1}^N (k - i(t))SVL(t-1)}{(1+k)^t}.$$

Similarly, the cost to the owner of maintaining the account for the required surplus is

$$NC(i, k, RS) = \frac{\sum_{t=1}^N (k - j)RS(t-1)}{(1+k)^t}$$

The total market value of the liability is then

$$MVL2 = \frac{\sum_{t=1}^N L(t)}{(1+k)^t + NC(i, k, SVL) + NC(j, k, RS)} \quad (2)$$

The equation (2) is the same result as Cummins (2004). Cummin’s results are obtained through the actuarial appraisal method and precise formulation of actuarial and financial theory.

3. CDO with further tranching

Currently, there is a trend that the life insurance companies try to further tranche the structure by introducing the capital market participation. This is often referred as life insurance securitization. Securitization is used by life companies to transfer risk, to increase capital efficiency, etc. One of the uses of securitization is to fund the redundant reserves of the level term life insurance and universal life insurance with secondary guaranteed. The statutory (XXX) reserve for term life and statutory (AXXX) reserve for UL with secondary guarantees are considered to have some degree of redundancy. That is to say that there is too much overcollateralization required of the CDO. A mezzanine (middle) tranche is introduced to fund this part of the overcollateralization.

The concept of economic reserve is introduced to provide a measure of appropriate level of the collateral required in the structure for the security of the policyholders tranche. In other words, it is a more realistic measure of the company’s liability to the policyholders. Let $EVL(t)$ be the economic reserve at time t . The difference of statutory and the economic

$$MVL3 = \frac{\sum_{t=1}^N L(t)}{(1+k)^t + NC(i-r+k, k, RVL) + NC(i, k, EVL) + NC(j, k, RS)} \quad (5)$$

The value created for the owner tranche is the $MVL3 - MVL2 = NC(r, k, RVL)$. The redundant reserve RVL is funded at $r(t)$ instead of the cost of the capital k . The saving is the dollar amount of negative carry from rate $r(t)$ to rate k .

4. Tranching with extra over collateralization

In most of the cases of life insurance securitization, the investor in the mezzanine tranche requires some extra overcollateralization. The owner are required

reserve is called the redundant reserves: $RVL(t) = SVL(t) - EVL(t)$. The RVL could be funded by the third party as a mezzanine tranche. The mezzanine tranche has less risk then the equity tranche, it has a lower interest rate.

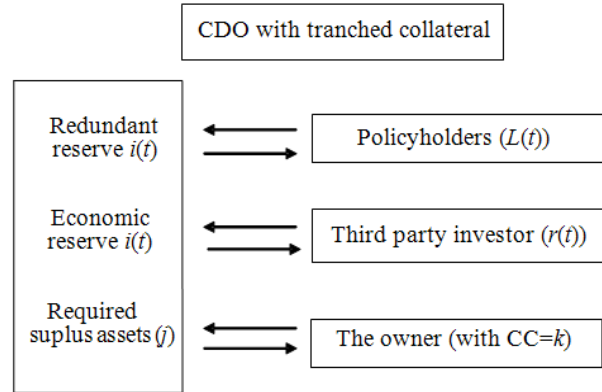


Fig. 3. CDO with tranching collateral

Under this structure, assuming the third party investor is secure that all the losses are still with the owner. The owner will have the loss of present value at

$$MVL2 = \frac{\sum_{t=1}^N L(t)}{(1+k)^t + NC(i, k, SVL) + NC(j, k, RS)},$$

$$NC(i-r+k, k, RVL) = \frac{\sum_{t=1}^N (r(t) - i(t))RVL(t-1)}{(1+k)^t} \quad (3)$$

The owner’s loss of maintaining the economic reserve is

$$NC(i-r+k, k, EVL) = \frac{\sum_{t=1}^N (r(t) - i(t))EVL(t-1)}{(1+k)^t} \quad (4)$$

The market value of the liability is then

$$MVL3 = \frac{\sum_{t=1}^N L(t)}{(1+k)^t + NC(i-r+k, k, RVL) + NC(i, k, EVL) + NC(j, k, RS)} \quad (5)$$

to, besides the economic reserves, maintain a minimal surplus (MS) level more than the required surplus (RS). The owner’s loss of up keeping the minimal surplus is

$$NC(j, k, MS) = \frac{\sum_{t=1}^N (k - j)MS(t-1)}{(1+k)^t} \quad (6)$$

The market value of the liability is

$$MVL4 = \frac{\sum_{t=1}^N L(t)}{(1+k)^t + NC(i-r+k, k, RVL) + NC(i, k, EVL) + NC(j, k, MS)} \tag{7}$$

The value added for owner by issuing the mezzanine tranche is then

$$VA = -MVL4 + MVL2 = NC(r, k, RVL) + NC(2k - j, k, MS - RS) \tag{8}$$

The $NC(r, k, RVL)$ is the saving of the financing the RVL with lower rate of $r(t)$. It is positive when $k > i(t)$. If $MS > RS$ and $k > j$, the term $NC(2k - j, k, MS - RS)$ will be negative. It is the extra cost of carrying MS instead of RS . The value added for the owner company will depend on the relative size of these two terms. The equation (8) obtains the same results as in Cummins (2004) when tax rate is 0.

5. An example

In this section, we will use an example to illustrate the components of the MVL as described above, and how securitization could help the company to regain profitability under the regulatory framework. The example provides further insight and intuition of the securitization process from a value prospective.

The example is based on a real life block of business. It is scaled and twisted with extra lapse to add some noise. The total face is \$35B. The initial statutory reserve is \$35M. The peak statutory reserve is \$464M. The initial economic reserve is \$98M. The peak economic reserve is \$185M. For simplicity, we assume the required surplus to be 10% of the statutory reserve. The total asset required to back the block of business is \$217M initially, and \$511M at the peak.

Table 1. Basic information

Face (\$M)	34,938				
Reserves (\$M)	Statutory	Economic	Redundant	Required surplus	Total asset
Initial	197	98	99	20	217
Peak	464	185	279	46	511

The statutory and economic reserve patterns are as follows.

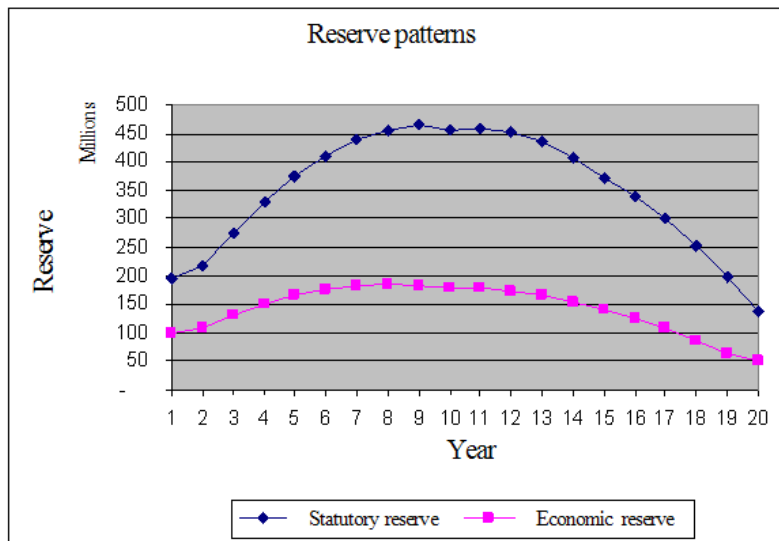


Fig. 4. Reserve patterns

We assume the following basic investment returns. In this low rate environment, we assume $WACC$ to be 7.0%. The return of the reserve assets is 4%, and return on the surplus assets is 5%. The note issued to investor on the redundant reserve has an interest rate of 4.5%.

Table 2. Basic investment assumption

Notation	Rates	Definition
k	7.0%	WACC
i	4.0%	Investment return of reserve assets
j	5.0%	Investment return of surplus assets
r	4.5%	Investment redundant reserve note

The net cash flow to the company, expected premium less expected death benefit is positive at early years, and slightly negative in later years. The PV of the net cash flow discounted at $WACC$ is \$80M. The block of the business is expected to produce positive fair value (FV) if there is no reserve requirement. Assuming the asset is zero, the $MVL1$ is negative (\$80M), see Table 3.

Note that the asset is accounted at the fair value; it will not impact the value of the block of the business. The FV of the block is the negative of the MVL .

The reality is that the company is required to hold the statutory reserve and the required surplus, which provides “collateral” to the policy holder. The total initial asset to be funded (at *WACC* of 7%) is \$217 (\$197 for reserve and \$20 for surplus). The reserve asset has return of 4%, which decreases the *FV* of the block by \$115M. The loss is due

to the higher funding cost and the lower investment return of reserve assets. As a result, the *MVL* increased by \$115M. The surplus asset has return of 5%, which increases the *MVL* by \$8M. The total *MVL* changes to *MVL2* at \$43M. The *FV* is negative (\$43M). It is no longer a profitable block of business.

Table 3. *MVL1* and *MVL2 NC(j, k, RS)*

Rows	Formula	Equ. #	Definition	<i>MVL</i>	<i>FV</i>
(1)			$\sum_{t=1}^N L(t)/(1+k)^t$	(80)	
(2)	= (1)	(1)	<i>MVL1</i>	(80)	80
(3)			<i>NC(i, k, SVL)</i>	115	
(4)			<i>NC(j, k, RS)</i>	8	
(5)	= (1) + (3) + (4)	(2)	<i>MVL2</i>	43	(43)

It is understood that the current statutory reserve requirement has certain redundancy. If we could fund the redundant reserve at a rate lower than the *WACC* through securitization, we could reduce the *MVL*. The economic reserve (*EVL*) continues to be funded at *WACC* of 7%, but the redundant reserve

(*RVL*) could be funded at 4.5% due to ring-fenced and lower risk. The increase of the *MVL* due to reserve requirement would be at \$11M for funding *RVL* and \$48M for funding *EVL*. The total *MVL* would be *MVL3* at negative (\$13M). The block is profitable again with *FV* at \$13M, see Table 4.

Table 4. *MVL3* and *MVL4*

Rows	Formula	Equ. #	Definition	<i>MVL</i>	<i>FV</i>
(6)			<i>NC(i - r + k, k, RVL)</i>	11	
(7)			<i>NC(i, k, EVL)</i>	48	
(8)	= (1) + (6) + (7) + (4)	(5)	<i>MVL3</i>	(13)	13
(9)			<i>NC(j, k, MS)</i>	10	
(10)	= (1) + (6) + (7) + (9)	(7)	<i>MVL4</i>	(11)	11
(13)			<i>NC(r, k, RVL)</i>	56	
(14)			<i>NC(2k - j, k, MS - RS)</i>	(2)	
(15)	= (13) + (14)	(8)	<i>VA = -MVL4 + MVL2</i>	54	

Sometimes, the inventor of the redundant reserve note negotiates for a minimum surplus (*MS*) above the required surplus, as addition cushion for risk. This will increase the *MVL*. In this example, we assume the *MS* to be 125% of the *RS*. The additional cost is \$2M. The *MVL* of block under this scenario *MVL4* is negative (\$11M). The *FV* reduced by \$2M comparing to *MVL3*.

Finally, with the surplus set at *MS*, the value enhancement due to securitization is \$54M, comparing *MVL4* of (\$11M) with *MVL2* of \$43M. It consists of two components, saving of funding *RVL* of \$56M, and additional funding cost for extra surplus assets (*MS-RS*) at \$2M.

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Conclusions

In this work, we had some brief discussion of using the CDO framework to value the life insurance liability, and to study value added with life insurance securitization. The author hopes that this new approach could provide some further insight to this topic.

We obtained formulas similar to those in Cummins (2004). Serious actuaries, mathematicians might prefer the approaches in Cummins (2004). Those approaches are more precise, and are confirmed by traditional actuarial literature. In this work, our approach is more intuitive.

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