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Population heterogeneity in defined contribution pension schemes

Abstract

Population ageing has implications on sustainability of pension systems and governments have acted in recent years to find more sustainable pension schemes. In this regard, in many countries a gradual shift from Defined Benefits (DB) to Defined Contributions (DC) system is occurring. According to the latter scheme, contributions paid during the working life are accumulated until the retirement age, when the amount is converted into a life annuity through annuitization factors taking into account the forecasted expectancy of life. In order to ensure actuarial fairness of pension contracts longevity risk has to be taken into account. Moreover, fairness is hindered by mortality heterogeneity. In fact, mortality rates are influenced by race, ethnicity, income, wealth, marital status and educational attainment but these aspects are not considered in DC pension plans and the same annuitization factors are applied for different individuals. The aim of this paper is to analyze the effect of mortality heterogeneity on DC plan, focusing on the resulting redistribution of wealth between different groups of individuals. Computationally, numerical analyses on the Italian dataset are provided.

Keywords: defined contribution scheme, longevity risk, pension system, transformation coefficient.

Introduction

Population ageing has implications on sustainability of pension systems, which need to be reformed and flanked by new investment products and private pension funds. In fact, pension plan sponsors face a myriad of risks, one of which is longevity risk that arises from the increasing life expectancy trends among pensioners. For this reason, plausible mortality forecasting techniques have to be used to consistently predict these trends. Governments have acted in recent years to find more sustainable pension systems as well. In many countries, for example, pension legislations has been reformed during the last decade, moving from Defined Benefits (DB) to Notional Defined Contributions (NDC) system, the last one considering particularly important the rules to take into account in the pension formulae life expectancies and their changes (Belloni, Maccheroni, 2006). In Italy, the switch from the DB to the NDC occurred with the 1995 pension reform. On the basis of this reform, different rules are applied to different workers and the differences are based on the seniority accrued in 1995. According to NDC, pensions are determined on a defined contribution basis, and notionally accumulated contributions are transformed into an annuity at retirement. In particular, contributions paid during the working life are accumulated at the expected economic growth rate until the retirement age, when the amount is converted into a life annuity taking into account the forecasted expectancy of life. If the forecasting of the GDP and that of mortality evolution are correct, both the macroeconomic and longevity risk are transferred by pension providers to workers.

New scheme is not applied to all workers at the same way. It is not applied, for example, to workers with at least 18 years of seniority in 1995, while it is fully applied to all who are employed from 1996 onward; younger workers are treated with a pro rata method.

The NDC pension formula is based on a computation given by two steps. The first step consists in computing the individual accrued fund at retirement. The second one allows to compute the pension benefit, multiplying the accrued fund at retirement times the transformation coefficient specific for the retirement age x. The transformation coefficient is given by the inverse of the expected present value of an annuity of one euro revertible to the spouse.

In the actual formulae differences between genders are averaged out and transformation coefficients are the same for males and females with the same age. This produces a redistribution of wealth from men, who have a shorter expectancy of life, to women. In fact, according to the current legislation, transformation coefficients are calculated on the basis of the entire population.

The aim of this paper is to catch mortality heterogeneity and to value its impact on the calculation of transformation coefficients and on pension system as well. We know that individuals are different with respect to mortality due to different race, ethnicity, income, wealth, marital status, geographical area and so on. Researchers have noted that individuals at the same age may differ substantially in their endowment for longevity, and that individual differences are important to population-based mortality studies. Consequently, the differences in mortality rates between the reference population and different groups in which the data set can be decomposed on the basis of specific factors, can compromise accuracy in the mortality projection. Thus actuarial valuation can be warped if heterogeneity is not considered.

The paper is organized as follows. Section 1 focuses on a particular issue regarding the mortality heterogeneity in the framework of defined contribution pension scheme. In order to deal with such issue, Section 2 introduces the Lee Carter model in its Poisson version. A numerical application of Italian transformation coefficients is provided in Section 3. The final section concludes.

1. The mortality heterogeneity in the framework of defined contribution pension scheme

In 1995 Italian Law 335/95 determined a shift from DB to NDC scheme, in which notional accumulated contributions on individual accounts were converted into an annuity at retirement. Unlike the preceding method, the latter takes into account the amount of contribution paid throughout the whole working life accumulated at the expected GDP (Gross Domestic Product) growth rate, the life expectancy of the pensioner at retirement age and the number of years that a survivor's benefit will be withdrawn by any widow or widower, according to actuarial equivalence principle. The NDC formula to compute pension benefits for retirement at age x is the following:

$$P(x) = \left[c_a + \sum_{i=1}^{a-1} c_i \prod_{j=i}^{a-1} (1 + \overline{g}_j) \right] \delta_x, \qquad (1)$$

where c_i is the contribution paid by the worker at seniority i, a is seniority at retirement, $\overline{g}_j$ is the geometric mean of nominal GDP growth rate calculated according the observations in the 5 years preceding the year in which seniority is j, δ_x is the transformation coefficient for retirement at age x (to deep the formula, see Piscopo, 2011). The term in brackets represents the total contribution accumulated by worker during the working life; it is transformed into pension benefit by the transformation coefficient δ_x, that is calculated according to the following formulas:

$$\delta_x = \left(\frac{\sum_{s=m,f} dir_{x,s} + ind_{x,s}}{2} - \gamma \right)^{-1} ; x \in [57, 65], \qquad (2)$$

$$dir_{x,s} = \sum_{t=0}^{(\Omega - x)} \frac{l_{x+t,s}}{l_{x,s}} \left(1 + g_f \right)^{-t}, \qquad (3)$$

$$ind_{x,s} = \theta \sum_{t=0}^{(\Omega - x)} \frac{l_{x+t,s}}{l_{x,s}} \left(1 - \frac{l_{x+t+1,s}}{l_{x+t,s}} \right) \left(1 + g_f \right)^{-(t+1)} a_{x+t+1}^{W}, \qquad (4)$$

where m stays for "male", f stays for "female", γ is an actuarial factor to take into account different frequencies in pension payment and is fixed by law, l_x is the number of survivors at age x, g_f is the long-run expected GDP growth rate, a_{x+t+1}^{W} is the ex-

pected present value of a unitary annuity paid to the widow or widower at time $x+t+1$, θ is the quota of pension revertible to the widow or widower[1].

Particularly important for the NDC systems are the rules which establish how to incorporate in the pension formulae life expectancies and their changes. If the population-based value of mortality is taken into account but its heterogeneity within the population is not, there is redistribution from shorter to longer living individuals. Although in some cases it can be considered desirable in a "solidarity" view of Public Pension Programs, in others it cannot. For this reason our research consists in providing more actual transformation coefficients for 2013. To this aim, we take into account the heterogeneity in the data depending on the geographical area. The degree of heterogeneity may significantly vary in different populations, depending on economic and social criteria. Wilkinson (1997) argues that the relation between individual income and mortality is primarily an effect of relative income. He states that absolute income levels are no longer important in the developed world, rather there is a relation with income distribution, whereby health is worse when there is greater inequality across the social gradient. He does acknowledge that within a small area of analysis, income is related to mortality and income distribution is not, due to the lack of heterogeneity in a small area. Kennedy et al. (1996) also suggest a significant positive relation between income distribution and life expectancy and infant mortality. These results were robust to adjustments for poverty, median household income and household size. Kennedy et al. (1996) found that the relation of income inequality to total mortality was higher for the black population but comment that the association between income inequality and mortality is not completely explained and that income distribution may be a proxy for other social indicators.

Because the differences in mortality rates can compromise accuracy in the mortality projection, we aim to derive different survival probabilities for different geographical areas (North, Center and South of Italy). The ultimate aim is to exploit the obtained survival probabilities for generating more reliable transformation coefficients and consequently more reliable pension benefits. In order to model and forecast mortality we will focus on the Lee Carter method in its Poisson version.

2. The Poisson Lee Carter model

The Lee Carter (LC) model combines the information about mortality level and age pattern to explain

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[1] Equations (2)-(4) form a simplied version of the formula estabilished by law, which takes into account also the probability for the window(er) to marry again and the reduction in the window(er)'s pension due to her(his) additional income.

the observed mortality rates. Since its introduction in 1992, this model has undertaken a leading role in the framework of mortality forecasting. In its original version, the LC model suggested a log-bilinear form for the death rate $m_{x,t}$ expressed by:

$$\ln(m_{x,t}) = \alpha_x + \beta_{x,t}k_t + \varepsilon_{x,t}, \quad (5)$$

where α_x is a parameter depending on the age and $\beta_{x,t}$ a parameter depending on the age and time, while k_t is a time-varying parameter. The final term $\varepsilon_{x,t}$ is the error term, assumed to be homoschedastic. The parameters α_x, $\beta_{x,t}$ and k_t can be estimated from historic data using the method of Singular Value Decomposition (SVD) (see LC (1992) for more details). The homoschedasticity hypothesis has been considered quite unrealistic: the logarithm of the observed force of mortality is much more variable at older ages than at younger ages because of the much smaller absolute number of deaths at older ages. For this reason, we consider the log-bilinear Poisson version of the Lee Carter model as in Renshaw and Haberman (2003), who keep unchanged the LC log-bilinear form for the central rate of death, but base their approach on heteroschedastic Poisson error structures. In this version, the LC parameters are estimated under the assumption that the number of deaths recorded at age x during year t, $D_{x,t}$ are distributed according to the Poisson distribution:

$$D_{x,t} \approx Poisson(E_{x,t}\mu_{x,t}), \ \mu_{x,t} \approx \exp(\alpha_x + \beta_x k_t), \quad (6)$$

where $E_{x,t}$ is the number of exposure to the risk of death from which $D_{x,t}$ occurred. To catch the heterogeneity in the data, we apply the Poisson LC model on sub-populations (North, Center and South of Italy) so that we work on more homogeneous groups.

3. Numerical application

As aforementioned, to catch the heterogeneity in the data, we consider sub-populations (North, Center, South) so that we work on more homogeneous groups. On this geographical stratified data, we implement an iterative regression methodology in R for the analysis of age-period mortality data based on the Poisson LC model. The LC-based modelling frameworks is viewed in the current literature as among the most efficient and transparent methods of modelling and projecting mortality improvements. Thus, we make use of the modelling approach discussed in Renshaw and Haberman (2003), which extends the basic LC model and proposes to make use of a tailored iterative process to generate parameter estimates based on Poisson likelihood.

Once we have fitted the Poisson LC model on North, Center and South mortality data, we produce forecasts of future mortality rates and compute the corresponding future life expectancy for the different geographical areas. Finally, we make use of the survival probabilities for area, to estimate transformation coefficients for 2013. We choose the year 2013, because since the 1st of January 2010, new transformation coefficients are applied to compute pension benefits and further revisions are made every three years.

In Figures 1 and 2, we show the fitted parameters for Italian male and female population. What we can notice are the differences in the parameter estimates between genders. On the basis of the results of this comparison, we carry on our analysis to check if there are differences also in the parameter estimates between areas. Thus, we fit the Poisson Lee Carter model on geographical stratified data.
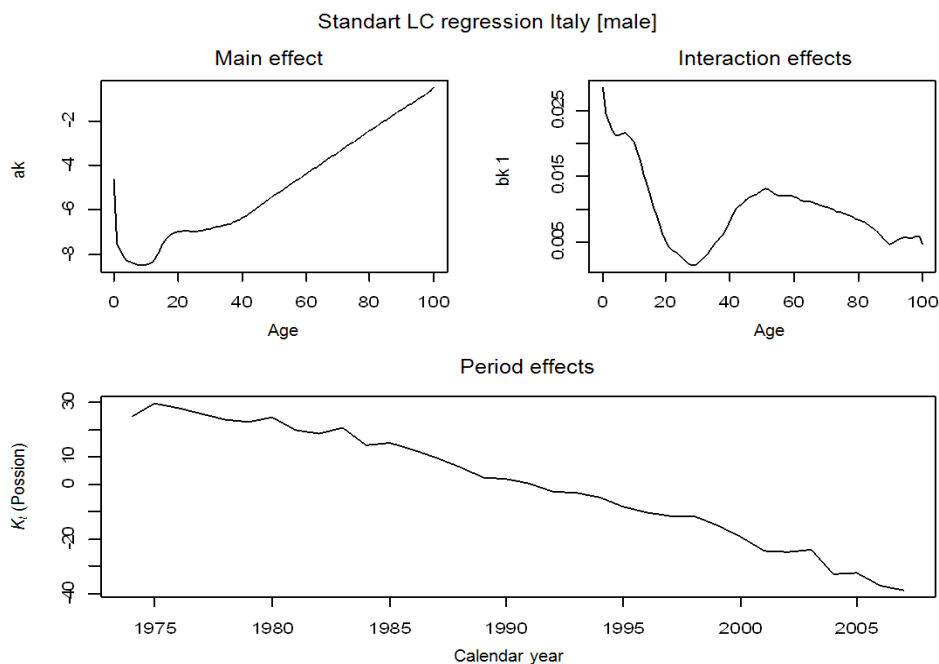

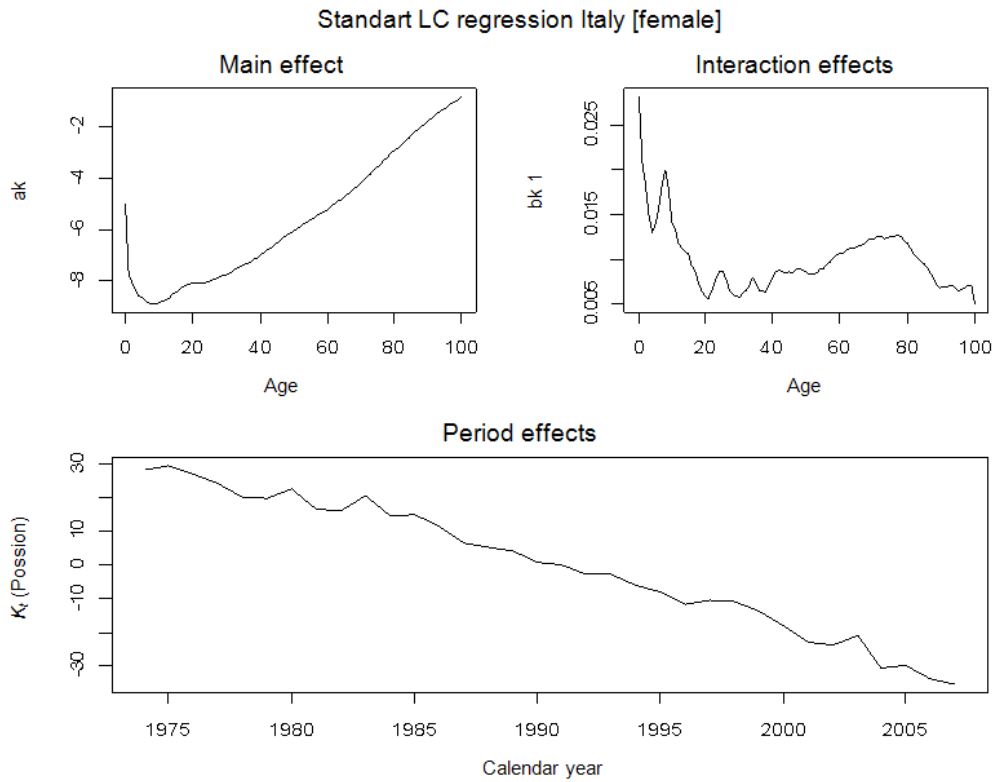**Fig. 1. The parameter estimates of LCP on Italian male mortality data**

Fig. 2. The parameter estimates of LCP on Italian female mortality data

The values of alpha, beta and kappa derived by the fitting of the Poisson LC model on male subpopulations are shown in Figures 3 and 4.
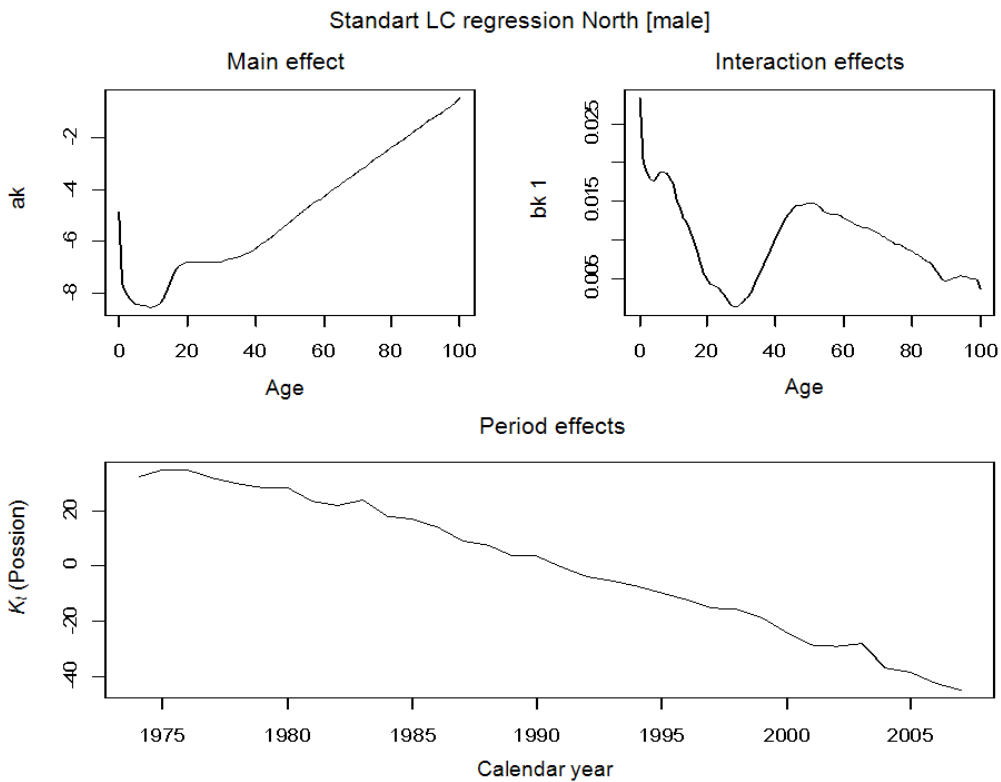

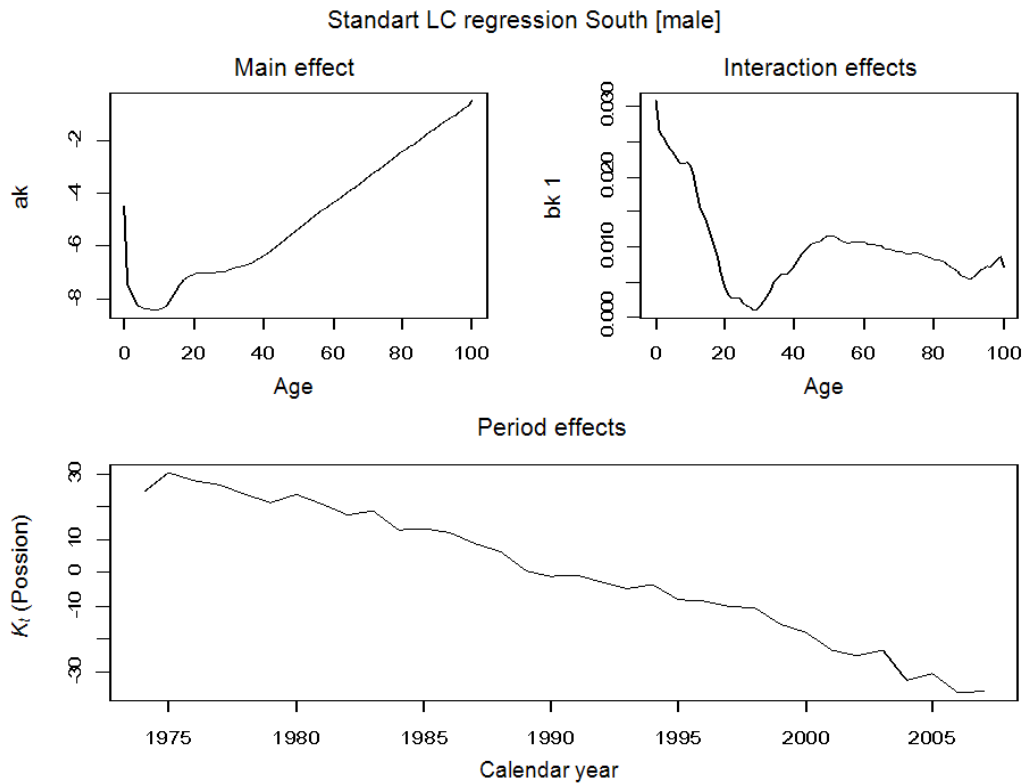**Fig. 3. The parameter estimates of LCP on North male mortality data**

Standart LC regression South [male]

Fig. 4. The parameter estimates of LCP on South male mortality data

In these figures, we show the results just for North and South, even if we have implemented the procedure also on Center data. We can notice the differences in the parameter estimates among North and South. For example, if we look at the estimated k_t for North, the pattern is essentially linear throughout its span. For South, this feature is much less pronounced, the trend being much more erratic.

This fitting supports our issue, to keep making forecast for area. These differences in the parameter estimates are even more pronounced for female (see for example the beta in Figures 5 and 6).
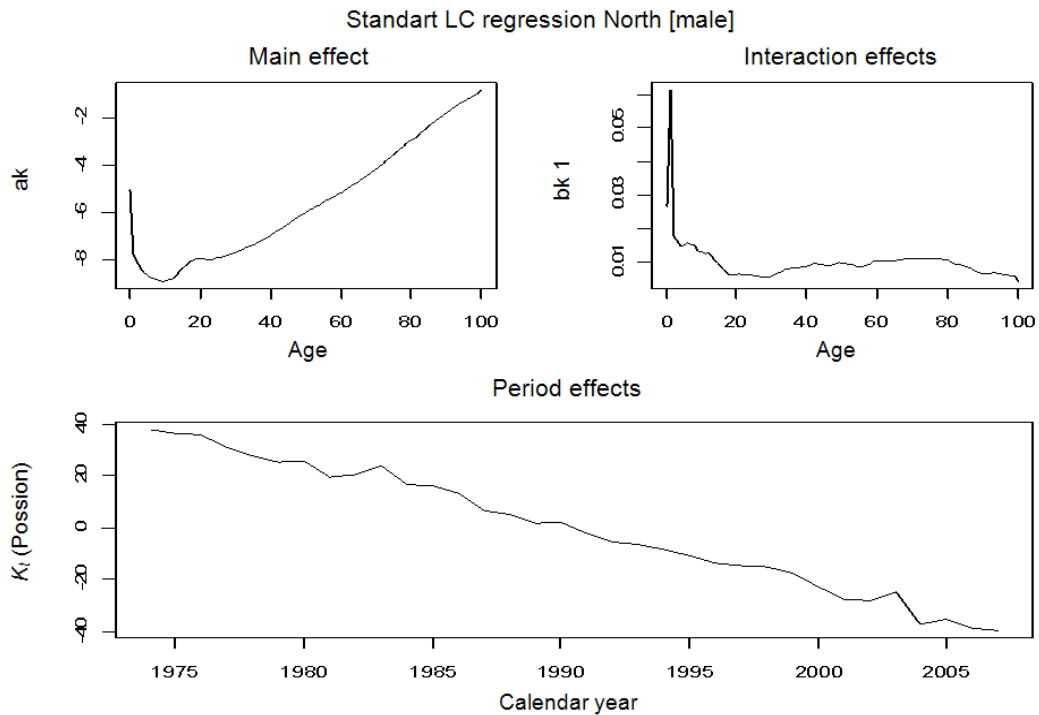
Standart LC regression North [male]

Fig. 5. The parameter estimates of LCP on North female mortality data
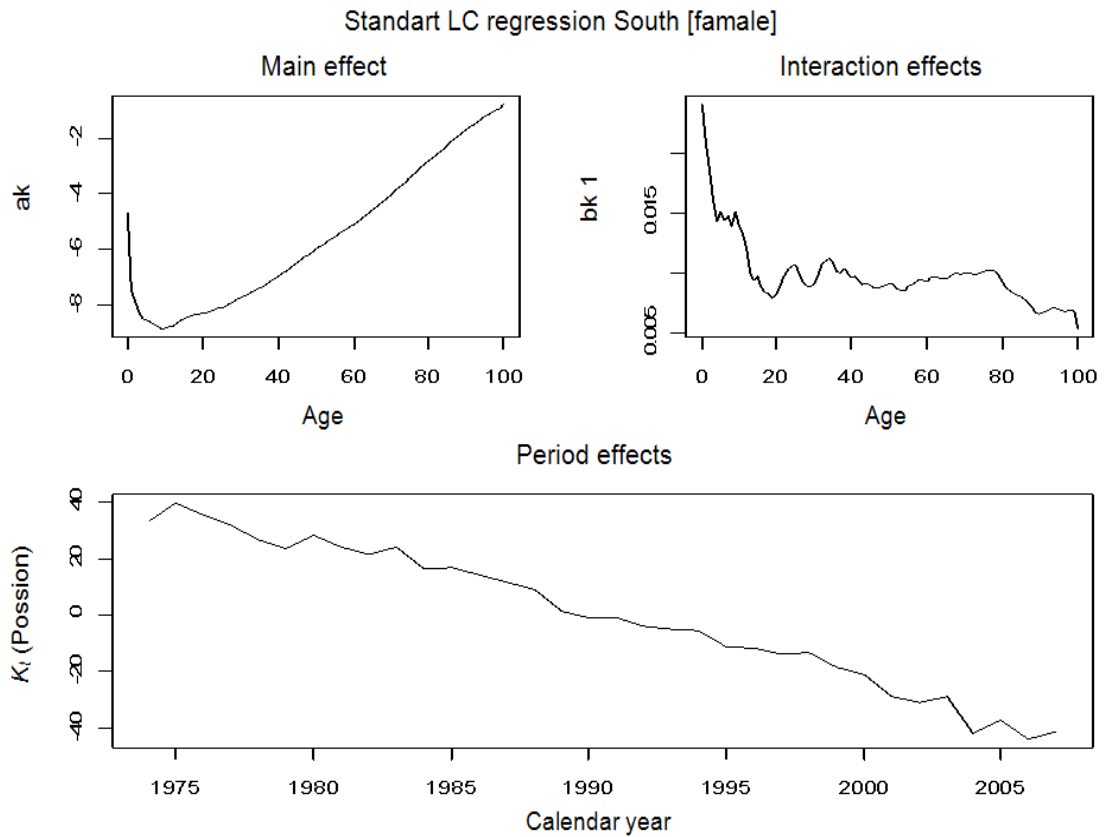
Fig. 6. The parameter estimates of LCP on South female mortality data

By way of illustration, in Figures 7 and 8, we show the estimated values of k_t for North and South, together with the 20-year forecasts and prediction intervals. We also display the life expectancy at 60 for both areas. If we look at the geographical results, male from the north has a life expectancy greater than the South (22,88 years for the North versus 22,22 years for the South).
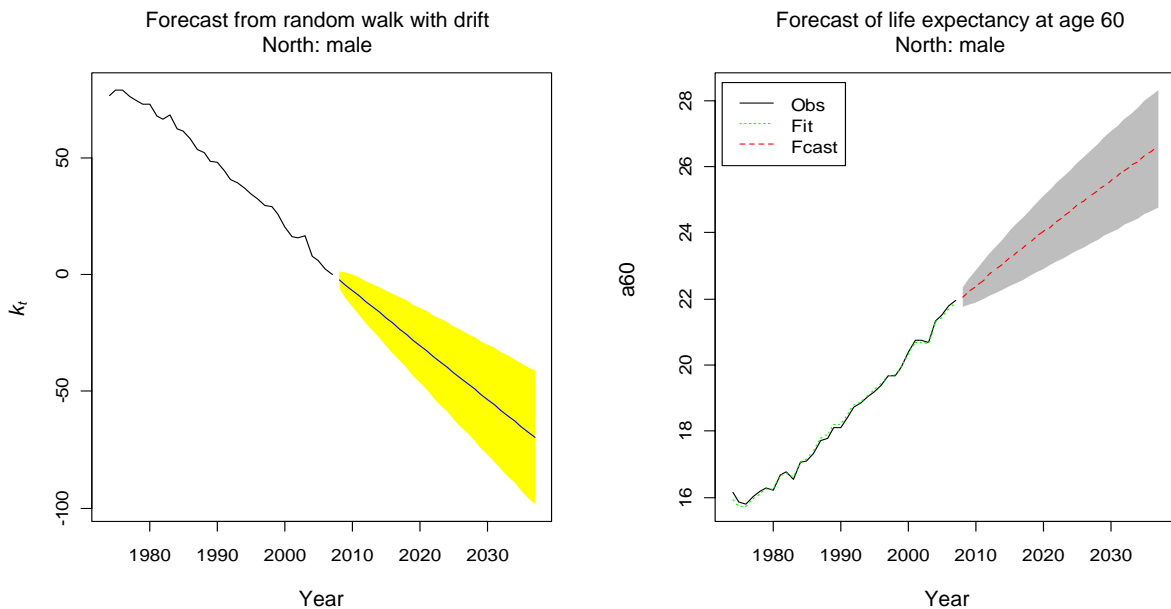

Fig. 7. Life expectancy at 60 for male (North population)
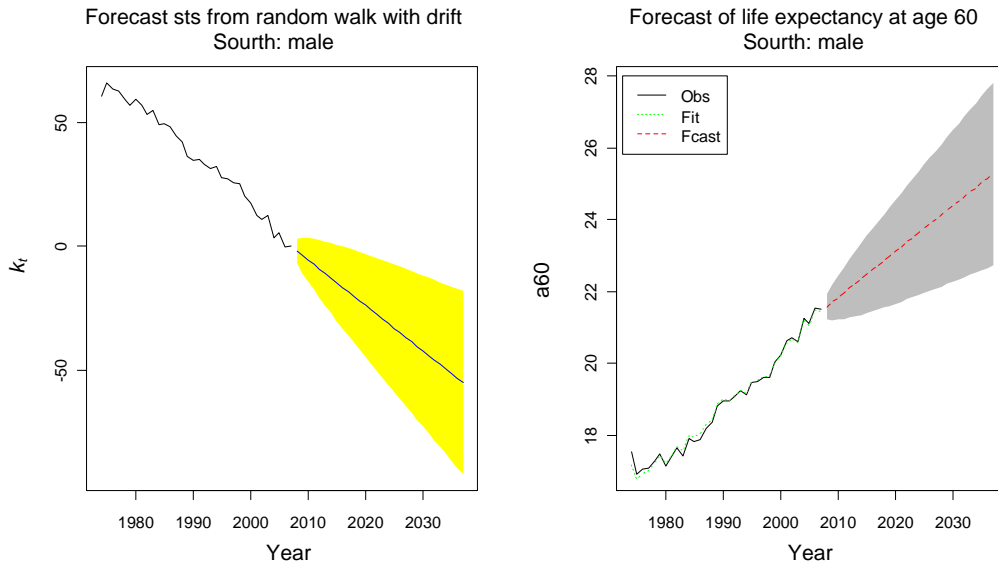
Fig. 8. Life expectancy at 60 for male (South population)

If we have a look at the results of the same analysis, but for female, this discrepancy between North and South is even more accentuated (Figures 9 and 10).
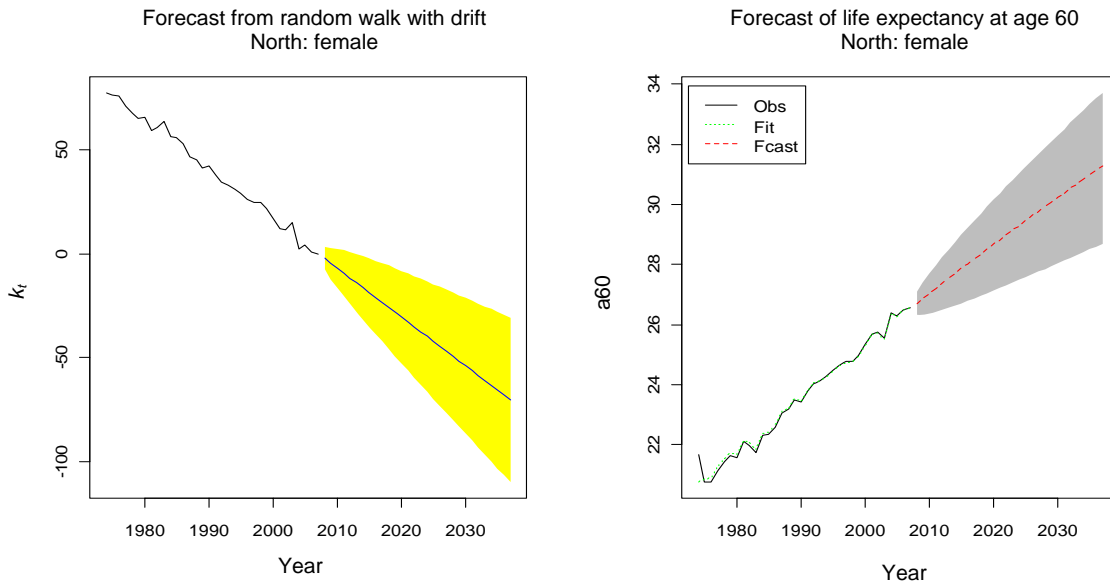

Fig. 9. Life expectancy at 60 for female (North population)
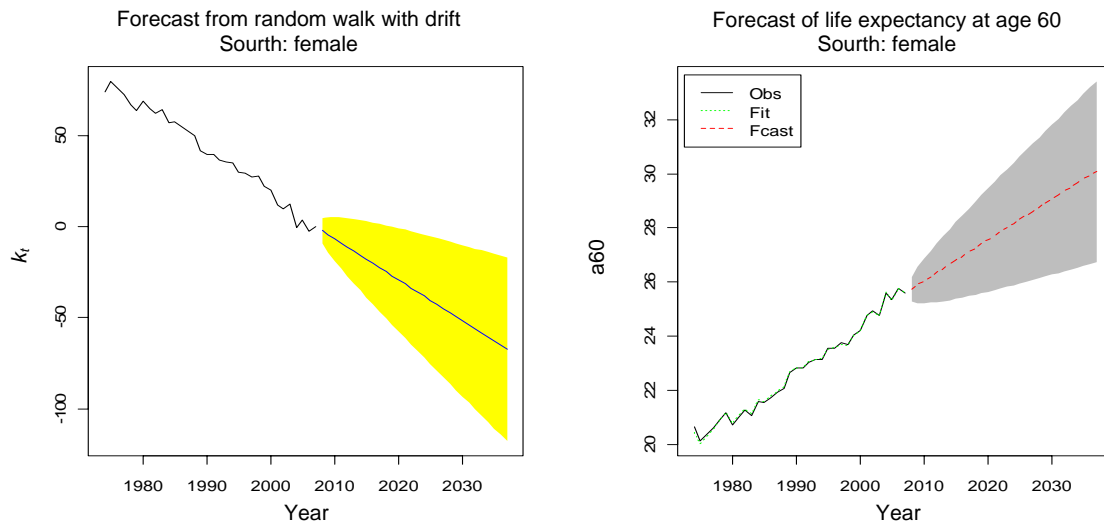

Fig. 10. Life expectancy at 60 for female (South population)

We have derived the estimations of transformation coefficients for area for 2013, setting $g_f = r =$ = 0.015, $\theta = 0.6$ and $\gamma = 0.42$, according to the Law 335/95. The results are summarized in Table 1.

Table 1. Transformation coefficients for area

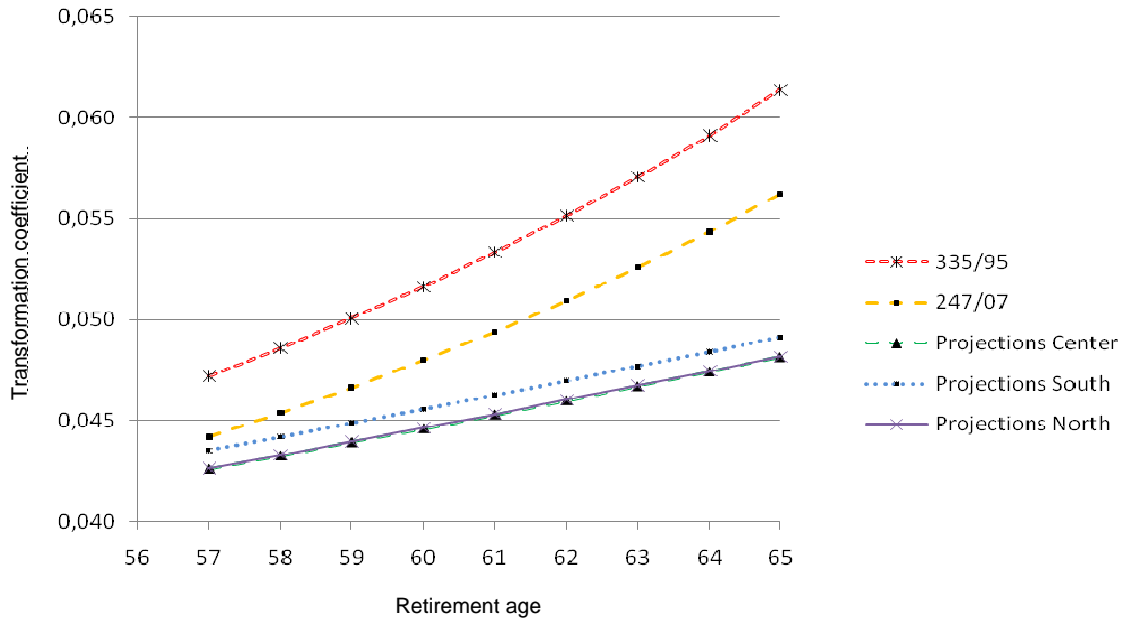| Level	Retirement age								
	57	58	59	60	61	62	63	64	65
North	0,04265	0,0433	0,04397	0,04465	0,04534	0,04603	0,04674	0,04745	0,04818
Center	0,04261	0,04326	0,04392	0,0446	0,04529	0,04599	0,0467	0,04742	0,04814
South	0,04354	0,0442	0,04488	0,04557	0,04627	0,04697	0,04768	0,0484	0,04912

Fig. 11. Comparison among transformation coefficients

Figure 11 presents a comparison with the coefficients established by the Law 335/95 and 247/07.

As it is clear from Figure 11, an increasing in life expectancy entails a reduction in the transformation coefficients. In fact, we can notice that transformation coefficient for North and Center would be lower than South. According to the principle of actuarial fairness, the worker who lives longer receives a smaller monthly pension but for a longer period. In particular, the reduction in the transformation coefficients is greater as the retirement age increases.

Conclusion

In this paper we have taken into account the current reform of the Italian public pension system and we have considered how the mortality heterogeneity impacts on the calculation of transformation coefficients, which are applied to accumulated balances in order to convert them into pension benefits. In the actual formulae differences between genders are averaged out and transformation coefficients are the same for males and females with the same age. This produce a redistribution of wealth from men, who have a shorter expectancy of life, to

women. Moreover, according to the current Law, transformation coefficients are calculated on the basis of the entire population. Running this analysis, we have notice that life expectancy is lower for South than North. We have gathered that if the heterogeneity within the population is not taken into account, there is a wealth redistribution from shorter to longer living individuals. Because the differences in mortality rates can compromise accuracy in the mortality projection, we have derived different survival probabilities for different geographical areas (North, Center and South of Italy). Then, we have exploited the obtained survival probabilities for generating more reliable transformation coefficients and consequently more reliable pension benefits. In order to model and forecast mortality we have used the Poisson version of the Lee Carter model. This model has been applied to male Italian data, divided by geographical areas. We have seen that if we take into account the heterogeneity in the data, higher transformation coefficients should be applied where there are shorter life expectancies in order to have higher pension for shorter period.

References

1. Belloni M., Maccheroni C. (2006). Actuarial Neutrality when Longevity increases: an Application to the Italian Pension System, CERP, Working Paper 47/06.
2. Bianchi C., Romanelli M., Vagliasindi P. (2003). Microsimulating the evolution of Italian Pension Benefits: The Role of Retirement Choices and Lowest Pensions Indexing, *Labour*, 17, pp. 139-173.
3. Borrella M. (2004). The Distributional Impact of Pension System Reforms: An Application to the Italian Case, *Fiscal Studies*, 25 (4), pp. 415-437.
4. Borrella M., Coda Moscarola F. (2005). Distributive Properties of Pension Systems: a Simulation of the Italian Transition from Defined Benefit to Defined Contribution, CERP, Working Paper 42/05.
5. Brouhns N., Denuit M., Vermunt J.K. (2002). A Poisson log-bilinear approach to the construction of projected lifetables, *Insurance: Mathematics and Economics*, 31, pp. 373-393.
6. Cossette H., Delwarde A., Denuit M., Guillot F., Marceau E. (2007). Pension plan valuation and dynamic mortality tables, *North American Actuarial Journal*, in press.
7. D'Amato V., Piscopo G., Russolillo M. (2009). Intensive Computational Forecasting Approach to the Functional Demographic Lee Carter Model, *Frontiers in Artificial Intelligence and Applications*, 204, pp. 177-186.
8. Franco D., Sartor N. (2006). NDCs in Italy: Unsatisfactory Present, Uncertain Future. In Pension Reform: Issues and Prospects for NDC Schemes, edited by Holzman R. and Palmer E., The World Bank Washington, D.C.
9. Haberman S., B. Zoltan (2010). Ilc: A Collection of R Functions for Fitting a Class of Lee-Carter Mortality Models Using Iterative Fitting Algorithm, Cass Actuarial Research Paper, N. 190.
10. Kennedy, Bruce P., Ichiro Kawachi, Deborah Prothrow-Stith (1996). Income distribution and mortality: cross sectional ecological study of the Robin Hood index in the United States, *British Medical Journal*, 312, pp. 1004-1007.
11. Lee, R.D., Carter, L.R. (1992). Modelling and Forecasting U.S. Mortality, *Journal of the American Statistical Association*, 87, pp. 659-671
12. Piscopo G. (2011). The impact of longevity risk on annuitization factors in the Italian state defined contribution pension scheme, *Insurance Markets and Companies: Analyses and Actuarial Computation*, 2 (1), pp. 31-39.
13. Renshaw, A.E. & Haberman, S. (2003). On the forecasting of mortality reduction factors, *Insurance: Mathematics & Economics*, 32, pp. 379-401
14. Wilkinson, R.G. (1997). Health inequalities: relative or absolute material standards, *British Medical Journal*, 314, pp. 591-595.