Evaristo Diz (Venezuela), Jeffrey Tim Query (USA)

Applying a Markov model to a plan of social health provisions

Abstract

This paper develops an actuarial statistical model that is applied to a plan of medical benefits to determine the meaningful foundation or constitution of a mathematical contingency. Knowing in advance the medical cost of per capita age, it is possible to measure different portfolios of medical policies and simultaneously quantify them alongside with the life insurance per age and or dead benefit specified previously. From a financial point of view, it is important to compare two or more profiles of cost projection for any set of interest rates, sensitizing the obligations to different bank rates. When it comes to cost, it is possible to run the model with various initial samples, which are exposed to detect the risk for future behaviors. In the event we assume that a fraction of those models open to a risk has been already retired, we can monitor the actuarial obligations and the retirement cash flow to this new contingency or eventuality.

Keywords: benefits and health in social prevention, Markov's chains, mathematical reserve, cash flow probability, mathematics of actuarial statistic.

Introduction

Markov models are often utilized in scenarios where a decision problem involves risk that is continuous over time. They are also useful when important events may happen more than once, and in situations where the timing of events is important. Using conventional decision trees to represent such clinical settings is complex and may force idealistic simplifying assumptions. With Markov models the overlying assumption is that a patient is constantly in one of a finite number of discrete states of health, referred to as Markov states.

One issue that invariably arises with an application of Markov such as this one regards the sensitivity of this type of model to specification errors. For example, what is the effect on results if the true probability of being in a specific state is one point higher or lower than estimated? Obviously changes in the estimations of the probability in a specific state also change the estimations in cost, although not necessarily proportional. There is always a risk that the probability that what we are estimating may not be the true state, but only in cases categorized as extremes. In a regular state the estimations look to be reasonable and stable in the short and medium term. There is also the possibility in the medium term of running the model again with different probabilities in order to correct any important deviations.

In general, changes in cost are directly proportional to the present value. If the cost of a given state is 10 percent higher or lower than estimated, for example, it will be relatively easy to determine the incremental cost by producing this change in the model ceteris paribus, for the remainder of the variables. It is also not a major problem when you add a new state in theory. For example if there are really 4 states instead of 3, you can model the different results in comparison with three states and then evaluate the differences in events by contingencies and also by cost-related changes. Because of these responses to changes in a given state, we contend that this type of modeling is practical for the social health industry.

Evaluating health care issues and delivery processes with Markov methods has been an area of interest for a number of scientific studies. While the examples below are not an exhaustive list, as doing a survey of papers related to this area of research is beyond the scope of this paper, they do illustrate the range of possibilities with the Markov model in health care.

Chen et al. (1975) incorporate empirically determined transition probabilities into a simple stochastic model to compute a quality-adjusted life expectancy to estimate a comprehensive social indicator for health. Bush et al. (1971) uses a stationary Markov chain to demonstrate how it can characterize a stochastic process of disease development. They use a module construction method that incorporates the benefits of the efficiencies of Markovian development without the downsides of pooling Markov states, which may not comprise a new Markov process. A Markov chain model for the examination of a centralized medical record system in a large general hospital is the focus of a study by Liu et al. (1991). Their model is used to describe the dynamic behavior of records aging in conjunction with patients' hospital visits which also provides a framework for the long-range planning of medical record storage facilities.

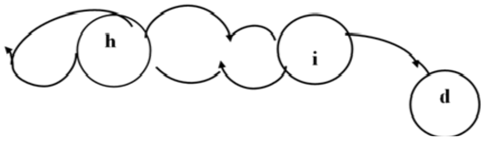
1. Model description

The model has pedagogical applications and implementation is relatively simple using Java Script. It is implemented with the purpose of determining the *cost of a contingent* as well as any type of illness, along with an evolution group of the same age from an initial $t = 0$ to a time age or final age $t = 20$, $t = 50$, $t = 100$, future periods. Obviously, these periods can be days, months, years, etc.

It is a simple model that includes multiple possible states {ill, healthy, and dead}. The idea is to determine which group size of any group should be used to fund and guarantee the initial payments of the debt caused by a particular individual with a conditional illness. It begins with the assumption that the deceased (or more accurately, the beneficiaries) would be paid with life insurance and it only models the volatility associated with the number of events that are sick, starting with these states: *health → ill, healthy → healthy, ill →ill, and ill →healthy*.

It is clear and obvious that for the remaining states *ill → dead*, *healthy → dead*, result into an absorbent condition, such that there is no return to the previous condition.

The main goal is to continue evaluating the development of the number of healthy, ill, and dead people in the course of 20, 50 or 100 periods. In particular, from an initial period at year $t = 0$; simultaneously determining the perspective previously described of the annual cost and the corresponding random cash flow, given a fixed interest rate. The points indicated previously would allow paying in an opportune way the debts of different ill individuals in the manner matching the way they occur. The model is summarized as follows.



Potential period to mode: 20 years.
Space state: {h,i,d}.
h: Healthy
i: Ill
d: Dead
Initial size of the group: 100 individuals.
Initial state vector: $\vec{V}_0$: (85, 15, 0).

Healthy:	85
Ill:	15
Dead:	0
Totals:	100

15 Ill
85 Healthy

Transition probabilities:

80% ($h \to h$)
20% ($h \to i$)
$P = f_{(x)}$ ($i \to d$)

Description:
Time fixed probabilities for the two first transitions
Variable probabilities as a function of age for transition $i \to d$

Being x: Ages / time future modeling for:
$x \in I; I = \{0,1,..., 20\}, I = \{0,1,..., 50\}, I = \{0,1,…, 100\}$

Fig. 1. Markovian graph

The model assumes an estimated dead probability for a given group $Pd = (1.0035^x) - 1$, which follows a basic model of potential mortality, derived from a equivalent rate computed like so:

$(1 + Pd) = (1 + 0.35\%)^x \ \forall x \in I$

In the most simple case, it assumes constant $r = 0.35\%$ as the rate of mortality for a period. Other models eventually would entail a more complex r, which could be a time function and eventually random under certain states.

Hypothesis and assumptions are the following:

- It assumes a fixed and constant interest rate for a year/time model equal to 10 percent.
- 80 percent of healthy people are more likely to be in the same condition, while the rest of them flow to the ill condition.
- Meanwhile, ill people go to a dead condition by an age probability generator or return back to a healthy condition with a given supplementary probability.
- Cost/contribution associated to the healthy people: $116.00.
- Cost/contribution associated to the ills: $931.00.

2. Java Script code

Java Script code is very versatile and relatively simple to learn. It easily allows for the creation of Markov trees. An elemental design following a Markov format can be converted to a relatively complex situation, with several states or scales. While our example has 3 states, it could easily be extended to 10-20 different states and 1000 future outcomes. Its true value is the simplicity of its functions and efficiency. The Java Script code used in this paper is presented below:

- stochastic present value: = npv(10%, random cash flow);
- random cash flow: = (clear, make list ((reset, mathematical reserve), *n*, 0, 100));
- mathematical reserve: = (markov, healthy*116 + ill*931);
- markov: = mkv(85, healthy, 15, ill, 0, dead);
- healthy: = mkv(80%, healthy, 20%, ill);
- ill: = mkv(1-dead probability, healthy, dead probability, dead);
- healthy: = mkv(80%, healthy, 20%, ill);
- ill: = mkv(1-dead probability, healthy, dead probability, dead);
- healthy: = mkv(80%,healthy,20%,ill)
- ill: = mkv(1-dead probability, healthy, dead probability, dead);
- dead: = 0%;
- healthy: = mkv(80%, healthy, 20%, ill);
- ill: = mkv(1-dead probability, healthy, dead probability, dead).

3. Tree of Markov modeling

The prior code allows us to automatically generate the Markov tree, which describes every subjacent logic, based on the structure (graph) of Markov and all hypotheses and settings used. It is important to emphasize that with small modifications, models' results can be configured in a more complex scenario. Likewise, we can increase the number of possible states, from three to a greater number in various events where the time transitions are crucial. Figure 2 below presents the Markov tree.

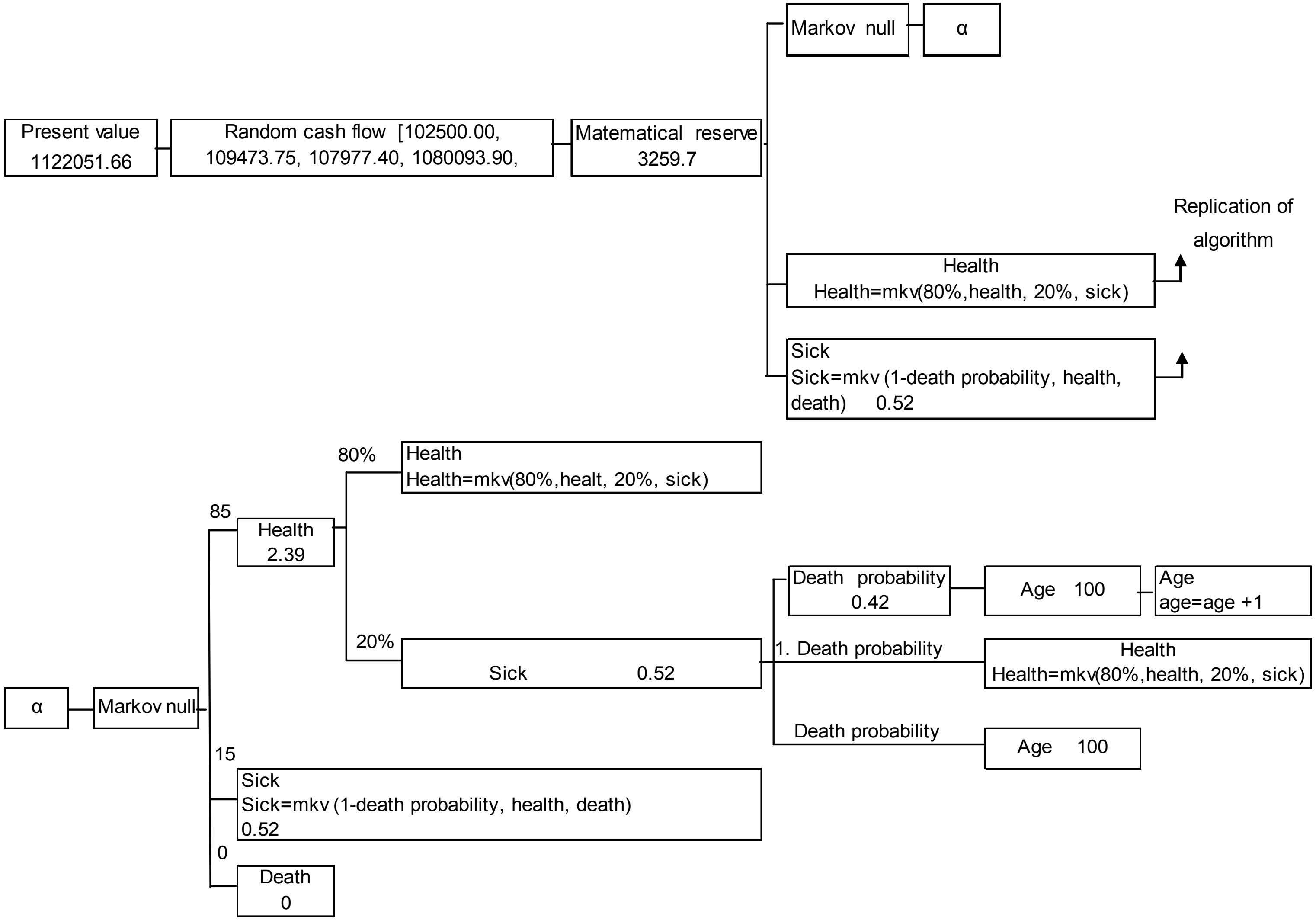


Fig. 2. Markovian modeling tree

4. Results

The model produces four types of output results which are presented below:

1. The states evolution table or succession of the vectors states space $\bar{V}_i$ for all i = 0-20, 0-50, 0-100.
2. The graph of the evolution of the components or the states within the time horizon.
3. The annual cost associated to each vector of the possible space states.
4. The random cash flow actualized and converted to a present value by a discounted rate in this constant state, or it could also be a variable in a time function and even random eventually.

For demonstrative purposes, the results of the plan cost starting from several basic options that have been described before are shown here. These options allow modeling on Markov's using the random flows and the obligation as a net present value on each stage or an advance time of the simulation sample.

Table 1. Results of prediction by period (year)

X	Annual proj. Cost (USD)	Healthy	Ill	Dead	Dead Prob. *X*	Cash flow Present val.	Accumulated Present val.
0	23,837.21	85.00	15.00	0.00		23,837.21	23,837.21
1	25,458.84	82.95	17.00	0.05	0.01	23,144.40	46,981.61
2	25,111.02	83.24	16.59	0.17	0.01	20,752.91	67,734.52
3	25,138.12	83.01	16.65	0.35	0.01	18,886.64	86,621.16
4	25,072.93	82.82	16.00	0.58	0.01	17,125.15	103,746.31
5	25,008.36	82.56	16.56	0.87	0.02	15,528.22	119,274.53
6	24,926.04	82.26	16.51	1.22	0.02	14,070.10	133,344.63
7	24,829.69	81.91	16.45	1.63	0.02	12,741.55	146,086.19
8	24,718.67	81.52	16.38	2.10	0.03	11,531.44	157,617.63
9	24,593.25	81.07	16.30	2.62	0.03	10,429.94	168,047.57
10	24,453.57	80.58	16.21	3.20	0.04	9,427.91	177,475.48
11	24,299.78	80.05	16.12	3.84	0.04	8,516.92	185,992.40
12	24,132.10	79.46	16.01	4.53	0.04	7,689.23	193,681.63
13	23,950.75	78.84	15.89	5.27	0.05	6,937.68	200,619.31
14	23,755.97	78.16	15.77	6.07	0.05	6,255.69	206,875.00
15	23,548.06	77.45	15.63	6.92	0.05	5,637.22	212,512.22
16	23,327.29	76.69	15.49	7.82	0.06	5,076.70	217,588.92
17	23,094.00	75.90	15.34	8.76	0.06	4,569.03	222,157.94
18	22,848.53	75.06	15.18	9.76	0.06	4,109.51	226,267.45
19	22,591.24	74.19	15.01	10.80	0.07	3,693.85	229,961.30
20	22,322.52	73.28	14.84	11.89	0.07	3,318.10	233,279.40
21	22,042.77	72.33	14.66	13.02	0.08	2,978.65	236,258.06
22	21,752.42	71.35	14.47	14.19	0.08	2,672.20	238,930.25
23	21,451.90	70.33	14.27	15.40	0.08	2,395.71	241,325.96
24	21,141.68	69.29	14.07	16.65	0.09	2,146.42	243,472.38
25	20,822.22	68.21	13.21	17.93	0.09	1,921.81	245,394.19
26	20,494.02	67.11	13.64	19.25	0.10	1,719.56	247,113.75
27	20,157.56	65.98	13.42	20.60	0.10	1,537.57	248,651.32
28	19,813.36	64.83	13.20	21.98	0.10	1,373.92	250,025.25
29	19,461.94	63.65	12.97	23.38	0.11	1,226.87	251,252.12
30	19,103.83	62.45	12.73	24.82	0.11	1,094.81	252,346.93
31	18,739.57	61.24	12.49	26.27	0.11	976.31	253,323.24
32	18,369.68	60.00	12.25	27.75	0.12	870.03	254,193.27
33	17,994.73	58.75	12.00	29.25	0.12	774.79	254,968.06
34	17,615.25	57.49	11.75	30.76	0.13	689.51	255,657.57
35	17,231.81	56.21	11.50	32.29	0.13	613.18	256,270.75
36	16,844.94	54.93	11.24	33.83	0.13	544.92	256,815.67
37	16,455.20	53.63	10.99	35.38	0.14	483.92	257,299.59
38	16,063.13	52.33	10.73	36.94	0.14	429.45	257,729.03
39	15,669.28	51.03	10.47	38.51	0.15	380.83	258,109.87
40	15,274.19	49.72	10.21	40.08	0.15	337.48	258,447.35
41	14,878.37	48.41	9.94	41.65	0.15	298.85	258,746.20
42	14,482.37	47.10	9.68	43.22	0.16	264.45	259,010.65
43	14,086.68	45.79	9.42	44.79	0.16	233.84	259,244.49

Table 1 (cont.). Results of prediction by period (year)

	Annual proj.				Dead	Cash flow	Accumulated
X	Cost (USD)	Healthy	Ill	Dead	Prob. *X*	Present val.	Present val.
44	13,691.82	44.49	9.16	46.36	0.17	206.63	259,451.12
45	13,298.27	43.19	8.90	47.92	0.17	182.44	259,633.56
46	12,906.51	41.90	8.64	49.47	0.17	160.97	259,794.53
47	12,517.01	40.61	8.38	51.01	0.18	141.92	259,936.45
48	12,130.22	39.34	8.12	52.54	0.18	125.03	260,061.48
49	11,746.57	38.08	7.87	54.05	0.19	110.07	260,171.55
50	11,366.49	36.83	7.62	55.56	0.19	96.83	260,268.38

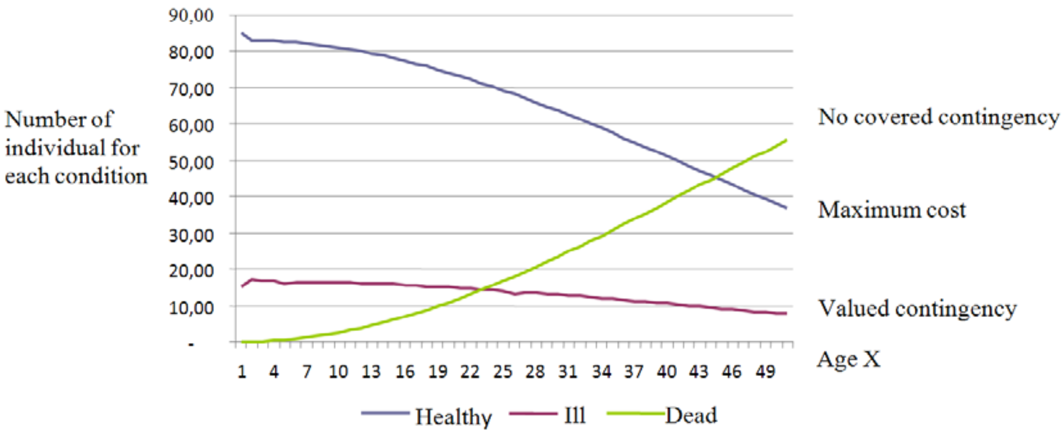


Fig. 2. State of evolutionary changes

The annual cost is a function of the number of employees or individuals that have been left with each future condition of age or simulation phase based on the scenarios or hypothesis utilized. A healthy person pays an equal amount of $116.00 and an ill person pays $931. With the passage of time, we see how the costs decrease as more people pass from a sick condition to a dead condition. In this particular plan, we do not recognize a benefit for the state of death. If it were recognized, it would be easier to include it in the sample and the cost would increase as the population increases. For example, if we assume that by the year 14, only an amount of $1,110.00 would be paid to the deceased people; the annual cost using this concept would be:

($1,110.00) (6.07) = $6,737.70 and its present value: PV= $ (6,737.70)/ $(1.10)^{14}$ PV=$1,774.25.

Therefore, this represents the amount which should be reserved today to pay future contingencies, on which six people have been estimated to produce the benefit.

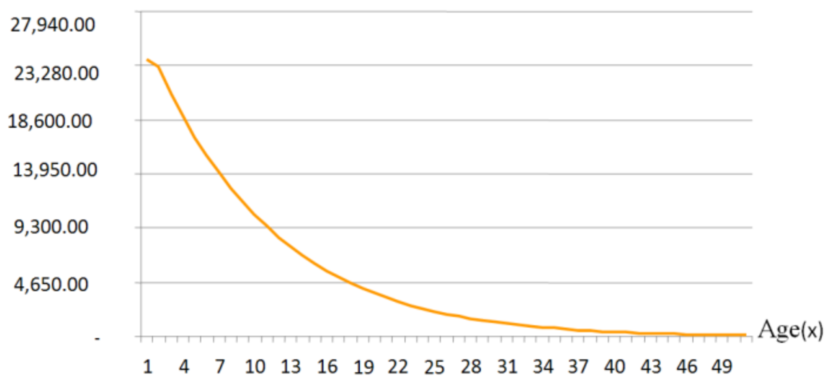


Fig. 3. Cash flow to present value

These values are the adjusted annual cost using an interest rate which discounts the annual cost from a time future year $x = 0$, assuming a fixed interest rate to discount the costs.

To account for expenses in year 5, the fund today must include a provision of a mathematical reserve which is in the order of $15,145.00 with the purpose of ensuring the payments of benefits for people that are in a condition at that moment.

The present value of the level fund to cover each and every one of the future values until the year 5, including the associated expense to year $x = 0$ which is in the order of $119,407.00 as it is seen in Figure 4.

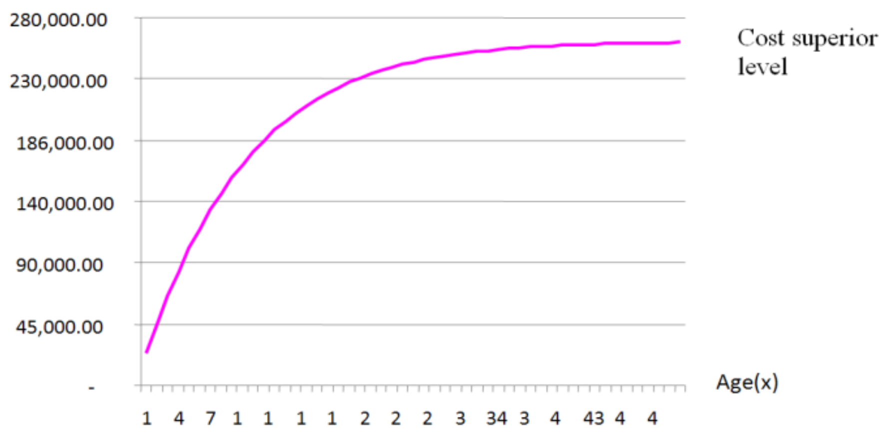


Fig. 4. Accumulated present value

The accumulated present value represents the total cost of each one of the present values associated with the annual cost of each different age. The present value of the fund is almost $260,568.00 to assist the whole population covered over a 100 year period, conditional upon to all population movements according of the states of the Markov model previously defined. About $279.000.00 could appear as a portion with enough cash flow coverage of the commitment, and the probability that it exceeds this limit is practically negligible.

The importance of these models in modern risk management, both actuarial and financial, is crucial. With $279,000.00 as a superior cost level, all obligations would be clearly covered with no problem. For proper budgeting, it is fundamental to know in advance the initial amount needed in the fund to accurately insure future liabilities and avoid insolvency. From a financial planning perspective, the ability to manage these reserves with sufficient time to address the issue makes a significant difference in this scenario. Receiving continuous service in a medical condition of high quality without interruptions due to lack of knowledge of the subjacent costs of its implementation is preferred to the alternative. Continued development and a deeper understanding of these types of models will help to ensure that a solvent fund is available for such future liabilities and responsibilities.

Conclusion and recommendations

There were made several conclusions:

1. Markov model allows to find various profiles for contingent costs of different long-term scenarios.
2. Knowing in advance the medical cost of per capita age, it is possible to measure different portfolios of medical policies and simultaneously quantify them alongside with the life insurance per age and or dead benefit specified previously.
3. Since the model is developed in Java Script is relatively easier to make changes in the initial parameters of the Markov process; generating different results with different thresholds.
4. In addition to having just three phases and following the same model philosophy, it is possible to extend the number of eventualities to a greater number, even categorizing those that are essential in different categories. For example, into the ill category, it is possible to define 3 sub-phases: (1) minor; (2) moderate; (3) critical or severe. Also, we can include the temporary and permanent disability as additional contingencies.
5. From a financial point of view, it is important to compare two or more profiles of cost projection for any set of interest rates, sensitizing the obligations to different bank rates.
6. When it comes to cost, it is possible to run the model with various initial samples, which are exposed to detect the risk for future behaviors. In other words, different groups can be modeled in separated ways to know the impact of each one individually and a consolidated manner.
7. In case we assume that a fraction of those scenarios open to a risk has been already retired, we can monitor the actuarial obligations and the retirement cash flow to this new contingency or eventuality.

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Appendix

Mathematical formulation of Markov model:

1. Space states: $\Omega = \{w_i | w_1 = h, w_2 = i, w_3 = d\}$.

2. Count of variables: $X_{w_i}(t)$: No of events that are in the state w_i on the time t.

3. States of vector: $\vec{V}_t$: $(x_{1,}x_2, x_3)$

with $x_1 = x_{w_1}{}^{(1)}$,

$X_2 = X_{w_2}{}^{(2)}$,

$X_3 = X_{w_3}{}^{(3)}$.

4. Propriety of a probabilistic vector: $\sum x_{w_j}(t) = 1, \forall j$ in each t.

5. Perspective of the prediction time: $t \in Z$ however, values can be found to $t + \Delta t \in R^+$ adjusting to each $x_{w_i}(t)$ of the vector state.

6. Closed population: The group is the same constant all the time and there is no enter and exit.

$\sum x_{w_j}(t) = \sum x_{w_i}(t + \Delta t)$ $\forall t$ and $\forall j$.

7. Description of the states: (1) healthy: individual of the group whose medical associate cost is major to \$116.00/period (*to keep healthy*); (2) ill: Individual of the group whose medical associate cost is major to \$116.00/period and in our example \$931/period (to solve the healthy *problem*); (3) dead: reached this state, it is not payment for this condition.

8. Plan of benefits: Coverage of expenses for healthy and ill people for \$116.00 and \$931.00/period, respectively. Obviously, it can be other structures of different benefits and greater complexity.

9. Probabilities of transition: Markov conditional probabilities[1]

$$P_{(w_2|w_1)} = P_1 = K_1,$$

$$P_{(w_1|w_1)} = P_2 = K_2,$$

$$P_{(w_2|w_2)} = P_3 = K_3,$$

$$P_{(w_3|w_2)} = f_{(t)} \neq K_4,$$

$$P_{(w_2|w_3)} = 0,$$

$$P_{(w_1|w_3)} = 0,$$

[1] These are obtained from historical estimates of similar groups. Those probabilities can be constant over time as in case of P_1 and P_2, or variables as in case $f_{(t)}$.

$P_{(w_1|w_2)} = 1 - f_{(t)}$,

Being $f_{(t)} = 1.0...^{t} - 1$.

10. Cost equation: It is an expected value of $X_{w_i}(t)$ con $(C_{w_i}(t) = K)$.

Annual cost: AC_t

$$AC_t = \sum X_{w_i}(t) \bullet C_{w_i}(t) \quad \forall w_i,\ \forall t,$$

where $C_{w_i}(t) = \begin{cases} K_1 = \$116.00 \quad \forall t \\ K_2 = 4K_1 \end{cases}$.

Given that $C_{w_i}(t)$ for i is constant along all the predicted perspective in a particular $K_3 = 0$ in the case of $C_{w_3}(t) \ \forall t$.

11. Present value: It is the actual value of the expected payments or annual costs t adjusted by an interest rate. For each $VP_{(0)} = \sum AC_t / (1+i)^t \forall t$.

Note that the exchange rate at the time of this study was: $1 USD equal to 4.30 Bs.