Maria Erminia Marina (Italy), Marina Ravera (Italy)

A note on the satisfaction levels of two agents subscribing an insurance policy

Abstract

Classical actuarial theory focuses on insurance problems and in particular on the determination of a premium for the insured risk. However, once a premium has been chosen, at the end of the insurance period it may happen that the policy has been disadvantageous either for the insurer or for the customer. In fact, the premium was not set high enough to cover the total claim amount or, vice versa, it was too high from the customer point of view. Our aim is to introduce, for each agent, a measurement in order to value how he is restrained in his possibilities. More precisely, the authors define two "satisfaction levels" that compare the increment in the expected utility that each agent has subscribing the insurance policy, with the increment in the expected utility that he could have if, unrealistically speaking, the insurer (customer) could withdraw from the contract in the case where the total claim amount is larger (smaller) than the premium, so that he never could have losses. Under assumptions, the authors show that the satisfaction levels are linked to the risk aversion of the agents, proving that inequalities comparing risk aversion of two insurers (customers) are related to inequalities between their satisfaction levels. Finally, the determination of a "fair" premium for an insurance contract is considered.

Keywords: risk aversion, expected utility, premium calculation principles, bargaining theory.

Introduction

We consider two agents: a customer, the owner of a risk X (a random variable), and an insurance company (insurer). The latter can accept to cover the risk against the payment of a premium from the customer. What should be an appropriate premium for the contract? Clearly, the premium cannot be too high because of competition between insurers and, at the same time, it cannot be too low because this would result in large losses for the insurer. However the premium should be also low enough so that the customer is willing to insure the risk. Several premium calculation principles have been proposed in actuarial sciences. All the proposed principles produce prices that are higher than the expected losses (in this context actuaries ignore loadings for expenses and profit). Calculation principles often used in practice are the expected value principle: $P=(1+\beta)E[X]$, the variance principle: $P=E[X]+\beta Var[X]$ and the standard deviation principle: $P=E[X]+\beta\sqrt{Var[X]}$. Alternative rules are the modified variance principle: $P=E[X]+\beta Var[X]/E[X]$ and the mixed principle: $P=E[X]+\alpha Var[X]+\beta\sqrt{Var[X]}$. Otherwise actuaries focused their attention on utility functions and their application to the risk evaluation (Daykin et al., 1994; Goovaerts at al., 1984; Rolski et al., 1999; Straub, 1988). In this context we consider a utility function u and an asset w. The compensating risk premium is defined as the unique solution of the equation $E[u(w+P-X)]=u(w)$, whereas the equivalent risk premium is defined as the unique solution of the equation $u(w-P)=E[u(w-X)]$. Related to the determination of the premium is the concept of risk aversion of an agent. Arrow (1965) and Pratt (1964) introduced the coefficient of risk aversion $r=-u''/u'$ and they showed that given a risk X and given two agents i_1 and i_2, both with initial asset w and with utility functions u_1 and u_2, respectively, if $r_1(x) > r_2(x)$, for each $x \in \Re$, then the compensating risk premium (equivalent risk premium) of the agent i_1 is larger than the compensating risk premium (equivalent risk premium) of the agent i_2.

In the following we start with the consideration that an insurance policy can be disadvantageous either for the insurer or for the customer. Therefore, we introduce two measurements called "satisfaction levels" that quantify the agents satisfaction level with respect to the chosen premium, basing on the comparison between increments in the expected utility. Under our assumptions we show that the satisfaction levels are linked to the risk aversion of the agents; that is, given two insurers (customers), inequalities concerning their different risk aversions (in the Arrow-Pratt sense) are related to inequalities between their different satisfaction levels.

Finally, we observe that classical actuarial theory mainly considers whether the premium is high enough to cover the risk, ignoring competition arising from the presence of the insurer and the customer. Considering the situation concerning the two economic agents (the insurer and the customer) that are willing to sign an insurance contract is a classical example of bargaining problem which we can tackle by means of a game theoretical approach. In fact, game theory analyses situations where agents have partially cooperative and partially conflicting goals. Otherwise, agents achieve greater benefits cooperating rather than not cooperating, and in the cooperation case they have to share benefits. In particular, Nash (1951) argued that cooperative actions are the results of some process of bargaining among the cooperating agents. We recall (Myerson, 1991) that "Nash's formulation of a two bargaining problem is based on the assumption that when two agents negotiate … the payoff allocations that the two agents ultimately get should depend only on the payoffs they would expect if the negotiation was to fail to reach a settlement and on the set of the payoff allocations that are jointly feasible for the two agents in the process of negotiation".

In the following, we want to show that the determination of a "fair" premium for the insurance contract can be obtained defining a suitable bargaining problem.

1. General framework

Let us start by giving a formal description of our problem. We consider an insurer *I* who would like to buy a risk *X*, receiving a premium *P*. The first question we address is what is a reasonable premium *P* from the insurer's point of view. The insurer is assumed to be risk averse expected utility maximizer; this means that he prefers one risk to another if the expected utility of the former exceeds the expected utility of the latter. We can describe the risk *X* by its cumulative distribution function $F_X : \Re \to [0,1]$, with $F_X(x) = Prob[X < x]$, left continuous and increasing (or strictly increasing), being $F_X(x) = 0$, for each $x \le 0$ and $F_X(x) > 0$, for each $x > 0$.

We denote by *u* the insurer's utility function on amounts of money. We assume that *u* is strictly increasing and continuous; we suppose moreover that *u* is a concave function (i.e. the insurer is a risk averse agent). We denote by *w* the insurer's asset. Let $P_I(u,w,X)$ (compensating risk premium) be the unique solution of the following equation:

$$E[u(w+P-X)]=u(w). \tag{1}$$

The Insurer is restrained in his possibilities since he does not know the losses over time. If he could know in advance the experienced losses he would buy the risk only in case the premium *P* was higher than the total claim amount. In this context we define, for each $P > 0$:

$$V_I(u,P,w,X) = \frac{E[u(w+P-X)] - u(w)}{\int_0^P u(w+P-x)dF_X(x) + \int_P^{+\infty} u(w)dF_X(x) - u(w)}. \tag{2}$$

$V_I(u,P,w,X)$ compares the increment in the expected utility $E[u(w+P-X)]-u(w)$ that the insurer with asset *w* obtains receiving the premium *P* and the risk *X*, with the increment $E[u(w+(P-X)^+)]-u(w)$. The latter is the increment in the expected utility that the Insurer would obtain if he could sign an advantageous contract, i.e. if the contract cover the total claim amount only if it is smaller than *P*, refunding *P* otherwise. We, therefore, refer to V_I as the "satisfaction level function". Does this function help the insurer in the choice of a reasonable premium?

Let us start by giving some properties of the function V_I. The satisfaction level does not vary under linear affine transformations of the utility function; that is $V_I(u', P, w, X) = V_I(u', P, w, X)$ if $u' = au + b$, with $a,b \in \Re, a > 0$. Moreover, the following proposition holds.

Proposition 1. For each utility function *u*, for each asset *w*, for each risk *X* having cumulative distribution function F_X, the function $V_I(u,\cdot,w,X):(0,+\infty) \to \Re$ is increasing.

Proof. Given a utility function *u*, for arbitrary *w* and *z*, define $v(w,z) = u(w+z) - u(w)$; for each $P > 0$, for each risk *X* we have by (2),

$$V_I(u,P,w,X) = \frac{\int_0^{+\infty} v(w,P-x)dF_X(x)}{\int_0^P v(w,P-x)dF_X(x)} = 1 + \frac{\int_P^{+\infty} v(w,P-x)dF_X(x)}{\int_0^P v(w,P-x)dF_X(x)}.$$

Moreover, if $0 < P < Q$:

$$\int_P^{+\infty} v(w,P\text{-}x)dF_X(x) = \int_P^Q v(w,P\text{-}x)dF_X(x) + \int_Q^{+\infty} v(w,P\text{-}x)dF_X(x) \le$$

$$\le \int_Q^{+\infty} v(w,Q\text{-}x)dF_X(x) \le 0$$

and

$$0 \le \int_0^P v(w,P\text{-}x)dF_X(x) \le \int_0^Q v(w,Q\text{-}x)dF_X(x).$$

Then, we have:

$$\frac{\int_P^{+\infty} v(w,P\text{-}x)dF_X(x)}{\int_0^P v(w,P-x)dF_X(x)} \le \frac{\int_Q^{+\infty} v(w,Q-x)dF_X(x)}{\int_0^P v(w,P-x)dF_X(x)} \le \frac{\int_Q^{+\infty} v(w,Q-x)dF_X(x)}{\int_0^Q v(w,Q-X)dF_X(x)}$$

and, consequently, the thesis holds.

Remark 1. For each utility function *u*, for each asset *w*, for each risk *X* it results that $0 \le V_I(u,P,w,X) < 1$, if $P_I(u,w,X) \le P < \text{ess sup} X$; moreover the function $V_I(u,\cdot,w,X):(0,\text{ess sup}X) \to \Re$

is strictly increasing. If $\operatorname{ess\,sup} X \in \Re$, then $V_I(u,P,w,X)=1$ for each $P \geq \operatorname{ess\,sup} X$ (in real life the insured risk X is a random variable of bounded range and $\operatorname{ess\,sup} X$ is the maximal possible claim).

1.1. Characterization. In this section, we study how the function V_1 can be used to characterize the risk aversion. More precisely, consider two insurers having utility functions u_1 and u_2.

Let $r_i = -u_i''/u_i'$, i=1, 2. It results that $r_1(x) \geq [>] r_2(x)$, for each $x \in \Re$ if and only if it is $u_1 = g \circ u_2$, being g a [strictly] concave function (Pratt, 1964; Mas-Colell et al., 1995, Ch. 6).

We observe that the above conditions are equivalent to the following:

$$\frac{u_1(y)-u_1(x)}{u_1(x)-u_1(v)} \leq [<] \frac{u_2(y)-u_2(x)}{u_2(x)-u_2(v)}, \tag{3}$$

$\forall v,x,y$ with $v<x<y$.

We have:

Theorem 1. Given two utility functions u_1 and u_2 the following conditions are equivalent, in either the strong form (indicated in brackets) or the weak form (with the bracketed material omitted):

(a) u_1 is a [strictly] concave transformation of u_2;

(b) $V_I(u_1,P,w,X) \leq [<] V_I(u_2,P,w,X)$, $\forall P,w,X$ with $P \in (0, \operatorname{ess\,sup} X)$.

Proof. Assume (a); again let $v_i(w,z) = u_i(w+z) - u_i(w)$, i =1,2, $\forall w,z \in \Re$.

We observe that for each w, for each X and for each $P \in (0, \operatorname{ess\,sup} X)$ we have $V_I(u_1,P,w,X) \leq$ $\leq [<] V_I(u_2,P,w,X)$ if and only if:

$$\frac{\int_P^{+\infty} v_1(w,P-y)\,dF_X(y)}{\int_0^P v_1(w,P-x)\,dF_X(x)} \leq [<] \frac{\int_P^{+\infty} v_2(w,P-y)\,dF_X(y)}{\int_0^P v_2(w,P-x)\,dF_X(x)}.$$

By means of the Fubini's theorem, the previous inequality can be written as:

$$\int_0^P \int_P^{+\infty} [-v_1(w,P-y)v_2(w,P-x)]\,dF_X(y)\,dF_X(x) \geq$$

$$\geq [>] \int_0^P \int_P^{+\infty} [-v_2(w,P-y)v_1(w,P-x)]\,dF_X(y)\,dF_X(x).$$

As we assume (3), then

$$\frac{v_1(w,P-x)}{-v_1(w,P-y)} \leq [<] \frac{v_2(w,P-x)}{-v_2(w,P-y)}, \forall y \in (P,+\infty);$$

consequently, (b) is proved.

Vice versa, suppose that (b) holds. Fixed a number $P>0$, let X be defined by the following cumulative distribution function $F_X : \Re \to [0,1]$:

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ 1/2 & 0 < x \leq y_0 \\ 1 & x > y_0 \end{cases},$$

where $y_0 \in (P,+\infty)$.

It results that, for every utility function u, it is:

$$V_I(u,P,w,X) = 1 + \frac{1/2\,(u(w+P-y_0)-u(w))}{1/2\,(u(w+P)-u(w))}.$$

From the condition (b) it follows that:

$$\frac{u_1(w+P)-u_1(w)}{u_1(w)-u_1(w+P-y_0)} \leq [<] \frac{u_2(w+P)-u_2(w)}{u_2(w)-u_2(w+P-y_0)}.$$

So (a) holds.

Remark 2. We recall that condition a) in Theorem 1 is equivalent to the following condition on the compensating risk premium (Pratt, 1964):

$$P_I(u_1,w,X) \geq [>] P_I(u_2,w,X), \forall w,X.$$

Moreover, we consider an agent having decreasing absolute risk aversion. Pratt (1964) proved that the function $r=-u''/u'$ is [strictly] decreasing if and only if, for every risk X, the function $P_I(u,\cdot,X)$ is [strictly] decreasing. We can easily prove that $P_I(u,\cdot,X)$ is a [strictly] decreasing function if and only if $V_I(u,P,\cdot,X)$ is a [strictly] increasing function, for each risk X and for each $P \in (0, \operatorname{ess\,sup} X)$.

We now consider a function u such that $r=-u''/u'=\alpha$, with $\alpha \in \Re$, $\alpha>0$; then $u(x) = -a\exp(-\alpha x)+b$, for each $x \in \Re$, with $a,b \in \Re$, $a>0$. For each asset w, for each risk X and for each $P>0$, we have, $V_I(u,P,w,X) = S(a,P,X)$ where

$$S(\alpha,P,X) = 1 + \frac{\int_P^{+\infty} (1-\exp(-\alpha(P-X)))\,dF_X(x)}{\int_0^P (1-\exp(-\alpha(P-X)))\,dF_X(x)}.$$

So the Insurer's satisfaction level does not depend on the asset w. Moreover, we have $S(a/\lambda, \lambda P, \lambda X) = S(a,P,X)$, for every $\lambda>0$; this property states, for example, that the agent with constant coefficient of absolute risk aversion α, buying the risk X and receiving the premium P, has the same satisfaction level of the agent with constant coefficient of absolute risk aversion 2α, buying the risk $X/2$ and receiving the premium $P/2$.

We provided theoretical characterizations of the function V_I so far. Moreover, the function V_I can give further information to the insurer. In fact, according to the utility theory, each premium P is acceptable if $P > P_I(u,w,X)$ and the Insurer would decide to set the contract at a premium P which give him at least a fixed satisfaction level.

We conclude this section with an example.

Example 1. Consider a situation in which we have six insurers with utility functions: $u_I(x) = \alpha_i^{-1}(1-\exp(-\alpha_i x))$, for each $x \in \Re$, i = 1,2,…6, being a_1 = 0.00006, a_2 = 0.00007, a_3 = 0.00008, a_4 = 0.00009, a_5 = 0.0001, and a_6 = 0.0002. For each asset w and for each risk X, it is $P_I(u_i,w,X) = \alpha_i^{-1}\ln(E[\exp(\alpha_i X)])$, i = 1,2,…6. As in Aumann and Serrano (2008), consider the risk X such that, $F_X(x) = 0$ if, $x \leq 0$ $F_X(x) = 0.999$, if $0 < x \leq 20000$ and $F_X(x) = 1$ if $x > 20000$, i.e. the risk consists in loosing $20,000 with 0.001% probability, like when buying loss damage waiver in a car rental. We have, if $P \leq 20000$.

$$V_I(u_I,P,w,X) = 1 + \frac{0.001\,(1-\exp(-\alpha_i(P-20000)))}{0.999\,(1-\exp(-\alpha_i(P)))},$$

$i = 1,2,...6$.

In Table 1 we calculate for each a_i, i = 1,2,…6, the compensating risk premium P_I (u_i, w, X) and the Esscher premium

$$P_I^E(u_i,w,X) = E[X\exp(\alpha_i X)]/E[\exp(\alpha_i X)]$$

(note that the Esscher premium is higher than the compensating risk premium (Buhlmann, 1980)). Table 2 shows, for different premiums, the induced values of V_I, at different values of the risk aversion. Note that if the Insurer 1 uses the Esscher premium, he obtains a satisfaction level higher than 40%. A premium of $100, as assumed in Aumann and Serrano (2008), gives him a satisfaction level higher than 60%.

Finally, we consider an insurer with utility function $u_{i,j}(x) = (1-\exp(-\alpha_i x))/2\alpha_i + (1-\exp(-\alpha_j x))/2\alpha_j$, i, j = 1,2,…6 and $i \neq j$; if $a_i < a_j$, then, $a_i < -u''_{ij}(x)/u'_{ij}(x) < a_j$ for each $x \in \Re$.

In particular (by previous Theorem and Table 2) if i = 1 and j = 2, a premium of $100 gives a satisfaction level higher than 56.5634 % and lower than 61.5088%.

Table 1. Compensating and Esscher's premiums for different values of the risk aversion

α	P_I	P_I^E
0.00006	38.6238	66.24863
0.00007	43.5792	80.85697
0.00008	49.3155	98.6706
0.00009	55.966	120.385
0.0001	63.6873	146.8429
0.0002	261.0556	1036.4132

Table 2. Induced values of V_I, for different premiums, at different values of the risk aversion

P	α					
	0.00006	0.00007	0.00008	0.00009	0.0001	0.0002
38.6238	0.113955					
43.5792	0.217269	0				
49.3155	0.310539	0.116613	0			
55.966	0.394390	0.221887	0.119185	0		
63.6873	0.417888	0.316531	0.226334	0.121665	0	
66.24863	0.523448	0.343053	0.256361	0.155759	0.038824	
80.85697	0.609873	0.462199	0.391251	0.308924	0.213236	
98.6706	0.615088	0.559748	0.501689	0.434324	0.356032	
100	0.680631	0.565634	0.508353	0.441891	0.364648	
120.385	0.738564	0.639614	0.592108	0.536993	0.472942	
146.8429	0.808623	0.705003	0.666137	0.621050	0.568661	
200	0.853887	0.784079	0.755662	0.722704	0.684415	
261.0556	0.964803	0.835169	0.813502	0.788379	0.7592	0
1036.4132	1	0.960357	0.955228	0.949299	0.942436	0.768067
20,000		1	1	1	1	1

2. A fair premium

In the following, we want to take into account the satisfaction level of the customer (the owner of the risk X) assuming that he has preferences according to the expected utility principle. Given a utility function u and an asset w, the equivalent risk premium $P_C(u, w, X)$ is defined as the unique solution of the following equation

$$u(w-P)=E\left[u(w\text{-}X)\right]. \tag{4}$$

As done previously for the insurer, we introduce the "customer's satisfaction level function", V_C. Let for each P, $0<P<\text{esssup}X$:

$$V_C(u,P,w,X)=\frac{u(w-P)-E[u(w\text{-}X)]}{\int_0^P u(w-x)dF_X(x)+\int_P^{+\infty} u(w-P)dF_X(x)-E[u(w-X)]}.$$

$V_C(u, P, w, X)$ compares the increment in the utility $u(w-P)-E[u(w-X)]$ that the customer with asset w obtains paying the premium P to insure against the risk X, with the increment $E\left[u\left(w-P+(P-X)^+\right)\right]-E[u(w-X)]$. The latter is the increment in the expected utility that the customer, paying the premium P, could obtain if the contract covered the total claim amount only if it was larger than P, refunding the difference between the premium and the losses otherwise.

Following ideas discussed in section 1, we can prove that the function $V_C(u,\cdot,w,X):[0,\text{esssup}X)\to\Re$ is strictly decreasing.

Furthermore it results that $P_c(u,\cdot,X)$ is a [strictly] decreasing function if and only if $V_C(u, P,\cdot,X)$ is a [strictly] decreasing function, for each risk X and for each $P\in[0,\text{esssup}X)$.

Moreover, given two utility functions u_1 and u_2, the following conditions are equivalent:

(a) u_1 is a [strictly] concave transformation of u_2;

(b) $V_C(u_1,P,w,X)\geq[>]V_C(u_2,P,w,X), \forall P,w,X$ with $P\in(0,\text{esssup}X)$.

In the following the question we address is what is a reasonable premium for the risk, as the premium should be acceptable with respect to the opposite interests of both the insurer and the customer.

We consider an insurer with utility function u_I and asset w_I, and a customer with utility function u_C and asset w_C. The customer is willing to insure the risk X. We suppose that both the agents have decreasing absolute risk aversion, $r_s, s = I,C$, with $r_C > r_I$, and $w_C < w_I$. It can be easily shown that $P_I(u_I,w_I,X)<P_C(u_C,w_C,X)<\text{esssup}X$.

Let $P_I = P_I(u_I, w_I, X)$, $P_C = P_C(u_C, w_C, X)$, $V_I(P) = V_I(u_I, P, w_I, X)$ and $V_C(P) = V_C(u_C, P, w_C, X)$ it results that, for each $P, P\in(P_I,P_C)$, the following inequalities are fulfilled:

$$u_C(w_C-P)>E[u_C(w_C\text{-}X)]$$

and

$$E[u_I(w_I+P\text{-}X)]>u_I(w_I).$$

What should be a "fair" price for the contract? We tackle the problem considering the satisfaction levels of the insurer and the customer, i.e. we want to determine a pair in the set γ, being $\gamma=\{(V_I(P),V_C(P)),P\in[P_I,P_C]\}$.

For example, we refer to the situation described in Example 1. We consider the risk X, consisting in loosing \$20,000 with 0.001% probability, an Insurer I and a customer C having utility functions $u_I(x) = (1-\exp(-0.00006x))/0.00006$ and $u_C(x) = (1-\exp(-0.0002x))/0.0002$, respectively. It results that $P_I = 38.6238$, $P_C = 261.0556$ and for each $P\in[P_I,P_C]$,

$$V_I(P)=1+\frac{0.001(1-\exp(-0.00006(P-20000)))}{0.999(1-\exp(-0.00006P))}$$

and

$$V_C(P)=1+\frac{0.999(1-\exp(-0.0002P))}{0.001(1-\exp(-0.0002(P-20000)))}.$$

Now, our insurance problem can be analyzed in the framework of the bargaining theory. In this context, the concept of fairness involves comparison with all possible gains of the agents from their all possible agreements, taking into account what the agents would get without an agreement.

In general (as it was introduced by Nash (1950), a two-person bargaining problem is a pair (F, v), where F is a closed convex subset of $\Re^2$, $v = (v_1, v_2)$ is the disagreement point and the set $\widetilde{F}=F\cap\{x\in\Re^2:x_1\geq v_1,x_2\geq v_2\}$ is nonempty and bounded. Many well known solutions have been proposed, with an axiomatic approach where desired properties of the solutions are satisfied; for a complete survey we refer to Thomson (1994). In particular, we make use of the Nash solution and the Kalai-Smorodinsky solution.

The Nash solution (Nash, 1950) is the unique point $x^N\in\widetilde{F}$ such that

$$x^N=\arg\max((x_1-v_1)(x_2-v_2)x\in\widetilde{F}). \tag{5}$$

The Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975) is the unique point $x^{KS} \in \widetilde{F}$ such that

$$\frac{x_2^{KS} - v_2}{x_1^{KS} - v_1} = \frac{m_2\left(\widetilde{F}, v\right) - v_2}{m_1\left(\widetilde{F}, v\right) - v_1} \tag{6}$$

being $m_i\left(\widetilde{F}, v\right) = \max_{y \in \widetilde{F}, y \geq v} y_i, i = 1,2.$

We can state a bargaining problem for our situation as follows:

Let $\widetilde{F} = \left\{ \begin{matrix} (x_1, x_2) \in \Re^2 : 0 \leq x_1 \leq V_I(P_C), \\ 0 \leq x_2 \leq V_C\left(V_I^{-1}(x_1)\right) \end{matrix} \right\}.$

The disagreement point is v = (0, 0); it represents the situation in which the two agents do not subscribe the contract. The set

$$\partial\widetilde{F} = \left\{ \begin{matrix} (x_1, x_2) \in \Re^2 : 0 \leq x_1 \leq V_I(P_C), \\ x_2 = V_C\left(V_I^{-1}(x_1)\right) \end{matrix} \right\} = \\ = \{(V_I(P), V_C(P)), P \in [P_I, P_C]\},$$

represents the set of feasible pairs of payoffs if the contract is subscribed at the premium $P \in [P_I, P_C]$. The set $\widetilde{F}$ can be justified assuming that the two agents can agree to jointly randomized strategies, i.e. planning to implement the contract at the premium P with probability θ, and not subscribe the contract otherwise.

According to (5) the Nash solution is $x^N \in \widetilde{F}$ such that $x^N = (V_I(P^N), V_C(P^N))$, where $x^N = (V_I(P^N), V_C(P^N))$ where $P^N = \arg\max \{V_I(P)V_C(P), P \in [P_I, P_C]\}$; according to (6) the Kalai-Smorodinsky solution is $x^{KS} = (V_I(P^{KS}), V_C(P^{KS}))$, where $V_C(P^{KS}) / V_C(P_I) = V_I(P^{KS}) / V_I(P_C)$. For our example, it results $V_I(P_C) = 0.8539$ and $V_C(P_I) = 0.8554$. Both the Nash and the Kalai-Smorodinsky solutions give the satisfaction levels higher than 61% corresponding to a premium $P \sim 100.93$.

Concluding remarks

Several premium calculation principles have been proposed in actuarial sciences. Clearly, the premium should not only be high enough to compensate the insurer for bearing the risk, it should be also low enough so that the customer accepts the policy. In this context, actuaries apply utility theory to the risk evaluation. Referring to their results, we argue that the premium has to be set higher than the compensating risk premium (defined by (1)) being the customer more risk averse than the insurer. In this note, we introduced two measurements called "satisfaction levels" that quantify the agent's satisfaction level with respect to the chosen price, basing on the comparison between increments in the expected utility. Keeping in mind the theorem in section 1 and the considerations made in section 2, we deduce that the agent a_1 with utility function u_1 is more risk averse than the agent a_2 with utility function u_2 (in the Arrow-Pratt sense, i.e. $r_1(x) \geq r_2(x)$, for each $x \in \Re$) iff $V_I(u_1, P, w, X) \leq V_I(u_2, P, w, X)$ or, equivalently, iff $V_C(u_1, P, w, X) \geq V_C(u_2, P, w, X)$ for each premium P, for each asset w, for each risk X, with $P \in (0, ess\sup X)$.

Moreover, in this note, we shown that the satisfaction levels are useful instruments for the choice of "fair" insurance premium.

Some authors argue that the Arrow-Pratt coefficients of risk aversion are generally too weak for making comparison between risky situations (Ross, 1981; Modica and Scarsini, 2005; Denuit and Eeckhoudt, 2010). In fact, Ross (1981), observed that the above result does not extend to the case in which the risk X is added to a random asset $\widetilde{w}$. Therefore, new stronger measures of risk aversion were proposed, for example, by Ross (1981) and, more recently, by Modica and Scarsini (2005) and Denuit and Eeckhoudt (2010). In particular, Ross (1981) defined a_1 strongly more risk averse than a_2 if there exists λ such that $\inf\ u''_1/u''_2 \geq \lambda \geq sup\ u'_1/u'_2$. This ordering is stronger than the Arrow-Pratt ordering: choose $u_1(x) = -\exp(-ax)$ and $u_2(x) = -\exp(-bx)$, with $x \in \Re$ and $a > b > 0$, it results that $r_1(x) > r_2(x)$, for each $x \in \Re$, but there exist x_1 and x_2 such that $u'_1{}'(x_1) / u'_1{}'(x_1) < u'_1(x_2) / u'_2(x_2)$. The new measure of risk aversion "provides necessary and sufficient conditions for more risk averse agents to have higher insurance premia".

Despite these considerations, in this note we have considered the Arrow-Pratt measure of absolute risk aversion, that continues to be used in the literature by several authors (we just recall the paper of Aumann and Serrano (2008), where the riskiness of a gamble is defined as the reciprocal of the absolute risk aversion of an individual with exponential utility function).

Finally, another remark considers the possibility that the insurer (as he has a portfolio of policies), could reduce the risk associated transferring all or part of his risk to a second (or more) insurance carrier(s). There are several applications of game theory to solve these insurance problems; in fact, it is possible to analyze the situation as an n-person game. We just recall Borch (1974), Lemaire (1991) and, more recently, Suijs et al. (1998) and Ambrosino et al.

(2006). A possible development of our paper is to perform an analysis of the satisfaction of the agents in this reinsurance problem.

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