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Premiums are not necessarily monotonic with interest and age

Abstract

In a previous paper [7] the author studied the impact of changes in the force of mortality and the force of interest on life annuities, and estimated the change in annuities, reserves, liabilities, and premiums under change in the force of interest and the force of mortality.

Dynamical life tables (DLT) uses force of mortality that varies with time. Life insurance plans and pension schemes are recently considering DLT and variable rate of interest.

Evaluation of annuities subject to DLT is quite complex. We suggested in [8] some approximations based on the estmates that we achieved in [7].

The study of the impact of changes in the rate of interest and the rate of mortality is classical and is carried out formely using different methods, e.g. [4], [5] and [6].

In [9] we proved the conjecture that premium decreases when interest increases for whole life assurance for some ages, whenever the life table is of a standard population.

In this paper we present a life table for which the conjecture is false, and we study the conjecture for the cases of term assurance and a pension scheme. We consider a second conjecture that premium decreases when age increases, for some fixed interest.

We present a life table that contradicts these two conjectures. The life table is typical for a population that is subject to some high risk within a given range of ages, a risk that fades away with time as the survivors of the risky period are cured, healthy, and regular.

Keywords: rates of interest, force of mortality, expectancy of life, annuities, the classical values $\overline{a}_x, \overline{A}_x, \overline{P}_x, {}_t\overline{V}_x$.

Introduction

It is general practice to assume that the two conjectures hold for whole life policies. The first, premium increases with increasing age, and the second, premium increases with interest decreasing. One may be tempted to argue that the conjectures hold, but these conjectures cannot be proved without reservations, as they may fail to hold.

In [9] we suggested that the second conjecture may fail to hold. It stems from our paper [7] where we studied changes in life annuities due to changes in the force of mortality and the force of interest, and where we proposed estimates for the change in annuities, liabilities of life assurance, premiums and reserves under change in the force of interest and the force of mortality.

Dynamical life tables (DLT) uses force of mortality that varies with time. Life insurance plans and pension schemes are recently considering DLT and variable rate of interest.

Evaluation of annuities subject to DLT is quite complex. We suggest approximations based on the estimates that we achieved in (7). We expanded the results in (8) to higher derivatives. This enabled us to achieve better estimates and to evaluate the size of the error of the estimated values.

In [9] we studied the first conjecture: premiums decreases when interest increases for whole life assurance and we proved that it is valid for a population that is subject to a life table that satisfies the weak decreasing assumption that is a life table for which the inequalities ${}_tp_x \geq {}_tp_{x+y}p_{x+y+t} = {}_{t+1}p_x$ hold for all x, y and t, where ${}_tp_x$ is the probability for a life aged x to survive for at least t years.

We refer to a population as a standard population if it is subject to a life table that satisfies the weak decreasing assumption; otherwise it is a non-standard population.

We intend to discuss the first conjecture: premiums decreases when interest increases for whole life assurance as well as for term assurance and pension schemes.

We observe a second conjecture: premiums increases with age for whole life assurance as well as for term assurance and pension schemes.

In this paper we suggest a life table that does not satisfy the weak decreasing assumption, that is the population that is subject to this life table is a non-standard population and we will investigate the conjectures on the premiums behavior under change of interest and age for the whole life assurance, for the term assurance and for the pension scheme.

This life table suggests that both conjectures may fail for non-standard population.

A non-standard population may arise when the population includes a range of age of high risk that fades away over some given period that is the survivors of the high risk period become “healthy”. Our table contains a young range of age under high risk, a risk that reduces over a decade to “normal” risk. The high risk fades away over a decade and the survivors are then subject to normal risk. For a life table in which the high risk occurs at a higher age range or occurs for several ranges of age the behavior of the premiums may be expected to behave “strangely” over large ranges of age.

In general life tables represent a standard population. A non-standard population occurs when the population includes a large group of high risk within a standard population.

Table 1 is a life table of a non-standard population that includes a group of a high rate of mortality in the range of ages of 35 to 45 and the rest fits to a standard population.

1. A non-standard life table

In [9] we studied the first conjecture: *premium increase when interest decrease*, and we established that the conjecture holds for a standard population that is a population that is subject to the inequality ${}_tp_x \geq {}_tp_{x+y}$ for all non-negative values of x, y and t.

We also observed in [9] that the inequality ${}_mp_x\, {}_{n-m}p_x \geq {}_np_x$ holds if the inequality ${}_tp_x \geq {}_tp_{x+y}$ holds for all non-negative values of x, y, and t.

Recall that a life table satisfies the weak decreasing assumption if ${}_tp_x \geq {}_tp_{x+y}p_{x+y+t} = {}_{t+1}p_{x+y}$ for all x, y, and t.

We proved in [9] that the first conjecture holds for a whole life assurance provided the underlying life table satisfies the weak decreasing assumption that seems to hold in most life tables

The natural question is: Do all life tables satisfy the weak decreasing assumption?

The life table in Table 1 (see Appendix) describes a population with high rates of mortality within the age range 35-45, and the survivors to age 45 are subject to “normal mortality”.

In Table 2 (see Appendix) we have the yearly premiums due for a whole life assurance for various ages.

In Table 3 (see Appendix) we have the yearly premiums due for a term assurance for various ages.

In Table 4 (see Appendix) we have the yearly premiums due for some pension scheme for various ages.

We proceed to verify the two conjectures for the various assurance cases.

2. A whole life assurance in a case of non-standard life table

Consider the life table in Table 1 that describes a population that is subject to high rates of mortality within the 35-45 age range, and for the survivors to age 45 are subject to “normal mortality”. We will see that this life table describes a non-standard population

Table 2 provides the yearly premiums due for a whole life assurance for various ages.

From Table 2 for the range of ages 35-45 it follows that for this non-standard population and for the case of whole life assurance the following hold.

Proposition 2.1: For a whole life assurance for non-standard population premiums may increase when interest increase, e.g. consider the range of ages 35-45 in Table 2.

Proposition 2.2: For a whole life assurance for a non-standard population premiums may decrease when age increase, e.g. consider the range of ages 35-45 in Table 2.

These results “contradict” both conjectures, and one can easily explain these phenomema by the high risk in the 35-45 range of age in the life table.

We proved in [9. Theorem 2] that the first conjecture is valid for a standard population, therefore it follows that:

Lemma 2.1: Table 1 describes a non-standard population.

3. A term assurance in a case of non-standard life table

Consider the life table in Table 1 that describes a population that is affected due to some cause that results in high rates of mortality within the 35-45 age range, so that those surviving the age of 45 overcame the cause and are subject to “normal mortality”.

In Table 3 we have the yearly premiums due for a term assurance for various ages.

From Table 3 for the range of ages 35-45 it follows that for this non-standard population and for the term assurance the following hold.

Proposition 3.1: For a term assurance for non-standard population premiums may increase when interest increase, e.g. consider the range of ages 35-45 in Table 3.

Proposition 3.2: For a term assurance for a non-standard population premiums may decrease when age increase, e.g. consider the range of ages 35-45 in Table 3.

These results "contradict" both fonjectures, and one can easily explain the behavior of premiums of the term assurance in the age range of 35-45 in case of a high risk in this age range as given in the life table.

4. A pension scheme in a case of non-standard life table

Consider the life table in Table 1 that describes a population that is affected due to some cause that results in high rates of mortality within the 35-45 age range, so that those surviving the age of 45 overcame the cause and are subject to "normal mortality".

In Table 4 (see Appendix) we have the yearly premiums due for some pension scheme for various ages.

From Table 4 for the range of ages 35-45 it follows that for this non-standard population and for the pension scheme the following hold.

Proposition 4.1: For a whole life assurance for non-standard population premiums increases when interest decreases, e.g. consider the range of ages 35-45 in Table 2.

Proposition 4.2: For a whole life assurance for a non-standard population premiums increases when age increases, e.g. consider the range of ages 35-45 in Table 2.

These results are "as expected" due to the fact that the liabilities are far beyond the age "abnormality" and that the high rates of mortality decreases the value of the liabilities.

These both conjectures are probably valid for the pension scheme as one can argue.

Conclusion

The results as obtained in sections 3-4 stem from the high rate of mortality in the 35-45 age range describing a highly non-standard population. Reducing the rates of mortality in the 35-45 age range affects the results in sections 3-4. This suggest the conjecture that there is no non-standard populations due to the young and young-adult mortality hump in a standard life tables, that is: the two conjectures on premiums monotonicity are valid for the population subject to a standard life table. The humps in the standard life tables are too small to create non-standard populations that are subject to the life table.

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Appendix

Table 1. Life table final age 115

Age *x*	*Lx*
35	9,940,000
36	5,964,000
37	3,757,320
38	2,517,404
39	1,762,183
40	1,286,394
41	964,795
42	752,540
43	639,659
44	575,693
45	520,750
46	519,375
47	517,825
48	516,100
49	514,200
50	512,125
51	509,875
52	507,450
53	504,850
54	502,075
55	499,125
56	496,000
57	492,700
58	489,225
59	485,575
60	481,750
61	477,750
62	473,575
63	469,225
64	464,700
65	460,000
66	455,125
67	450,075
68	444,850
69	439,450
70	433,875
71	428,125
72	422,200
73	416,100
74	409,825

Age *x*	*Lx*
75	403,375
76	396,750
77	389,950
78	382,975
79	375,825
80	368,500
81	361,000
82	353,325
83	345,475
84	337,450
85	329,250
86	320,875
87	312,325
88	303,600
89	294,700
90	285,625
91	276,375
92	266,950
93	257,350
94	247,575
95	237,625
96	227,500
97	217,200
98	206,725
99	196,075
100	185,250
101	174,250
102	163,075
103	151,725
104	140,200
105	128,500
106	116,625
107	104,575
108	92,350
109	79,950
110	67,375
111	54,625
112	41,700
113	28,600
114	15,325

Table 2. Whole life assurance (even premium due yearly for a whole life assurance for 1)

Age	Interest				
	1%	2%	3%	4%	5%
35	0.214	0.233	0.248	0.260	0.270
36	0.161	0.179	0.194	0.207	0.218
37	0.119	0.133	0.147	0.159	0.169
38	0.088	0.099	0.110	0.120	0.129
39	0.066	0.073	0.080	0.088	0.096
40	0.049	0.053	0.058	0.063	0.069
41	0.037	0.038	0.040	0.043	0.046
42	0.028	0.027	0.027	0.028	0.029
43	0.023	0.021	0.020	0.020	0.020
44	0.020	0.018	0.016	0.015	0.015
45	0.018	0.015	0.013	0.011	0.009
46	0.019	0.016	0.013	0.011	0.010
47	0.019	0.016	0.014	0.012	0.010
48	0.020	0.017	0.014	0.012	0.011
49	0.020	0.017	0.015	0.013	0.011
50	0.020	0.018	0.015	0.013	0.012
51	0.021	0.018	0.016	0.014	0.012
52	0.022	0.019	0.016	0.014	0.013
53	0.022	0.019	0.017	0.015	0.013
54	0.023	0.020	0.017	0.015	0.014
55	0.023	0.020	0.018	0.016	0.014
56	0.024	0.021	0.018	0.016	0.015
57	0.024	0.021	0.019	0.017	0.015
58	0.025	0.022	0.019	0.017	0.016
59	0.026	0.023	0.020	0.018	0.016
60	0.026	0.023	0.021	0.019	0.017
61	0.027	0.024	0.021	0.019	0.018
62	0.028	0.025	0.022	0.020	0.018
63	0.028	0.025	0.023	0.021	0.019
64	0.029	0.026	0.024	0.021	0.020
65	0.030	0.027	0.024	0.022	0.020
66	0.031	0.028	0.025	0.023	0.021
67	0.032	0.029	0.026	0.024	0.022
68	0.033	0.029	0.027	0.025	0.023
69	0.033	0.030	0.028	0.025	0.024
70	0.034	0.031	0.029	0.026	0.024

Table 3. Term assurance (even premium due yearly for a term assurance to the age of 70 for 1)

Age	Interest				
	1%	2%	3%	4%	5%
35	0.2531	0.2616	0.2689	0.2751	0.2804
36	0.2021	0.2105	0.2182	0.2251	0.2313
37	0.1580	0.1650	0.1718	0.1783	0.1844
38	0.1239	0.1291	0.1345	0.1399	0.1452
39	0.0972	0.1003	0.1038	0.1076	0.1116
40	0.0768	0.0779	0.0795	0.0816	0.0841
41	0.0606	0.0598	0.0596	0.0600	0.0607
42	0.0486	0.0464	0.0446	0.0434	0.0426
43	0.0424	0.0393	0.0366	0.0344	0.0327
44	0.0393	0.0356	0.0323	0.0296	0.0272
45	0.0364	0.0322	0.0284	0.0250	0.0221
46	0.0381	0.0338	0.0300	0.0266	0.0235
47	0.0399	0.0355	0.0317	0.0282	0.0251
48	0.0418	0.0374	0.0335	0.0300	0.0268
49	0.0439	0.0395	0.0355	0.0319	0.0287
50	0.0461	0.0417	0.0377	0.0341	0.0308
51	0.0486	0.0442	0.0401	0.0364	0.0330
52	0.0513	0.0468	0.0427	0.0390	0.0355
53	0.0543	0.0498	0.0456	0.0418	0.0383
54	0.0577	0.0531	0.0489	0.0450	0.0414
55	0.0615	0.0568	0.0526	0.0486	0.0449
56	0.0657	0.0610	0.0567	0.0526	0.0489
57	0.0706	0.0658	0.0614	0.0573	0.0534
58	0.0761	0.0713	0.0668	0.0626	0.0587
59	0.0826	0.0777	0.0732	0.0689	0.0648
60	0.0902	0.0853	0.0806	0.0762	0.0721
61	0.0993	0.0943	0.0896	0.0851	0.0808
62	0.1104	0.1054	0.1005	0.0959	0.0915
63	0.1243	0.1191	0.1142	0.1094	0.1049
64	0.1421	0.1368	0.1317	0.1268	0.1221
65	0.1658	0.1603	0.1550	0.1499	0.1451
66	0.1988	0.1932	0.1877	0.1824	0.1773
67	0.2484	0.2425	0.2367	0.2311	0.2256
68	0.3309	0.3245	0.3183	0.3122	0.3063
69	0.4958	0.4885	0.4815	0.4745	0.4678
70	0.9901	0.9804	0.9709	0.9615	0.9524

Table 4. Pension scheme (even premium due yearly for a pension of 1 p.a. due starting at age 70)

Age	Interest				
	1%	2%	3%	4%	5%
35	0.192	0.128	0.085	0.057	0.039
36	0.262	0.178	0.122	0.084	0.058
37	0.333	0.232	0.162	0.114	0.080
38	0.402	0.285	0.203	0.146	0.105
39	0.465	0.335	0.243	0.178	0.130
40	0.522	0.382	0.281	0.208	0.155
41	0.573	0.424	0.316	0.237	0.179
42	0.617	0.462	0.348	0.264	0.201
43	0.657	0.495	0.376	0.288	0.221
44	0.694	0.527	0.403	0.311	0.241
45	0.731	0.558	0.430	0.334	0.261
46	0.768	0.590	0.457	0.357	0.281
47	0.808	0.624	0.486	0.382	0.303
48	0.851	0.661	0.519	0.410	0.327
49	0.900	0.703	0.554	0.441	0.354
50	0.953	0.749	0.594	0.475	0.384
51	1.012	0.799	0.638	0.514	0.417
52	1.078	0.856	0.687	0.557	0.455
53	1.151	0.920	0.743	0.605	0.498
54	1.234	0.992	0.805	0.660	0.546
55	1.329	1.073	0.877	0.723	0.602
56	1.437	1.167	0.959	0.795	0.666
57	1.563	1.276	1.054	0.879	0.740
58	1.709	1.403	1.165	0.978	0.828
59	1.883	1.554	1.298	1.095	0.933
60	2.092	1.736	1.457	1.237	1.059
61	2.348	1.959	1.653	1.410	1.215
62	2.669	2.238	1.899	1.629	1.411
63	3.081	2.598	2.216	1.911	1.664
64	3.633	3.078	2.640	2.289	2.004
65	4.406	3.753	3.235	2.819	2.481
66	5.566	4.766	4.130	3.617	3.200
67	7.503	6.456	5.623	4.951	4.402
68	11.379	9.841	8.614	7.622	6.812
69	23.013	20.003	17.595	15.647	14.052