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A two-period model of an insurer with catastrophic loss and capacity constraint

Abstract

In this paper, a two-period cash flow model of an insurer is constructed to explore how catastrophic loss can influence insurance price and industrial organization under capacity constraint. The model focuses on studying the impact of loss payment on an insurer's optimal underwriting and capital rising strategies in the next period. The study confirms the previous finding of the ambiguous relationship between losses and next-period premiums. In the situation of tight capital supply and high insurance demand, a positive relationship between catastrophic loss and insurance price and a negative relationship between catastrophic loss and insurance coverage capacity can be observed. The paper contributes to show an insurer's solvency ratio plays an important role in the interaction between its ability to sell new business and to raise external capital. The paper can also help find out in which condition an insurer can make extra profit in the case of catastrophic loss payment, and this condition further implies that one catastrophic event could act as a trigger, splitting insurers into high-quality ones and low-quality ones with respect to their underwriting efficiencies and capital raising abilities.

Keywords: two-period model, property-liability insurer, capacity constraint, catastrophic loss.

Introduction

Both the capacity constraint theory (see Winter, 1988, 1991; Gron, 1994) and the related risk over hang theory (see Gron and Winton, 2001) suggest that sharp price increases and large capacity swings will follow capital shocks, such as those caused by a large natural disaster or a significant macroeconomic event. This is, in part, due to relatively high capital adjustment costs[1]. In the property-liability insurance market, the mismatch between an unexpected catastrophe loss and limited capital could cause capital shortfall, coverage reduction, and premium increases for the entire insurance industry[2]. At the firm level, after a catastrophe event, insurers turn out to have different post-catastrophe performances. For example, eleven property-liability insurers[3] became insolvent resulting from Hurricane Andrew occurring in 1992, and some of the state's largest homeowners insurers had to obtain resources from their parent companies and others had to use their surplus to pay large claims[4]. Considering the large effect of catastrophe events, it is important to understand how insurers and the insurance market respond. In this paper, I construct a two-period cash flow model with catastrophic loss for an insurer to explore whether and how catastrophic shocks can influence insurance price and industrial organization in the property-liability insurance market.

Many empirical literatures support the capacity constraint theory. Doherty and Garven (1995) find that unanticipated decreases in the insurance industry capacity can cause higher profitability and prices. Doherty, Lamm-Tennant, and Starks (2003) check the temporal and cross-sectional variation in insurance company stock prices after 9/11 and find insurers suffering the lowest losses with less leverage are able to exploit the post-loss hard market. This implies that some insurers could make profits from underwriting catastrophic risk if they can develop successful catastrophic risk intermediation strategies with respect to underwriting and leverage. In this paper, the model also focuses on analyzing an insurer's underwriting capability and capital rising strategy in post-loss market. Grace and Klein (2009) show that insurers have substantially raised insurance rates and reduced their exposures after the intense hurricane seasons of 2004 and 2005, and indicate that there has been substantial market restructuring in Florida but significantly less so in other states. In this paper, the model results are able to imply and confirm their finding that catastrophes can influence the insurance industrial organization.

Several models have been built to study the relationship between shocks and capitalization. Froot, Scharfstein, and Stein (1993) develop a portfolio model of corporate risk management to show that capital-market imperfections can make risk-neutral insurers appear to be risk averse. This portfolio model is extended to research shocks in the insurance industry. Cagle and Harrington (1995) and Cummins and Danzon (1997) both develop models to predict an

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[1] Especially, in property-liability insurance market, the short-run insurance industry's supply curve is upward sloping when a capacity constraint becomes binding, and it is costly for insurers to raise new capital immediately following a negative capital shock because of agency and bankruptcy costs. Negative shocks to claims or industry capital can substantially reduce industry capacity, shifting the supply curve to the left to push up the price (see Winter, 1988, 1991; Gron, 1994).

[2] Taking Hurricane Katrina (2005) for instance, some insurers stopped insuring homeowners in the disaster area, or raised homeowners insurance premiums to cover their risk.

[3] Ten in Florida and one in Louisiana.

[4] Allstate, as an example, paid out $1.9 billion, $500 million more than the profits it had made from all types of insurance and investment income over the 53 years it had been in business in Florida. See "Catastrophes: Insurance Issues", Insurance Matters, May 2013.

ambiguous relationship between the insurance price and a loss shock (also see Grace, Klein and Kleindorfer, 2004). Specifically, Cagle and Harrington (1995) construct a one-period cash flow model for the insurance market equilibrium with the costly capital market assumption. Cummins and Danzon (1997) build a two-period risky debt model for an insurer with the assumption that the new equity is endogenously issued in the second period. Different from these two models, I involve both a reinsurance market and an external capital market in a two-period cash flow model and emphasize on exploring an insurer's response to capacity constraints when the capital market is informational inefficiency. Specifically, I study the impact of catastrophic loss payment on the insurer's next-period optimal strategic choices of underwriting capacity quantity and capital structure under two different environments of capital market, without capacity constraint and with capacity constraint. Moreover, I incorporate different levels of loss incurred into the model. It provides one way to compare the profitability of an insurer in different cases and find out in which condition an insurer can make extra profit when catastrophic loss payment occurs.

The model confirms the finding in previous literature that, in the situation of tight capital supply and high insurance demand, a positive relationship between catastrophic losses and insurance price and a negative relationship between losses and insurance-coverage capacity can be observed. The model contributes by showing that the insurer has an optimal capital structure with capital constraint in costly capital market. Further, I find that the firm's solvency ratio plays an important role in the interaction between its ability to sell new business and to raise external capital. I also derive the condition in which an insurer can gain additional profit from catastrophic risk underwritings: when it is able to take advantage of the insurance price increase and the insured's loyalty after catastrophic shocks. This implies that one catastrophic event could act as an accelerated trigger, splitting insurers into high-quality ones and low-quality ones due to different underwriting efficiencies and capital raising abilities.

This paper is structured as follows. Section 1 develops a two-period cash flow model with catastrophic loss and capacity constraint for an insurer. In section 2, the model is solved to analyze the insurer's optimal catastrophic risk intermediation strategy in two different cases: without capacity constraints and with capacity constraints. The final section concludes the study.

1. Two-period cash flow model

In this section, I develop a two-period cash flow model with different levels of losses for an insurer to explore its optimal catastrophic risk intermediation strategies of underwritings and capital rising. In this model, the insurer originally has retained earnings e_0 as initial endowment and one catastrophe event may occur during the first period. Figure 1 shows the time line.

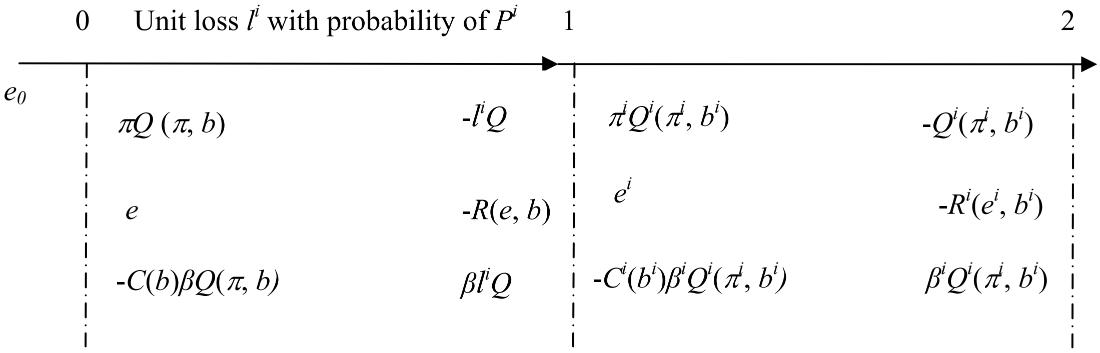

Fig. 1. Time line of the two-period model

At the beginning of each of these two periods, the insurer collects annual premium πQ from the insured, where π is the insurance premium per unit of coverage and Q is insurance coverage. The premium π is assumed to be exogenously determined in the first period, while the insurer will choose its own optimal post-losses premium in the second period. The insurer raises external capital in each period. Here I treat external capital as the one-period debt[1], which is issued by the insurer at an amount of e at the beginning of each period and is repaid with the total cost of R at the end of each period. Meanwhile, the insurer transfers its coverage of βQ to reinsurers at the beginning of each period, where β is the ratio of the ceded coverage to its total insurance coverage. Here β is between 0 and 1, and $\beta = 0$ means no reinsurance while $\beta = 1$ means full reinsurance. The reinsurance premium per unit of coverage is denoted by C. At the end of each period, the insurer indemnifies the insured for covered losses lQ and receives the

[1] In real world, the insurer can raise the capital both from debt holders with interest rate cost and equity holders with agency cost and adjustment cost. Here I use debt holders as the representative in order to simplify the calculation of capital cost in each period.

reimbursement of βlQ from the reinsurer. Here l can be interpreted as unit loss that is the loss incurred per dollar covered, so loss incurred, L, should be equal to lQ.

In this model, the covered event that occurs during the first period can cause different levels of losses: l^iQ with probability of P^i, and $l^i < l^j$, if $i < j$, where $i, j = 1, 2, \ldots I$. Here l^i is the loss incurred per dollar covered in state i, and loss incurred in state i can be $L^i = l^iQ$. Correspondingly, each economic variable in the second period would have different states with superscript "i". The expected value of loss incurred in different states should be equal to the total insurance coverage, which is $\sum_{i=1}^{I} P^i l^i Q = Q$. If we set $\overline{L}$ to be the threshold for catastrophic loss, a catastrophe event in this model could refer to the event, which loss, L^i, is more than $\overline{L}$.

In this model, b is the ratio of assets to liabilities that can be referred as solvency ratio. In Cummins and Danzon (1997), the ratio of assets to liability can be described as capital structure. Here, b is assumed to impact the insurance coverage Q, the external capital cost R, and the reinsurance premium C in the first period, and thus b^i would have impact on Q^i, R^i, and C^i during the second period. The assumptions with regards to some functions are as follows. I assume that the insured will purchase more insurance with a lower premium π and a higher solvency ratio b. Therefore, the demand function for insurance coverage Q (π, b) is a concave function with $Q_\pi < 0$, $Q_{\pi\pi} < 0$, $Q_b > 0$, $Q_{bb} < 0$, where subscripts denote partial derivatives. The cost function of reinsurance per unit of coverage, C (b), is a convex function with $C_b < 0$, $C_{bb} > 0$. My assumption here is that the reinsurance premium will increase if the insurer has a lower solvency ratio, and the increase can be faster as the solvency ratio reduces further (see Froot, 2001). In addition, the cost function of external capital, R (e, b), is a convex function with $R_e > 0$, $R_{ee} > 0$, $R_b < 0$, $R_{bb} > 0$. Considering deadweight costs should be an increasing function of the amount of external capital, I assume that R will go up, at an increasing rate, when the insurer issues a larger amount of debt e. I also assume that issuing debt will be more costly when the insurer is more likely to be insolvent, and the cost changes at a rising speed. The capital market will not be informational efficient if the external capital cost is assumed to be affected by a loss shock.

Then the insurer's expected cash inflows during the first period include the total premium collected from policyholders, πQ (π,b), capital raised from the capital market, e, and the end-of-period loss reimbursement from reinsurers, $\beta\sum_{i=1}^{I} P^i l^i Q(\pi,b)$. The insurer's expected cash outflows during the first period consist of the ceded premium to reinsurers, C (b)βQ (π,b), the expected end-of-period payment to policyholders, $\sum_{i=1}^{I} P^i l^i Q(\pi,b)$, and also to capital holders, R(b, e). Therefore, the insurer's expected net present value of cash flows in the first period should be $\left\{e_0 + [\pi - C(b)\beta - (1-\beta)\sum_{i=1}^{I} P^i l^i r_f^{-1}]Q(\pi,b) + e - R(b,e)r_f^{-1}\right\}$, where r_f is the risk-free rate. In the second period, following the same logic, the expected net cash flow of the insurer in state i (with the probability of P^i) in the model economy should be $\{[\pi^i - C^i(b^i)\beta^i - (1-\beta^i)\, r_f^{-1}]Q^i(\pi^i, b^i) + e^i - R^i(b^i,e^i)r_f^{-1}\}$.

To maximize the profit in two periods (expected net cash flow), the insurer would choose the optimal amount of external capital $\{e, e^i\}$ and reinsurance ratio $\{\beta, \beta^i\}$ in both periods, and set up the optimal premium $\{\pi^i\}$ for each state i in the second period. The optimization problem can be as follows:

$$Max\, e_0 + [\pi - C(b)\beta - (1-\beta)\sum_{i=1}^{I} P^i l^i r_f^{-1}]Q(\pi,b) + e - R(b,e)r_f^{-1} + r_f^{-1}\sum_{i=1}^{I} P^i \begin{Bmatrix} [\pi^i - C^i(b^i)\beta^i - (1-\beta^i)r_f^{-1}] \\ Q^i(\pi^i,b^i) + e^i - R^i(b^i,e^i)r_f^{-1} \end{Bmatrix}; \quad (1)$$

s.t.

$$b = \frac{e_0 + (\pi - C\beta)Q + e - Rr_f^{-1}}{(1-\beta)Qr_f^{-1}}, \quad (2)$$

$$b^i = \frac{\left[r_f(\pi - C\beta)Q - (1-\beta)L^i\right] + \left[r_f(e_0 + e) - R\right] + [(\pi^i - C^i\beta^i)Q^i + (e^i - R^i r_f^{-1})]}{(1-\beta^i)Q^i r_f^{-1}}. \quad (3)$$

2. The insurer's optimal strategy analysis

In this section, I discuss the solutions for the optimization model above in two different cases: we want to look at the insurer's choices of catastrophic risk intermediation strategy in the costly external capital market, but let us look at the risk free capital market at first.

2.1. Case one: risk free capital market. In the first case, the cost of capital is assumed to be equal to risk-free rate, so conditions (4) and (5) below will hold for the marginal cost of reinsurance and external capital.

$$C(b) = C^i(b^i) = r_f^{-1}, \tag{4}$$

$$R_e(b,e) = R^i_{e^i}(b^i, e^i) = r_f. \tag{5}$$

These two conditions imply that the insurer can choose any reinsurance ratio β^i between 0 and 1 and raise any feasible external capital e^i without any extra charge. In other words, there is no need for the insurer to reserve funds to prepare for future loss payments. Based on the first order conditions (FOCs) and the comparative statics analysis of the optimization problem under these two conditions, the following results can be obtained,

$$Q_b = C_b = R_b = Q^i_{b^i} = C^i_{b^i} = R^i_{b^i} = 0. \tag{6}$$

$$E_{Q^i\pi^i} = -\frac{\pi^i}{\pi^i - r_f^{-1}}. \tag{7}$$

Equation (6) describes the fact that the solvency ratio of the insurer, *b*, has no impact on the insurance demand *Q*, the reinsurance cost *C*, or the external capital cost *R*, because the insurer can always raise revenues as high as it needs with no extra charge for risky assets. Equation (7) is the price elasticity of insurance demand in each state during the second period. It implies that the second-period premium will be determined by the specific price elasticity in each state and has nothing to do with the previous loss payment. As a result, in a risk free economy, the insurer's solvency position does not matter and a catastrophic shock has no effect on the insurer's underwriting and capital rising strategies. In this case, neither the capacity constraint theory nor the risk over hang theory has any effect at all.

2.2. Case two: costly capital market. In the second case, the capital market is costly, and then conditions (4) and (5) should be changed into inequities (8) and (9) as follows:

$$C(b), C^i(b^i) > r_f^{-1}, \tag{8}$$

$$R_e(b,e), R^i_{e^i}(b^i, e^i) > r_f. \tag{9}$$

From the FOCs and the comparative statics analysis of the optimization problem, the following results with regards to the optimal catastrophic risk intermediation strategy can be derived. Note that $T^i = \pi^i - C^i\beta^i - (1-\beta^i)\, r_f^{-1}$ and $i < j$ for all the equations below.

$$MP^i_b = \frac{\partial Profit}{\partial b^i} = T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}, \tag{10}$$

$$MP^i_b = -\frac{Q^i + T^i Q^i_{\pi^i}}{b^i_{\pi^i}} = \frac{(C^i - r_f^{-1})Q^i}{b^i_{\beta^i}} = \frac{r_f^{-1} R^i_{e^i} - 1}{b^i_{e^i}}, \tag{11}$$

$$MP^i_\pi = MP^i_b b^i_{\pi^i} = -Q^i - T^i Q^i_{\pi^i}, \tag{11.1}$$

$$MP^i_\beta = MP^i_b b^i_{\beta^i} = (C^i - r_f^{-1})Q^i, \tag{11.2}$$

$$MP^i_e = MP^i_b b^i_{e^i} = r_f^{-1} R^i_{e^i} - 1. \tag{11.3}$$

Equations (10) and (11) indicate the optimal strategies of underwriting, reinsurance purchasing and capital rising are jointly determined by the interactions among these three markets, the primary insurance market, the reinsurance market, and the external capital market. They also imply that the insurer's optimal solvency position plays an important role in the interaction between the insurer's ability to sell new business and its ability to raise capital from reinsurance market or external capital market. The insurer with a good solvency position could have relatively high marginal profit, MP^i_b, and thus it can obtain advantages in both the ability to sell new business and the ability to raise external capital. It is consistent with the finding in Cummins and Danzon (1997) that optimal capital structure is implied.

Equations (11.1) to (11.3) describe the equilibrium within three markets, respectively. Equation (11.1) shows that the marginal profit with respect to the insurance premium, MP^i_π, is equal to the marginal cost of setting up the premium π^i in the second period, $-Q^i - T^i Q^i_{\pi^i}$. Equation (11.2) states that the optimal β^i is the reinsurance ratio when the marginal cost of purchasing such reinsurance, $(C^i - r_f^{-1})Q^i$, is equal to the marginal profit of reinsurance, MP^i_β. In addition, equation (11.3) implies that the optimal e^i is chosen when the marginal cost of raising such capital, $r_f^{-1} R^i_{e^i} - 1$, is equal to the marginal profit of external capital, MP^i_e.

$$\frac{d\pi^i}{dL^i} = \frac{\beta^i C^i_{b^i} Q^i_{\pi^i} - (Q^i_{b^i} + T^i Q^i_{\pi^i b^i} + b^i_{\pi^i} MP^i_{b^i})}{|SOC| * |b^{i^{-1}}_{L^i}|}. \tag{12}$$

Equation (12) describes the effect of losses in the last period on the next period insurance price, the sign of which is determined by cross-partial derivative $Q^i_{\pi^i b^i}$ and the derivative of solvency ratio with respect to premium $b^i_{\pi^i}$ (SOC is second order condition). Firstly, let us assume $C^i_{b^i} = 0$ in order to check the sign without the reinsurance market. If the insurance demand becomes more price elastic in response to a lower solvency ratio, with $Q^i_{\pi^i b^i}$ and $b^i_{\pi^i}$ being both positive, the effect of losses on premium will be negative. This situation can be applicable when people turn to buy insurance products at the same cost from insurers with higher solvency ratio, or when people can make use of other effective mechanisms, such as government's financial relief programs, to mitigate risks rather than purchase insurance. If $Q^i_{\pi^i b^i}$ is negative, which means the insurance price elasticity of demand will be lower in response to a lower solvency ratio, the relationship between previous losses and future premiums can be positive. This situation can be valid when there is a supply shock in the insurance industry, and people cannot find any other effective risk management solutions. In this situation, the insurer can increase its own premium and the insured is willing to purchase higher priced insurance products from insurers with relatively higher solvency ratio. So the model confirms the ambiguous relationship between losses and premiums in previous literature (see Cagle and Harrington, 1995; Cummins and Danzon, 1997). Moreover, the model can show the positive effect of previous losses on future premium can be stronger if $b^i_{\pi^i}$ is also negative. Based on the definition of b^i in the optimization problem above, the negative $b^i_{\pi^i}$ should be induced by a large shortfall of insurance coverage Q. Therefore, in the extreme case with tight capital supply and high insurance demand in the market, the positive relationship of shock losses and premium can be observed. Let us now check this effect in an economy involving the reinsurance market. The positive relation between the previous loss payment and next period premium would be greater when $C^i_{b^i}$ is more largely negative (costly). This indicates that price spikes after a shock would be larger when the reinsurance rate is more sensitive to the insurer's solvency ratio during the period of tight reinsurance market.

In addition, I find that the positive effect of losses on the next period premium can shrink when $Q^i_{b^i}$ is larger, which represents the insured is more sensitive to solvency ratio. It implies that the price spike can be limited for those insurers with relatively low solvency ratio after the shock so that it is more likely for those insurers to encounter insolvency problem after a catastrophic event. This also means the insurer's solvency prospect matters as well-capitalized insurers have the pricing advantage over weakly-capitalized insurers and they can keep more consumers in business after catastrophes. This equation can further show that the next period premium will be affected not only by its previous losses but also by changes in solvency regulation of the insurance market. Considering the fact that insurance regulation can force the insurer with weak solvency status to either raise more external capital or cede more insurance to the reinsurer when it suffers the catastrophic loss, insurance regulation can have large impact on an insurer's optimal capital rising strategy. In such situation, the insurer may encounter the problem of raising additional capital, beyond its optimal level in the initial equilibrium, with a higher cost, and then its marginal profit, MP^i_b, should be adjusted to reach a new equilibrium. Equations (11) and (12) imply that the next period premium of an insurer can be influenced by the newly built equilibrium with changes of insurance regulation.

$$\frac{de^i}{dL^i} = \frac{r_f^{-1} R^i_{e^i b^i} - b^i_{e^i} MP^i_{b^i}}{|SOC| * |b^{i^{-1}}_{L^i}|}. \qquad (13)$$

Equation (13) illustrates the effect of losses on the external capital, and the sign of the effect is determined by the cross-partial derivative, $R^i_{e^i b^i}$, and the first derivative of solvency ratio with respect to external capital, $b^i_{e^i}$. If $R^i_{e^i b^i}$ is negative, with the external capital cost being more sensitive to the capital amount in response to a lower solvency ratio, the relationship between losses and external capital can be negative. In this case, the external capital market is too tight so the insurer tends to decrease its external capital, or otherwise the insurer should make its solvency ratio as high as possible to attract external capital. If $R^i_{e^i b^i}$ is positive, the external capital market is not tight yet and the insurer will be able to directly access more external capital to cover possibly large loss payment in future.

$$\frac{d\beta^i}{dL^i} = \frac{(C^i - r_f^{-1}) Q^i_{b^i} + Q^i C^i_{b^i} - b^i_{\beta^i} MP^i_{b^i}}{|SOC| * |b^{i^{-1}}_{L^i}|}. \qquad (14)$$

Equation (14) provides the effect of losses on the next period reinsurance ratio. It shows that the effect will be small if the marginal cost of reinsurance, $C^i_{b^i}$, is largely negative (costly). This implies that the insurer would avoid reinsurance solutions to transfer risks when the reinsurance market is tight.

$$E_{Q^i \pi^i} = -\frac{(Q^i + MP^i_b b^i_{\pi^i}) \pi^i}{T^i Q^i}. \qquad (15)$$

Equation (15) is the price elasticity of coverage demand in the costly external capital market. It shows that the insurance premium in each state, i, in the costly capital market is determined not only by its price elasticity but also by its overall marginal profit and its solvency position. It means that changes in premium in the costly capital market can be induced by changes in the insurer's solvency position[1].

[1] If we let $MP^i_b = 0$, this equation will be the same as equation (7) derived in the risk-free capital market, in which the insurer's solvency ratio does not matter at all.

$$\Delta^{ij} = (T^j Q^j - T^i Q^i) - [r_f^{-1}(R^j - R^i) - (e^j - e^i)] - \\ -(1-\beta)(L^j - L^i). \quad (16)$$

The difference between the insurer's overall profit in two states, *i* and *j*, can be shown by equation (16). The first term in equation (16), $(T^jQ^j - T^iQ^i)$, can be interpreted as underwriting premium spikes after the larger loss, L^j; and the term $[r_f^{-1}(R^j - R^i) - (e^j - e^i)]$ is the extra external capital cost due to the larger loss. The term $(1-\beta)(L^j - L^i)$ is the difference of loss payment between these two states. There is a chance for the insurer with good solvency ratio to have larger expected profit all through these two periods when larger loss payment incurs in the first period, which can be described by a positive profit difference, $\Delta^{ij} > 0$. This equation shows that a positive profit difference between these two states can be more likely to be obtained if the highly solvent insurer can take advantage of price spikes and the insured's loyalty in post-catastrophe insurance market, and also mitigate the effect of penalties of a costly reinsurance rate and high external capital costs aftershocks. This is also the condition in which the insurer can benefit from catastrophic risk coverage across two periods in this model economy.

One can imagine that, once a catastrophe occurs, the demand expansion and the supply reduction turn out to cause premiums to grow sharply and then gradually moderate as the insurance industry becomes sufficiently recapitalized. During this process, insurers with a comparative advantage in intermediating catastrophic risks may take advantage of the market price increase and relatively low cost of external capital, while other insurers may encounter insolvency or significant financial stress resulting from capital insufficiency. Thus, equation (16) can also imply that one catastrophic event could act as a trigger, splitting insurers into high-quality ones and low-quality ones with respect to different underwriting efficiencies and capital raising capabilities. Meanwhile, new investors, who would supply capital to incumbent insurers and new insurers, may enter the insurance market after the event. With incumbent insurers categorized by their ability to withstand serial catastrophes and new comers continually entering into the market, changes in the insurance industry are sequentially occurring. Equation (16) also suggests the larger the losses incurred by a catastrophe event, the stronger the splitting effect between high-quality insurers and low-quality insurers can be found.

One should be aware that, when the insurer cannot, because of insurance regulation, adjust its post-catastrophe premium, or when the capital market responses to the insurer's great losses due to the catastrophe in a largely negative way, no insurer can benefit from catastrophic risk underwritings since a positive profit difference in equation (16) can never be obtained. This is consistent with the finding in Chen and Hamwi (2012). From equation (16), the negative effect of the possibly large difference between losses in two states, $(1-\beta)(L^j - L^i)$, also supports their finding that the huge unexpected costs associated with disaster insurance can significantly contribute to the failure of the disaster insurance market.

Conclusions and discussions

In this paper, I study an insurer's optimal strategy in a two-period cash flow model with capacity constraints given the possibility of catastrophic loss. The static effect of loss payment on an insurer's optimal underwriting strategy and capital raising strategy in the next period is analyzed in the model. I find that the insurer with a good solvency position could obtain advantageous position in both the ability to sell new business and the ability to raise external capital. The model further implies that catastrophic shocks can impact industrial organization of the property-liability insurance market in the post-catastrophe period.

The model developed in this paper contributes to find the interaction between the insurer's capital rationing and coverage underwriting, in which the solvency ratio plays an important role. I also discuss what kind of insurers, to some extent, can benefit from the catastrophic risk underwriting. This paper also sheds light on empirical tests of capacity constraint theory to find more about the impact of catastrophic shocks on insurer performance and insurance industrial organization and to further study the relation between the capital market and the insurance industry.

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Appendix: Optimization solutions for the two-period cash flow model

FOCs with β, β^i, e, e^i, π^i are as follows:

$$(TQ_b - \beta QC_b - r_f^{-1}R_b)b_\beta + (r_f^{-1} - C)Q + r_f^{-1}\sum_{i=1}^{I} P^i (T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}) b^i_\beta = 0, \quad (1)$$

$$(T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}) b^i_{\beta^i} - (C^i - r_f^{-1)})Q^i = 0, \quad (2)$$

$$(TQ_b - \beta QC_b - r_f^{-1}R_b)b_e + (1 - r_f^{-1}R_e) + r_f^{-1}\sum_{i=1}^{I} P^i (T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}) b^i_e = 0, \quad (3)$$

$$(T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}) b^i_{e^i} + (1 - r_f^{-1} R^i_{e^i}) = 0, \quad (4)$$

$$(T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i}) b^i_{\pi^i} + Q^i + T^i Q^i_{\pi^i} = 0, \quad (5)$$

where $T = \pi - C\beta - (1-\beta) r_f^{-1}$ and $T^i = \pi^i - C^i\beta^i - (1-\beta^i) r_f^{-1}$. Case one: risk free capital market with $C(b) = C^i(b^i) = r_f^{-1}, R_e(b,e) = R^i_{e^i}(b^i, e^i) = r_f$,

Then, $Q_b = Q^i_{b^i} = 0$, and $TQ_b - \beta QC_b - r_f^{-1}R_b = T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i} = 0$, then, $Q^i + T^i Q^i_{\pi^i} = 0$ according to (5), equivalently, it is $E_{Q^i\pi^i} = -\dfrac{\pi^i}{\pi^i - r_f^{-1}}$. Case two: costly capital market with

$C(b), C^i(b^i) > r_f^{-1}, R_e(b,e), R^i_{e^i}(b^i, e^i) > r_f$. (2), (4), and (5) can derive that:

$$T^i Q^i_{b^i} - \beta^i Q^i C^i_{b^i} - r_f^{-1} R^i_{b^i} = -\frac{Q^i + T^i Q^i_{\pi^i}}{b^i_{\pi^i}} = \frac{(C^i - r_f^{-1})Q^i}{b^i_{\beta^i}} = \frac{r_f^{-1} R^i_{e^i} - 1}{b^i_{e^i}}.$$

Then (5) can show that: $E_{Q^i\pi^i} = -\dfrac{(Q^i + MP^i_b b^i_{\pi^i})\pi^i}{T^i Q^i}$, next, according to comparative statics analysis, one can get

$$\frac{d\pi^i}{dL^i} = \frac{\frac{d(5)}{dL^i}}{|SOC|}, \frac{de^i}{dL^i} = \frac{\frac{d(4)}{dL^i}}{|SOC|}, \text{ and } \frac{d\beta^i}{dL^i} = \frac{\frac{d(2)}{dL^i}}{|SOC|}.$$