

Olga Kozmenko (Ukraine), Yevgen Balatskiy (Ukraine), Viktor Oliynyk (Ukraine),
Olga Kuzmenko (Ukraine)

Peculiarities of optimization of insurance portfolio of companies in the countries with transition economies

Abstract

This article deals with effective management of companies' insurance portfolios. It carries out mathematical formalization of stages of this process in general, which makes it possible to use the proposed scientific and methodical approach to optimizing the insurance portfolio of any company in a developing country. It conducts practical implementation of this methodology for insurers in Ukraine. Considerable attention is given to identification of the relevant indicators of the optimal insurance portfolio.

Keywords: optimization, insurance company, insurance portfolio, regression analysis, center of mass.

Introduction

Problem statement. Insurance companies play the leading role in the formation and development of countries with transition economies, their ability to neutralize adverse effects of various kinds, to act as active investors on the financial market and to stimulate transformation processes in the economies of these countries. At the same time, an effective functioning of insurance companies themselves is linked to the implementation of a significant number of administrative processes, the basis of which is the formation of optimal insurance portfolio. Only through a balanced ratio of the share of each type of insurance in the portfolio's structure the insurance company is able to remain financially sustainable over time. The problem of finding an individual methodology of insurance portfolio optimization in countries with transition economies is caused by the peculiarity of specific risks that are insured by domestic companies in these countries. Thus, the formation of a balanced insurance portfolio should be based on the optimal consideration of individual characteristics of each type of insurance.

The main results of the study. Stressing the need to develop an adequate system of indicators for the types of insurance during the first stage we will concentrate on the input array. It is proposed to use five different relative indicators for the types of insurance: profitability of insurance, the level of riskiness for insurer, the adequacy of insurance reserves, insurance concentration and dependence on reinsurance.

We will consider these indicators in more detail and determine the level of their importance for any type of insurance. Thus, the profitability of a certain type of insurance describes the share of profit (which remains with the insurer) in the total amount of insurance premiums received from insuring specific risks. This is the profitability of a certain type of insurance in the context of optimizing insurance portfolio reflecting its profitability.

The next indicator is the level of riskiness of insurance, which is more difficult to calculate. Thus, in order to obtain adequate results, the share of insurance payments to the share of insurance premiums for certain types of insurance must be adjusted to a specific value of probability of the insured event. This step is due to the fact that for each of the different types of insurance a different level of risk is acceptable as more profitable types of insurance are more risky while an optimal portfolio cannot consist only of low-risk types of insurance, as in this case, the company will receive no profits (Boiko, A.O., 2011).

The adequacy of insurance reserves is a very important indicator for the characteristic of the optimal insurance portfolio, as the inability to pay for its obligations to customers will lead to the bankruptcy of the insurer, even if from the point of view of profitability and riskiness the insurance portfolio will be balanced. Therefore, the ratio of insurance reserves formed for a certain type of insurance to net insurance premiums for this type of insurance should be at a high level.

In terms of the optimality of insurance portfolio, the concentration of insurance is important, i.e. the maximum proportion of a certain type of insurance in one region. The concentration of risks in a limited area increases the probability of a loss by the insurer of financial stability subject to natural or man-made disasters that lead to the accumulation of insurance claims for all types of insurance contracts in the region (Roienko, V.V., 2011).

© Olga Kozmenko, Yevgen Balatskiy, Viktor Oliynyk, Olga Kuzmenko, 2015.

Olga Kozmenko, Dr., Prof., Ukrainian Banking Academy, Ukraine.

Yevgen Balatskiy, Dr., Associate Professor, Ukrainian Banking Academy, Ukraine.

Viktor Oliynyk, Ph.D., Associate Professor, Ukrainian Banking Academy, Ukraine.

Olga Kuzmenko, Ph.D. student, Ukrainian Banking Academy, Ukraine.

The last fifth indicator of the optimal insurance portfolio is the coefficient of dependency on reinsurance, that is, the ratio of premiums transferred to reinsurance for a specific type of insurance to total premiums for this type of insurance. Of course, a balanced insurance portfolio cannot be formed without the use of reinsurance, but the use of this instrument should be prudent and not threaten the financial stability of the insurer in the

context of its high dependence on the solvency of reinsurers and the timely execution of their obligations (Pakhnenko, O.M., 2010).

The conventional formalization of information database for the study of optimization of insurance portfolio is presented in Table 1. Within the classification of insurance its most common types in the countries with transition economies were chosen.

Table 1. Introduction of conventional signs in terms of input data for the optimization of portfolios of insurance companies

Type of insurance	Profitability of insurance	The level of insurance risks		Adequacy of insurance reserves	Insurance concentration	Dependence on reinsurance
		Value	Probability			
Car insurance (CASCO, MTPL)	DS_1	RS_1	p_1	DSR_1	KS_1	KZR_1
Green card	DS_2	RS_2	p_1	DSR_2	KS_2	KZR_2
Property insurance	DS_3	RS_3	p_1	DSR_3	KS_3	KZR_3
Insurance from fire risks	DS_4	RS_4	p_1	DSR_4	KS_4	KZR_4
Insurance of financial risks	DS_5	RS_5	p_1	DSR_5	KS_5	KZR_5
Life insurance	DS_6	RS_6	p_1	DSR_6	KS_6	KZR_6
Medical insurance	DS_7	RS_7	p_1	DSR_7	KS_7	KZR_7
Insurance of freight and baggage	DS_8	RS_8	p_1	DSR_8	KS_8	KZR_8
Third party liability insurance	DS_9	RS_9	p_1	DSR_9	KS_9	KZR_9
Credit insurance	DS_{10}	RS_{10}	p_1	DSR_{10}	KS_{10}	KZR_{10}
Accident insurance	DS_{11}	RS_{11}	p_1	DSR_{11}	KS_{11}	KZR_{11}
Aviation insurance	DS_{12}	RS_{12}	p_1	DSR_{12}	KS_{12}	KZR_{12}
Insurance of medical expenses	DS_{13}	RS_{13}	p_1	DSR_{13}	KS_{13}	KZR_{13}
Insurance of transport accidents	DS_{14}	RS_{14}	p_1	DSR_{14}	KS_{14}	KZR_{14}
Other types of insurance	DS_{15}	RS_{15}	p_1	DSR_{15}	KS_{15}	KZR_{15}
Total	$\sum_{i=1}^{15} DS_i$	$\sum_{i=1}^{15} (RS_i \times p_i)$		$\sum_{i=1}^{15} DSR_i$	$\sum_{i=1}^{15} KS_i$	$\sum_{i=1}^{15} KZR_i$

During the second stage, it is important to determine mathematical instruments for the realization of the proposed scientific and methodological approach. Based on the fact that the main purpose of forming a balanced insurance portfolio of companies that operate in countries with transition economies is not only the consideration of all relevant characteristics of each type of insurance, but also the determination of their optimal combination taking into account the importance of these five indicators. Therefore, for the implementation of this scientific and methodological approach it is proposed to use the following:

- ♦ regression analysis to describe dependencies of the indicators of insurance portfolio on the shares of each of insurance type;
- ♦ geometric modeling to identify a generalized criterion for optimization of an insurance company's portfolio, which determines the

structure of certain types of insurance. We use mathematical instruments to identify the center of mass of the pentagon, the sides of which are five indicators of characteristics of the studied insurance portfolio. The center of mass, in this case, will describe the best compromise value of the indicator of each type of insurance, which forms the portfolio of an insurance company.

After determining a statistical base of the research and mathematical instruments for the implementation of the methodology, *during the third stage*, we proceed directly to the implementation of scientific and methodical approach to the optimization of insurance companies' portfolios in countries with transition economies. We conduct the normalization of indicators (through relative method) given in Table 1 and define the structure of insurance portfolio by calculating a relative indicator of the structure on the basis of the following formulas:

$$\begin{aligned}
SDS_i &= \frac{DS_i}{\sum_{i=1}^{15} DS_i} \times 100\%, & SRS_i &= \frac{RS_i \times p_i}{\sum_{i=1}^{15} (RS_i \times p_i)} \times 100\%, \\
SDSR_i &= \frac{DSR_i}{\sum_{i=1}^{15} DSR_i} \times 100\%, & SKS_i &= \frac{KS_i}{\sum_{i=1}^{15} KS_i} \times 100\%, \\
SKZR_i &= \frac{KZR_i}{\sum_{i=1}^{15} KZR_i} \times 100\%,
\end{aligned} \tag{1}$$

where SDS_i is a share of the i -th type of insurance in the structure of the insurance portfolio according to the indicator “profitability of insurance”; DS_i – profitability of the i -th type of insurance; SRS_i – the share of the i -th type of insurance in the structure of insurance portfolio according to the indicator “insurance risk level”; RS_i – risk level of the i -th type of insurance, $SDSR_i$ – the share of the i -th type of insurance in the structure of insurance portfolio according to the indicator “adequacy of insurance reserves”; DSR_i – adequacy of insurance reserves of the i -th type of insurance; SKS_i – the share of the i -th type of insurance in the structure of insurance portfolio according to the indicator “insurance concentration”; KS_i – concentration of the i -th type of insurance; $SKZR_i$ – the share of the i -th type of insurance in the structure of insurance portfolio according to the indicator “coefficient of dependence on reinsurance”; KZR_i – coefficient of dependence on reinsurance for the i -th type of insurance.

During *the fourth stage* of mathematical implementation of the scientific and methodical approach we carry out formalization of dependence of the above indicators of insurance portfolio on each type of insurance.

- ♦ Regression equation of the dependence of profitability of all types of insurance on their shares in the structure of insurance portfolio (Asanga, S.);

$$\begin{aligned}
\sum_{i=1}^{15} DS_i &= a_0 + a_1 \times SDS_1 + a_2 \times SDS_2 + \dots \\
&+ a_{15} \times SDS_{15} = a_0 + \sum_{i=1}^{15} (a_i \times SDS_i).
\end{aligned} \tag{2}$$

where $\sum_{i=1}^{15} DS_i$ is the total value of profitability for all types of insurance in the portfolio; a_0, a_1, \dots, a_{15} – coefficients of regression equation, constants.

- ♦ Regression equation of the dependence of

insurance portfolio risk levels weighted by the probability of their occurrence for all types of insurance on their shares in the structure of insurance portfolio.

$$\begin{aligned}
\sum_{i=1}^{15} (RS_i \times p_i) &= b_0 + b_1 \times SRS_1 + b_2 \times SRS_2 + \dots \\
&+ b_{15} \times SRS_{15} = b_0 + \sum_{i=1}^{15} (b_i \times SRS_i),
\end{aligned} \tag{3}$$

where:

$\sum_{i=1}^{15} (RS_i \times p_i)$ – the total value of the insurance portfolio risk levels weighted by the probability of their occurrence; b_0, b_1, \dots, b_{15} – coefficients of regression equation, constants.

- ♦ Regression equation of the dependence of adequacy of insurance reserves for all types of insurance on their shares in the structure of insurance portfolio (Rardin, Ronald L., 1997).

$$\begin{aligned}
\sum_{i=1}^{15} DSR_i &= c_0 + c_1 \times SDSR_1 + c_2 \times SDSR_2 + \dots \\
&+ c_{15} \times SDSR_{15} = c_0 + \sum_{i=1}^{15} (c_i \times SDSR_i).
\end{aligned} \tag{4}$$

Where:

$\sum_{i=1}^{15} DSR_i$ – the total value of insurance reserves’ adequacy for all types of insurance; c_0, c_1, \dots, c_{15} – coefficients of regression equation, constants.

- ♦ Regression equation of the dependence of insurance concentration for all types of insurance on their shares in the structure of insurance portfolio (Kuzmenko, O.V., Kyrkach S., 2014).

$$\begin{aligned}
\sum_{i=1}^{15} KS_i &= d_0 + d_1 \times SKS_1 + d_2 \times SKS_2 + \dots \\
&+ d_{15} \times SKS_{15} = d_0 + \sum_{i=1}^{15} (d_i \times SKS_i),
\end{aligned} \tag{5}$$

where $\sum_{i=1}^{15} KS_i$ – the total value of insurance concentration for all types of insurance; d_0, d_1, \dots, d_{15} – coefficients of regression equation, constants.

- ♦ Regression equation of dependence of the coefficients of dependence on all types of insurance on their shares in the structure of insurance portfolio (MacKie-Mason J.K., 1992).

$$\sum_{i=1}^{15} KZR_i = e_0 + e_1 \times SKZR_1 + e_2 \times SKZR_2 + \dots + e_{15} \times SKZR_{15} = e_0 + \sum_{i=1}^{15} (e_i \times SKZR_i), \quad (6)$$

where $\sum_{i=1}^{15} KZR_i$ – the total value of the coefficients of dependence on all types of insurance in the portfolio; e_0, e_1, \dots, e_{15} – coefficients of regression equation, constants.

The second component of mathematical realization of the proposed scientific and methodical approach during **the fourth stage** is a graphic interpretation of optimization of the insurance portfolio. We will compare the above indicators of the insurance portfolio with the corresponding sides of the pentagon, which are designated as follows:

$$AB = \sum_{i=1}^{15} DS_i, \quad BC = \sum_{i=1}^{15} (RS_i \times p_i), \quad CD = \sum_{i=1}^{15} DSR_i,$$

$$R_1 = BM = \frac{AB \times BF \times AF}{\sqrt{(AB + BF + AF) \times (-AB + BF + AF) \times (AB - BF + AF) \times (AB + BF - AF)}}. \quad (7)$$

$$\begin{aligned} R_1 &= \frac{AB}{2 \sin \alpha_{AB}} = \frac{BF}{2 \sin \alpha_{BF}} = \frac{AF}{2 \sin \alpha_{AF}} \Rightarrow \\ \Rightarrow \alpha_{AB} &= \arcsin\left(\frac{AB}{2R_1}\right), \alpha_{BF} = \arcsin\left(\frac{BF}{2R_1}\right), \alpha_{AF} = \arcsin\left(\frac{AF}{2R_1}\right) \Rightarrow \\ \Rightarrow \alpha_{AB} + \alpha_{BF} + \alpha_{AF} &= 180 \Rightarrow \arcsin\left(\frac{AB}{2R_1}\right) + \arcsin\left(\frac{BF}{2R_1}\right) + \arcsin\left(\frac{AF}{2R_1}\right) = 180. \end{aligned} \quad (8)$$

Solution to the system of equations:

$$\begin{cases} R_1 = \frac{AB \times BF \times AF}{\sqrt{(AB + BF + AF) \times (-AB + BF + AF) \times (AB - BF + AF) \times (AB + BF - AF)}} \\ \arcsin\left(\frac{AB}{2R_1}\right) + \arcsin\left(\frac{BF}{2R_1}\right) + \arcsin\left(\frac{AF}{2R_1}\right) = 180, \end{cases} \quad (9)$$

will help find the center of mass of the triangle ABF, the radius of the circle circumscribed around it $R_1 = BM$ and the unknown side BF.

We will consider the triangle DBC, for which we

$FD = \sum_{i=1}^{15} KS_i$, $AF = \sum_{i=1}^{15} KZR_i$. We consider the process of determining the optimal structure of insurance portfolio as a task of finding the center of mass of the pentagon (Figure 1), the sides of which are the specified indicators.

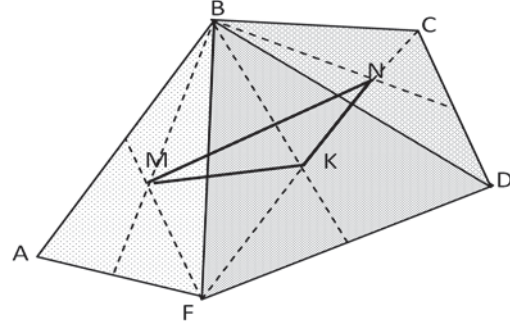


Fig. 1. Geometric interpretation of finding an optimal structure of insurance portfolio

The sequence of search for the optimal structure of insurance portfolio can be presented in the form of the following steps:

Step 1. We divide the pentagon ABCDF into three triangles ABF, FBD, DBC, for each of whom we will find the center of mass (the points of intersection of the medians – M, N, K) with the quantitative characteristics – the radius of the circle circumscribing the corresponding triangle – BM, BK, BM, respectively.

We will consider the triangle ABF:

$$\begin{cases} R_2 = \frac{DB \times BC \times DC}{\sqrt{(DB + BC + DC) \times (-DB + BC + DC) \times (DB - BC + DC) \times (DB + BC - DC)}} \\ \arcsin\left(\frac{DB}{2R_2}\right) + \arcsin\left(\frac{BC}{2R_2}\right) + \arcsin\left(\frac{DC}{2R_2}\right) = 180. \end{cases} \quad (10)$$

We will consider the triangle FBD, for which we define the center of mass, the radius of the circle circumscribed around it $R_3 = BK$:

$$R_3 = \frac{FB \times BD \times FD}{\sqrt{(FB + BD + FD) \times (-FB + BD + FD) \times (FB - BD + FD) \times (FB + BD - FD)}} \quad (11)$$

Step 2. We will consider the triangle MNK built on the center of mass. For this triangle we also find the center of mass by analyzing the triangles ABF, FBD,

DBC, which are the components of the pentagon ABCDF. The quantitative characteristic of the pentagon is the radius of the circumscribed circle:

$$R = \frac{MN \times NK \times MK}{\sqrt{(MN + NK + MK) \times (-MN + NK + MK) \times (MN - NK + MK) \times (MN + NK - MK)}}. \quad (12)$$

2.1. Determination of the unknown side MN:

$$\begin{cases} \left(\frac{AF}{2}\right)^2 = \left(\frac{3R_1}{2}\right)^2 + R^2 - 3R_1 \times R \times \cos \angle MBF \Rightarrow \angle MBF = \arccos \frac{\left(\left(\frac{3R_1}{2}\right)^2 + R^2 - \left(\frac{AF}{2}\right)^2\right)}{3R_1 \times R} \\ \left(\frac{CD}{2}\right)^2 = \left(\frac{3R_2}{2}\right)^2 + R^2 - 3R_2 \times R \times \cos \angle NBD \Rightarrow \angle NBD = \arccos \frac{\left(\left(\frac{3R_2}{2}\right)^2 + R^2 - \left(\frac{CD}{2}\right)^2\right)}{3R_2 \times R} \\ FG^2 = BF^2 + BD^2 - 2BF \times BD \times \cos \angle FBD \Rightarrow \angle FBD = \arccos \left(\frac{BF^2 + BD^2 - FG^2}{2BF \times BD}\right). \end{cases} \quad (13)$$

$$MN = \sqrt{R_1^2 + R_2^2 - 2R_1 \times R_2 \times \cos(\angle MBF + \angle NBD + \angle FBD)}. \quad (14)$$

2.2. Determination of the unknown side KN:

$$\begin{aligned} KN &= \sqrt{R_2^2 + R_3^2 - 2R_2 \times R_3 \times \cos(\angle NDK)}, \\ \angle NDK &= \arccos \left(\frac{\left(\left(\frac{3R_3}{2}\right)^2 + BD^2 - \left(\frac{BC}{2}\right)^2\right)}{3R_3 \times BD} \right) + \arccos \left(\frac{\left(\left(\frac{3R_3}{2}\right)^2 + BD^2 - \left(\frac{BF}{2}\right)^2\right)}{3R_3 \times BD} \right). \end{aligned} \quad (15)$$

2.3. Determination of the unknown side MK:

$$\begin{aligned} MR &= \sqrt{R_1^2 + R_3^2 - 2R_1 \times R_3 \times \cos(\angle MFK)}, \\ \angle MFK &= \arccos \left(\frac{\left(\left(\frac{3R_1}{2}\right)^2 + BF^2 - \left(\frac{AB}{2}\right)^2\right)}{3R_1 \times BF} \right) + \arccos \left(\frac{\left(\left(\frac{3R_1}{2}\right)^2 + BF^2 - \left(\frac{BD}{2}\right)^2\right)}{3R_1 \times BF} \right). \end{aligned} \quad (16)$$

The next **fifth stage** of the scientific and methodical approach to the balancing of insurance portfolio is a regression analysis of dependence of the generalizing indicator of the

optimal structure of portfolio (center of mass of the pentagon) on the shares of each of the i -th type of insurance. This, accordingly, will provide an opportunity to determine an optimal share of each type of insurance that will ensure effective functioning of the insurer. For that the following steps are needed:

The building of a multiple linear regression equation of dependence of the generalizing indicator of the optimal structure of insurance portfolio (R) on the sum of absolute values of the indicators of its characteristics in the context of the studied time interval:

$$R = f_0 + f_1 \times \sum_{i=1}^{15} DS_i + f_2 \times \sum_{i=1}^{15} (RS_i \times p_i) + f_3 \times \sum_{i=1}^{15} DSR_i + f_4 \times \sum_{i=1}^{15} KS_i + f_5 \times \sum_{i=1}^{15} KZR_i. \quad (17)$$

Building of a standardized equation of dependence of the generalizing indicator of the optimal portfolio structure (R) on the shares of certain types of

insurance. So, if $SDS_i = SRS_i = SDR_i = SKS_i = SKZR_i = s_i$, the equations (2) - (6) take the following form (Pindyck, R., Rubinfeld, D., 1991):

$$\begin{aligned} \sum_{i=1}^{15} DS_i &= a_0 + \sum_{i=1}^{15} (a_i \times s_i), \sum_{i=1}^{15} (RS_i \times p_i) = b_0 + \sum_{i=1}^{15} (b_i \times s_i), \sum_{i=1}^{15} DSR_i = c_0 + \sum_{i=1}^{15} (c_i \times s_i), \\ \sum_{i=1}^{15} KS_i &= d_0 + \sum_{i=1}^{15} (d_i \times s_i), \sum_{i=1}^{15} KZR_i = e_0 + \sum_{i=1}^{15} (e_i \times s_i). \end{aligned} \quad (18)$$

Taking into account the introduced conventional signs, the formula (17) takes the following form:

$$\begin{aligned} R &= f_0 + f_1 \times \sum_{i=1}^{15} DS_i + f_2 \times \sum_{i=1}^{15} (RS_i \times p_i) + f_3 \times \sum_{i=1}^{15} DSR_i + f_4 \times \sum_{i=1}^{15} KS_i + f_5 \times \sum_{i=1}^{15} KZR_i = f_0 + f_1 \times \left(a_0 + \sum_{i=1}^{15} (a_i \times s_i) \right) + \\ &+ f_2 \times \left(b_0 + \sum_{i=1}^{15} (b_i \times s_i) \right) + f_3 \times \left(c_0 + \sum_{i=1}^{15} (c_i \times s_i) \right) + f_4 \times \left(d_0 + \sum_{i=1}^{15} (d_i \times s_i) \right) + f_5 \times \left(e_0 + \sum_{i=1}^{15} (e_i \times s_i) \right) = \\ &= (f_0 + f_1 \times a_0 + f_2 \times b_0 + f_3 \times c_0 + f_4 \times d_0 + f_5 \times e_0) + (f_1 \times a_1 + f_2 \times b_1 + f_3 \times c_1 + f_4 \times d_1 + f_5 \times e_1) \times s_1 + \\ &+ (f_1 \times a_{15} + f_2 \times b_{15} + f_3 \times c_{15} + f_4 \times d_{15} + f_5 \times e_{15}) \times s_{15} = (f_0 + f_1 \times a_0 + f_2 \times b_0 + f_3 \times c_0 + f_4 \times d_0 + f_5 \times e_0) + \\ &+ \sum_{i=1}^{15} (f_1 \times a_i + f_2 \times b_i + f_3 \times c_i + f_4 \times d_i + f_5 \times e_i) \times s_i. \end{aligned} \quad (19)$$

The standardized regression equation of dependence of the generalizing indicator of the optimal portfolio's structure on the share of the i -th type of insurance will take the following form (Oyatoye, E.O.):

$$R^* = \sum_{i=1}^{15} w_i \times s_i^*, R^* = \frac{R - \bar{R}}{\sigma_R}, s_i^* = \frac{s_i - \bar{s}_i}{\sigma_{s_i}}, \quad (20)$$

$$w_i = (f_1 \times a_i + f_2 \times b_i + f_3 \times c_i + f_4 \times d_i + f_5 \times e_i) \frac{s_i}{\bar{R}},$$

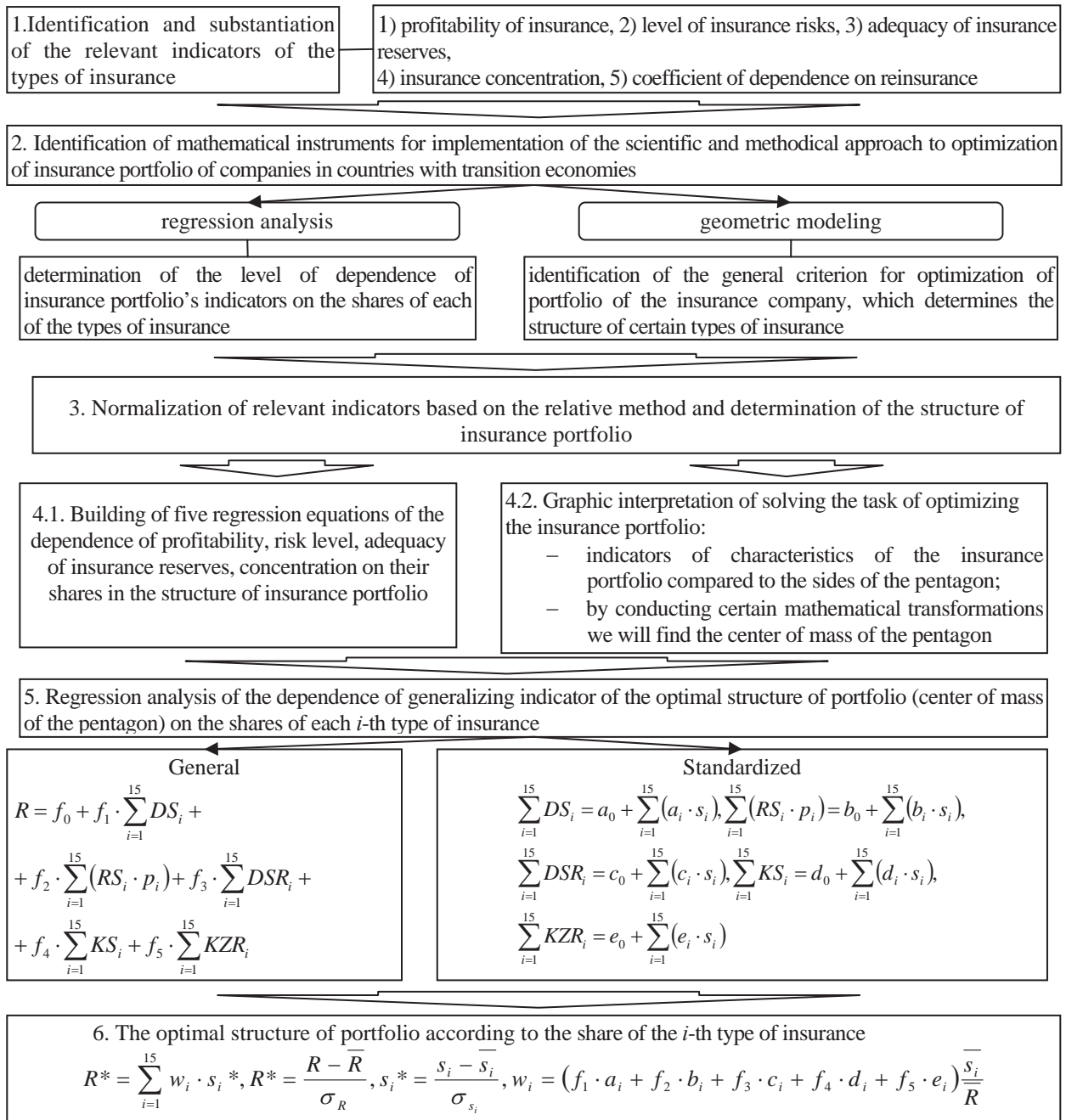
where R^* (s_i^*) – dependent (respectively, independent) variable of the standardized regression equation of dependence of the indicator of the optimal portfolio's structure on the share of the i -th type of insurance;

w_i – parameters of standardized regression equation of dependence of the generalizing indicator of the optimal structure of insurance portfolio on the share of the i -th type of insurance. This indicator

determines the optimal structure of insurance portfolio.

f_0, f_1, \dots, f_5 – coefficients that determine the impact of each of the five relevant indicators on the indicator of the optimal structure of insurance portfolio; a_i, b_i, c_i, d_i, e_i – coefficients that determine the impact of the portfolio's structure for each of the relevant indicators on the generalizing indicator.

Having formed all the necessary basic principles and the algorithm of theoretical realization of the scientific and methodical approach to the optimization of companies' insurance portfolios in countries with transition economies (Fig. 2) we consider the results of its practical implementation in Ukraine illustrated by the example of two insurance companies: public joint-stock company "Oranta" and PJSC "TAS Insurance Group".



Note: R – generalized indicator of characteristics of the optimal structure of insurance portfolio; \bar{R} – average value of generalizing indicator of the optimal structure of insurance portfolio during the reporting period; σ_R – standard deviation of generalizing indicator of the optimal structure of insurance portfolio during the reporting period; S_i – the share of the i -th type of insurance in the structure of insurance portfolio; \bar{s}_i – average value of the shares of the i -th type of insurance in the structure of insurance portfolio during the reporting period; σ_{s_i} – standard deviation of the i -th type of insurance in the structure of insurance portfolio during the reporting period; $\sum_{i=1}^{15} DS_i$ – the total value of profitability for all types of insurance in the portfolio; $\sum_{i=1}^{15} (RS_i \cdot p_i)$ – the total value of risk levels of the insurance portfolio weighted by the probability of their occurrence; $\sum_{i=1}^{15} DSR_i$ – the total value of the adequacy of insurance reserves for all types of insurance in the portfolio; $\sum_{i=1}^{15} KS_i$ – the total value of insurance concentration for all types of insurance in the portfolio; $R^* (s_i^*)$ – dependent (respectively, independent) variable of standardized regression equation of the dependence of generalized indicator of the optimal structure of portfolio on the share of the i -th type of insurance; w_i – parameters of standardized regression equation for the dependence of the generalizing indicator of the optimal structure of insurance portfolio on the share of the i -th type of insurance; f_1, f_2, \dots, f_5 – coefficients that determine the impact of each of the five relevant indicators on the generalizing indicator of the structure of insurance portfolio; a_i, b_i, c_i, d_i, e_i – coefficients that determine the impact of the structure of portfolio for each of the relevant indicators of the types of insurance on the generalizing indicator.

Fig. 2. Scientific and methodical approach to the optimization of insurance portfolio of companies in countries with transition economies

Data calculation for the two companies was carried out on the basis of data for public joint-stock company “Oranta”: 2012-2014 and for PJSC “TAS Insurance Group”: 2013-2014. These actions are related to the

leveling of random changes in the structure of insurance portfolio for one-year study period. Consequently, the normalized data for the calculation of the optimal structure of insurance portfolio are shown in Table 2.

Table 2. Normalized values of indicators for the optimal insurance portfolio of the joint-stock company “Oranta” during the period 2012-2014

Types of insurance	Indicators of the optimal insurance portfolio														
	Profitability of insurance			Level of insurance risks			Adequacy of insurance reserves			Insurance concentration			Coefficient of dependence on reinsurance		
	2012	2013	2014	2012	2013	2014	2012	2013	2014	2012	2013	2014	2012	2013	2014
Insurance of ground transportation (s_1)	1.00	1.17	0.64	0.04	0.03	0.03	0.20	0.08	0.14	0.91	0.81	0.70	0.27	0.03	0.30
Insurance of other property (s_2)	0.84	1.07	0.63	1.00	0.68	0.85	0.24	0.06	0.06	0.74	0.94	0.66	0.51	0.07	0.47
Car insurance (internal) (s_3)	0.70	1.00	0.62	0.17	0.04	0.07	0.31	0.09	0.12	0.83	1.00	0.70	0.53	0.02	0.03
Green card (s_4)	0.82	1.29	0.85	0.32	0.10	0.04	0.18	0.10	0.25	0.89	0.79	1.00	0.55	0.02	0.04
Voluntary medical insurance (s_5)	0.81	1.00	0.62	0.03	0.01	0.01	0.22	0.04	0.05	0.68	0.89	0.93	0.60	1.00	0.83
Other private insurance (s_6)	0.77	1.01	0.62	0.58	0.27	0.29	0.21	0.05	0.06	1.00	0.94	0.85	1.00	0.73	1.00
Other types of insurance (s_7)	0.98	1.43	1.00	0.80	1.00	1.00	1.00	1.00	1.00	0.80	0.91	0.85	0.29	0.02	0.03
Total	5.92	7.96	4.98	2.94	2.12	2.29	2.35	1.41	1.68	5.85	6.28	5.69	3.74	1.89	2.69

Characteristic of the types of insurance within the five indicators for Ukrainian companies is based on the information provided by the state regulator (National Commission for State Regulation of Financial Services Markets).

Within the study of the balanced structure of insurance portfolio of PJSC “TAS Insurance Group” the normalized values of indicators of the optimal insurance portfolio are presented in Table 3.

Table 3. Normalized values of indicators of the optimal insurance portfolio of PJSC “TAS Insurance Group” in 2013-2014

Types of insurance	Indicators of the optimal insurance portfolio									
	Profitability of insurance		Level of insurance risks		Adequacy of insurance reserves		Insurance concentration		Coefficient of dependence on reinsurance	
	2013	2014	2013	2014	2013	2014	2013	2014	2013	2014
Insurance of ground transportation	1.16	0.73	0.05	0.04	0.26	0.20	0.76	0.72	0.05	0.87
Insurance of other property	1.18	0.88	1.00	0.76	0.28	0.28	0.65	0.78	0.05	0.05
Car insurance (internal)	1.00	0.72	0.06	0.06	0.43	0.40	0.76	0.64	0.72	1.00
Green card	1.70	1.00	0.36	0.09	0.67	0.46	1.00	0.66	0.02	0.04
Voluntary medical insurance	1.00	0.72	0.03	0.02	0.21	0.15	0.68	0.77	0.09	0.08
Other private insurance	1.01	0.72	0.39	0.29	0.22	0.17	0.65	1.00	1.00	0.13
Other types of insurance	1.08	0.76	0.82	1.00	1.00	1.00	0.75	0.80	0.10	0.19
Total	8.12	5.54	2.70	2.26	3.06	2.66	5.25	5.37	2.03	2.36

Normalized data for indicators of the optimal insurance portfolio for public joint-stock company “Oranta” and PJSC “TAS Insurance Group” provide an opportunity to form a structure of insurance

portfolio, conduct a regression analysis of dependence of insurance portfolio's indicators on the shares of each type of insurance and find a general characteristic of the optimal structure of portfolio – R (Table 4).

Table 4. Information basis for building a standardized regression equation of dependence of generalizing indicator of the optimal structure of portfolio (center of mass of the pentagon) on the shares of the i -th type of insurance for public joint-stock company “Oranta”

Year	Indicator	Type of insurance							Integral indicator R
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	
2014	Profitability of insurance	12.87	12.71	12.44	17.02	12.44	12.44	20.07	0.76
	Level of insurance risks	1.21	36.98	3.05	1.78	0.62	12.63	43.74	0.76
	Adequacy of insurance reserves	8.14	3.81	7.02	15.13	2.98	3.37	59.55	0.76
	Insurance concentration	12.33	11.52	12.33	17.57	16.34	14.95	14.95	0.76
	Coefficient of dependence on reinsurance	11.20	17.47	0.93	1.38	30.93	37.12	0.98	0.76
2013	Profitability of insurance	14.68	13.43	12.57	16.15	12.62	12.64	17.91	0.42
	Level of insurance risks	1.20	32.17	1.75	4.59	0.33	12.74	47.22	0.42
	Adequacy of insurance reserves	5.57	4.29	6.08	6.76	2.72	3.68	70.90	0.42
	Insurance concentration	12.96	14.99	15.93	12.54	14.16	14.99	14.43	0.42
	Coefficient of dependence on reinsurance	1.67	3.76	0.80	1.09	53.01	38.86	0.81	0.42
2012	Profitability of insurance	16.90	14.13	11.90	13.92	13.73	12.93	16.49	0.46
	Level of insurance risks	1.35	34.06	5.85	10.84	0.96	19.63	27.31	0.46
	Adequacy of insurance reserves	8.64	10.16	13.00	7.52	9.42	8.74	42.52	0.46
	Insurance concentration	15.54	12.66	14.24	15.19	11.59	17.09	13.68	0.46
	Coefficient of dependence on reinsurance	7.19	13.52	14.18	14.70	16.00	26.75	7.65	0.46

Based on the data presented in Table 4, standardized regression equation will take the following form (Magnanti, Thomas L., 1989):

$$R = 0.0168 \times s_1 + 0.0073 \times s_2 - 0.0209 \times s_3 + 0.0188 \times s_4 + 0.0056 \times s_5 + 0.0036 \times s_6 + 0.0052 \times s_7. \quad (21)$$

The adequacy of the received calculations confirms the value of determination coefficient at 0.94. The relevant systematization of the structure of insurance portfolio of PJSC “TAS Insurance Group” and the integral indicator of the optimal portfolio structure – R is shown in Table 5.

Table 5. Information basis for building a standardized regression equation of dependence of generalized indicator of the optimal structure of portfolio (center of mass of the pentagon) on the shares of the i -th type of insurance for PJSC “TAS Insurance Group”

Year	Indicator	Type of insurance							Integral indicator R
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	
2014	Profitability of insurance	13.17	15.95	13.02	18.06	13.02	13.02	13.76	0.76
	Level of insurance risks	1.81	33.44	2.71	4.12	1.01	12.71	44.20	0.76
	Adequacy of insurance reserves	7.45	10.46	15.18	17.29	5.68	6.28	37.66	0.76
	Insurance concentration	13.36	14.57	11.96	12.33	14.31	18.64	14.84	0.76
	Coefficient of dependence on reinsurance	37.05	2.31	42.38	1.52	3.53	5.30	7.92	0.76
2013	Profitability of insurance	14.28	14.51	12.32	20.90	12.32	1.41	13.27	0.42
	Level of insurance risks	1.69	37.08	2.13	13.32	1.04	14.42	30.32	0.42
	Adequacy of insurance reserves	8.37	9.13	14.00	21.89	6.72	7.19	32.71	0.42
	Insurance concentration	14.47	12.40	14.47	19.05	13.02	12.40	14.20	0.42
	Coefficient of dependence on reinsurance	13.17	15.95	13.02	18.06	13.02	13.02	13.76	0.42

Standardized regression equation ($R^2 = 0.87$) for PJSC “TAS Insurance Group” will take the following form (Franco, G):

$$R = 0.0095 \times s_1 - 0.0072 \times s_2 - 0.0091 \times s_3 + 0.0036 \times s_4 - 0.0068 \times s_5 + 0.0071 \times s_6 + 0.0068 \times s_7. \quad (22)$$

Having the coefficients that characterize the types of insurance and transforming them into the structure indicator we obtain the value of a balanced insurance portfolio (Table 6).

Table 6. Optimal insurance portfolio for public joint-stock company “Oranta” and PJSC “TAS Insurance Group”

Types of insurance	PJSC "ORANTA"		PJSC "TAS Insurance Group"			
	Current value		Optimal value	Current value		Optimal value
Insurance of ground transportation	11.96		21.47	20.72		18.91
Insurance of other property	18.78		9.31	7.52		14.34
Car insurance (internal)	54.28		26.74	39.08		18.20
Green card	6.06		24.05	16.64		7.17
Voluntary medical insurance	2.14		7.23	8.80		13.54
Other private insurance	4.16		4.56	5.51		14.21
Other types of insurance	2.62		6.64	1.73		13.63
Total	100		100	100		100
Insurance of ground transportation	11.96	21.47	9.51	20.72	18.91	-1.81
Insurance of other property	18.78	9.31	-9.47	7.52	14.34	6.82
Car insurance (internal)	54.28	26.74	-27.54	39.08	18.2	-20.88
Green card	6.06	24.05	17.99	16.64	7.17	-9.47
Voluntary medical insurance	2.14	7.23	5.09	8.8	13.54	4.74
Other private insurance	4.16	4.56	0.4	5.51	14.21	8.7
Other types of insurance	2.62	6.64	4.02	1.73	13.63	11.9

The conducted optimization makes it possible to assert that in the present conditions of the insurance market in Ukraine, in particular, and the financial system of the country as a whole, the insurance portfolio of public joint-stock company “Oranta” and PJSC “TAS Insurance Group” is unbalanced. The most problematic for both companies is a significant share of mandatory civil liability of vehicle owners. In “Oranta”, for example, this type of insurance accounts for more than half of the total insurance portfolio. Therefore, despite the profitability and significant demand for this type of insurance, its share should be limited in order to preserve financial stability of the company.

At the same time, for PJSC “Oranta” a promising field of insurance is “Green Card”, while for PJSC “TAS Insurance Group” this type of insurance is threatening and its share should be reduced.

The adjustment of all other types of insurance at the two companies should occur within the range of 12%, with the balanced insurance portfolio of PJSC “TAS Insurance Group” being more uniform and the optimal insurance portfolio of PJSC “Oranta” focused on

property insurance. This trend can be explained by the specific nature of the objects of study. Thus, PJSC “TAS Insurance Group” is part of the financial and industrial group, which can insure specific industrial and financial risks, while PJSC “Oranta” is a classic insurance company that provides services to the general population.

Conclusions

It would be fair to note that the choice of relevant indicators of insurance types reflecting the criteria of optimality of insurance portfolio; regression analysis of the relationship between the relevant indicators and types of insurance, as well as determination of insurance portfolio optimization criterion on the basis of establishing the center of mass, creates scientifically grounded preconditions for making effective management decisions to balance the volumes of insurance of different types of risks. Consequently, the proposed scientific and methodical approach makes it possible for insurance companies in the developing countries to form an adequate system of financial stability.

References

1. Asanga, S. Portfolio optimization under solvency constraints: a dynamical approach in S. Asanga, A. Asimit, A. Badescu, S. Haberman. Access mode: <http://www.actuaries.org/ASTIN/Colloquia/Hague/Papers>.
2. Franco, G. Portfolio Optimization for Insurance Companies. Access mode: <http://www.air-worldwide.com/Publications/AIR-Current>.
3. MacKie-Mason J.K. (1992). Econometric software: A user's view, *Journal of Economic Perspectives*, 6 (4), pp. 165-187.
4. Magnanti, Thomas L. (1989) Twenty years of mathematical programming, *Contributions to Operations Research and Economics: The twentieth anniversary of CORE* (Papers from the symposium held in Louvain-la-Neuve, January 1987). Cambridge, MA: MIT Press, pp. 163-227.
5. Oyatoye, E.O. Optimization Models for Insurance Portfolio Optimization in the Presence of Background Risk in E.O. Oyatoye, K.K. Arogundade, *British Journal of Management & Economics*, Vol. 1 (2), pp. 114-127.
6. Pindyck, R., Rubinfeld, D. (1991). *Econometric models and economic forecasts*. Mc. Grow- Hill, Inc. USA, 596 p.
7. Rardin, Ronald L. (1997). *Optimization in operations research*. Prentice Hall. p. 919.
8. Kuzmenko, O.V., Kyrkach, S. (2014). The use of regression analysis in the financial planning of banks, mathematical formalization of the stages of financial planning in banks. Access mode: https://scholar.google.com.ua/citations?view_op=view_citation&hl=ru&user=Gpd0708AAAAJ&ccstart=20&citation_for_view=Gpd0708AAAAJ:YFjsv_pBGBYC.
9. Kozmenko, S.M., Lieonov, S.V. (2010). Rozvytok metodychnykh pidkhodiv do formalizatsii struktury vitchyznianoho finansovoho rynku, access mode: http://dspace.uabs.edu.ua/jspui/bitstream/123456789/1303/1/Leonov_14.pdf.
10. Boiko, A.O. (2011). Identyfikatsiia finansovykh potokiv strakhovoi kompanii, poviazanykh z upravlinniam finansovoiu stiikistiu strakhovyka. Access mode: http://dspace.uabs.edu.ua/jspui/bitstream/123456789/6177/1/176_boiko.pdf.
11. Roienko, V.V. (2011). Svitovi tendentsii investytsiinoi diialnosti strakhovykh kompanii .Access mode: http://lib.uabs.edu.ua/library/Zbirnik/1119_2011.pdf#page=323.
12. Pakhnenko, O.M. (2010). Finansovi metody upravlinnia strakhovymy ryzykamy na zovnishniomu strakhovomu rynku. Access mode: http://dspace.uabs.edu.ua/jspui/bitstream/123456789/6265/1/29_04_04.pdf.