Probabilistic behavior of well rate decrease laws through time. Universal dependence of Weibull–Makeham law

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Abstract

Analytical methods were used for engineering design of oil and gas fields in olden times (old fields). Nowadays numerical methods based on computer geological and mathematical models (new fields) have leading positions. While using numerical methods it is necessary to undergo a stage of history match to real condition with a switch to a live simulation model. With analytical methods a great part of information was not received, and there was no need in it. Nowadays it is impossible to build a computer geological and mathematical model and it is not of economic benefit. But in both cases concerning new fields as well as the old ones there is a need of additional study of the fields at the final stage of the development and assessment of reservoir behavior in future on the basis of development history statistical data.

Well rate (oil wells, gas wells, injection wells, water wells) changes with the pace of time due to different reasons, particularly due to reservoir conditions change, bottomhole zone damage as well as decrease of production equipment efficiency. This paper presents a universal law of well rate decrease using statistical laws of Makeham and Weibull and united Weibull–Makeham law according to which well rate decrease is characterized by four parameters which makes better description of well behavior possible. The deduced law is based on the law of probability. This law provides for getting logarithmic, harmonic, hyperbolic and parabolic laws, and in some cases several other laws can be deduced. The examples of law application are provided.

Key Words: *forecasting development, oil calculations, reservoir development*.

Introduction

In practice of reservoir engineering of "new" oil and gas fields rightful place is occupied by numerous methods, based on computer geological and mathematical models [1]. These methods ensure acquisition of acceptable (for the moment being) and the most accurate technological characteristics of hydrocarbon deposit development but only in case of complete and adequately accurate information which makes possible building of digitized likelihood geological model of this deposit. Such a model must be gradually specified for the moment of the next project document preparing [2]. The reason for this is that in the course of exploration, wide well spacing patterns are used, while for development, spacing patterns are a bit closer though also wide. Stroke of reservoir characteristics random changing of deposit is usually 5-10 times less than the distance between two offset exploration wells and equal to or half as large as the distance between exploration wells. Thus, as a result of drilling of each new well, model in the zone of this well can be drastically changed. Since in terms of reservoir study comprehensiveness, volume of drilling is usually a given value, then real geological model won't be built at all, instead of that we'll have its approximation [1]. So, model must be adapted according to the results of previous study of object under investigation with the transition to the new improved model, so we speak about constantly acting geological mathematical model on the basis of which project engineer and industrial worker can make stipulated engineering decisions, concerning design of works aimed at optimization of development process [2].

As for the "old" developed fields, there was no much information with satisfactory accuracy, besides there was no need to determine a number of parameters for analytical design [3], thus it is practically impossible and economically unsound to build a modern geological model for them. It's possible only to establish approximately actual picture of depletion of oil reserves and evaluate prospective characteristics of deposit development.

But in both cases, for "new" as well as for "old" fields there is a need for additional investigation of deposits on the final stage of development and evaluation of the reservoir behavior in future.

From this it follows that the issue of wells production rates forecasting is very urgent as well as forecasting of parameters of deposits development on the later stage both for the "new" and "old" fields on the basis of actual statistical data of fields development. After learning of history (retrospective) it's possible to formulate conclusions concerning further development of the main development parameters (perspective). At the same time peculiarities of the system and development technology being used are automatically taken into consideration.

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The analysis of modern investigations and publications

Statistical dependences of change of well production rate (group of wells, deposit, field) through time according to the actual exploitation data were built in a graphical form at the beginning of oil industry development, later with regard to production rate decrease period, mathematical dependencies were selected from the known mathematical formulas or derived on account of different physical (theoretical) considerations. These mathematical dependencies were as follows: depletion characteristics or in other words curves (laws) of exploitation, first exploitation curves, production decrease curves, chronological curves (L.S. Leibenzon, S.M. Czarnocki, K.G. Bell, Ch.S. Larki). Among them in practice of predicting of technological characteristics of oil fields development there are empirical (correlation) formulas of the following type [3, 4, 8, 14]:

$$q(t) = a(1+cft)^{-1/c}, \qquad (1)$$

$$q(t) = a(d+ft)^{-c}, \qquad (2)$$

$$q(t) = ae^{-bt}, \qquad (3)$$

$$q(t) = at^{b},$$

$$q(t) = \frac{a}{t},$$

$$q(t) = \frac{a}{b+t},$$

$$q(t) = ab^{ct},$$

$$q(t) = a + b^{ct},$$

$$q(t) = ab^{t}c^{t^2},$$

$$q(t) = \frac{a}{d+be^{-ct}},$$

$$q(t) = a + bt + ct^2 + dt^3 + \ldots,$$

where $q(t)$ is the current production rate; a, b, c, d, f are the fixed coefficients (constants), which are determined by correspondent treatment of actual data.

Change of current well production rate $q(t)$ in the course of time t is characterized by [5, 6]:

a) nominal speed of production rate decrease (or negative tangent of slope angle of line of dependence of ln q from time t)

$$D = -\frac{d\ln q}{dt} = -\frac{dq/dt}{q};$$

b) effective speed of production rate decrease (or relative production rate decrease during certain period – one month or one year)

$$D_e = \frac{q_0 - q_1}{q_0},$$

where q_0 is the initial well production rate for this period; q_1 is the production rate in a month (then monthly production rate decrease can be calculated) or in a year (analogically annual production rate decrease).

According to the nominal speed, three types of curves (laws) of production rate decrease [5, 6] with correspondent characteristics are singled out:

of equal percentage-based production rate decrease law (or law of logarithmic production rate decrease)

$$D = idem,$$

$$q = q_0 e^{-Dt}, \qquad (4)$$

$$Q_н = \frac{q_0 - q}{D};$$

of harmonic production rate decrease law

$$D = bq,$$

$$q = q_0 \frac{1}{1+bt}, \qquad (5)$$

$$Q_н = \frac{q_0}{D_{\text{orig}}} \ln\frac{q_0}{q},$$

where constant b is determined on the basis of initial conditions

$$b = \frac{D_{\text{orig}}}{q_0};$$

of hyperbolic production rate decrease law

$$D = bq^n,$$

$$b = \frac{D_{\text{orig}}}{q_0^n},$$

$$q = q_0 \frac{1}{(1+nbt)^{1/n}},$$

$$Q_н = \frac{q_0^n}{(1-n)D_{\text{orig}}}\left(q_0^{1-n} - q^{1-n}\right),$$

where n is the constant value; $Q_н$ is the accumulated volume of oil extraction.

These laws are derived on condition that nominal speed of production rate decrease D is correspondingly constant, proportional to current production rate and power function of current production rate.

Formulas (1), (2) and (3) describe cases of correspondingly hyperbolic ($0 \le c \le 1$), harmonic ($a = 1$, $c = 1$) and logarithmic production rate decrease, while $a = q_0$, $c = n$, $t = b$ and $t = D$ [6].

Nominal speed of production rate decrease as a continuous function can be used mainly to build different mathematical correlations, while on practice effective speed of production rate decrease D_e is mostly used. Then, re-calculation can be done according to the written laws in the following way:

$$D = -\ln(1 - D_e),$$

$$D_{\text{orig}} = \frac{D_{\text{e orig}}}{1 - D_{\text{e orig}}},$$

$$D_{\text{orig}} = \frac{1}{n}\left[(1 - D_{\text{e orig}})^{-n} - 1\right].$$

To determine production rate decrease speed these curves are smoothed out by rebuilding them in correspondent coordinates.

In case of equal percentage production rate decrease ln dependencies $q - t$ and $q - Q_н$ are built; in both cases straight line slope ration is equal to nominal speed D.

In case of harmonic production rate decrease curve $q - Q_н$ becomes a straight line in semilogarithmic coordinates ($\ln q - Q_н$), and nominal speed is equal to production rate multiplied by tangent of slope angle of line. Dependence of reciprocal value of production rate $1/q$ from time t, is also expressed by a straight line.

As to hyperbolic production rate decrease, $q - t$ and $q - Q_н$ correlations can become straight-line after inversion - x-shift in logarithmic scale; for curves of hyperbolic production rate decrease special coordinate paper with charts is suggested which allows for plotting either time or accumulated volume in linear scale and obtaining straight-line correlations.

The analysis of a large number of actual production rate decrease curves has shown [6] that most curves belong to hyperbolic type with power exponent n in the range of 0–0.7 (most of all from 0 to 0.4) and this can be connected with the fact that decrease law unlike the other two mentioned is more complicated and characterized by two parameters. Curves with harmonic production rate decrease (n = 1) are very rare.

Clearly, the task is to define one of the two unknown values [6]: either residual productive time or ROIP (Remaining Oil In Place), that's why productive time or accumulated volume of oil extraction are chosen as an independent variable and are plotted against x-axis. Meanwhile dependent variable must have a known end point and an end point of curve is defined on the basis of the known or foreseen production expenses as an economically recoverable production rate. To determine the unknown value, production rate decrease dependence is extrapolated to the end point.

Production rates of wells (oil, gas, injection, water supply) are changed in the course of time due to different reasons, such as:

change of downhole pressure, gas factor (especially in case of dissolved gas regime in the deposit), water encroachment into oil and gas, gas accumulation in a part of reservoir pore spaces or water fillup, deposit energy drain, in other words change of reservoir conditions;

change of productivity index degree of excellence (quality) of well as a result of change of physical conditions at the bottomhole or in bottomhole zone, e.g. paraffin colmatation of the bottomhole zone and paraffin accumulation in wellbore, formation of gas hydrates in the bottomhole zone and wellbore of a gas well, liquid accumulation (water, gas condensate) at the bottomhole of oil and gas wells, colmatation of the bottomhole zone with mechanical parts, deposition of salts or asphaltens from the produced liquid, accumulation of quick sand, aleurite or clay (formation of sand clay plugs), formation of caverns in bottomhole zone as a result of damage to the bottomhole zone;

decline of performance or effectiveness of work of production equipment which ensures lifting of the output (oil, water, gas, gas condensate) on the surface.

If we are interested in the forecast of cumulative oil production and determination of residual oil deposits, then production rate decrease curve must represent only the change of reservoir conditions. That means that production equipment (gaslift valves, pumps. pipes etc.) must be correctly adjusted and bottomhole zone quality must be ensured. In other cases only those deposits can be evaluated which can be produced only in the existing and mostly not effective bottomhole zone conditions and equipment state. Production rate decrease curve will represent all the mentioned reasons for production rate change, that are not easy to be defined. But often it's not possible to describe character of well work adequately with the help of these formulas.

Paper objective

To find on the basis of probability theory provisions general dependence (law) of well production rate decrease in the course of time, taking into account all the mentioned reasons for production rate change simultaneously.

Coverage of the main material of the study

Let's assume that a part of cross section of oil and gas flow at a certain distance ξ from well and at a certain time t is disturbed according to the above mentioned reasons (e.g. because of gas accumulation or paraffin colmatation to any extent) then overall oil and gas flow consumption to the well

$$q(t) = q_0 P(t) + q_1 Q(t),$$

where q_0 is the flow consumption in non colmatated part of cross section; q_1 is the flow consumption in partially colmatated cross section part at any time t; $P(t)$ is the reliability function [7] of colmatation absence; $Q(t)$ is the probability of cross section colmatation at the moment t, while $Q(t) = 1 - P(t)$. Here any of the mentioned reasons for production rate decrease can be named, correspondingly rephrasing text.

Function of reliability of colmatation absence is formulated by Makeham law, which is well-known in probability theory [9]:

$$P_1(t) = e^{-\int_0^t \delta(t)dt},$$

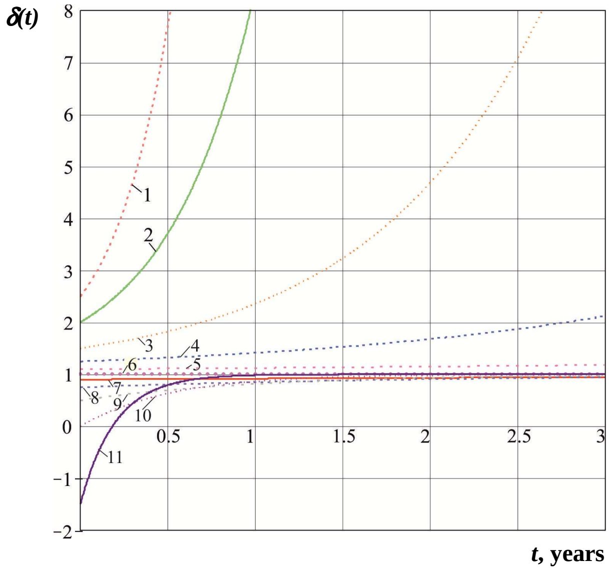
failure risk – function $\lambda_1(\xi)$ – is written as:

$$\delta(t) = \alpha + \gamma\beta e^{\gamma t}, \quad (6)$$

where α, β, γ are the constant values (coefficients).

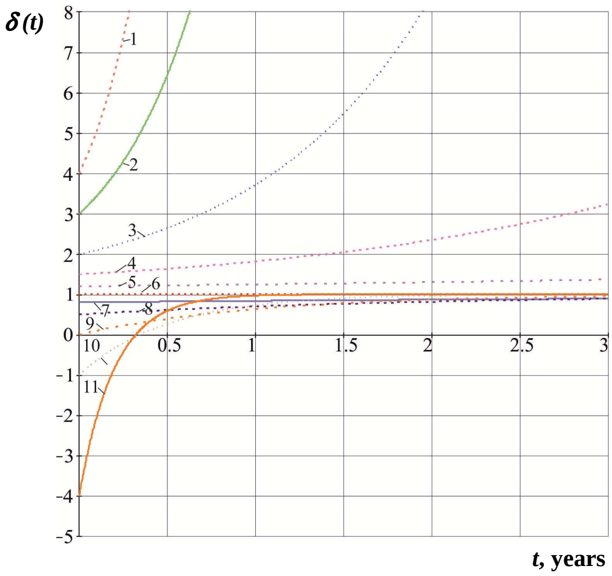
The analysis shows that parameter $\Phi\big|_{L^*} = \Phi(P^*) = \Phi^*$ according to Makeham law (6) grows linearly with growth of α and β, and character of change $\delta(t)$ depending on γ and t for other set values is shown in Fig. 1–3. Meanwhile for dependence $\tilde{\rho}(\Phi) = \rho(p(\Phi))$ minimum of function will be in point $\gamma = t^{-0.5}$, horizontal line $t = \alpha$ is an asymptote, if $\gamma << 0$. To build graphs let's assume that α = 1, and change of α leads to shift of the beginning of graphs building along Y-axis. Influence of β at acceptable values is not sufficient.

Makeham assumption means that the process takes place under the influence of two constituents: a) the first



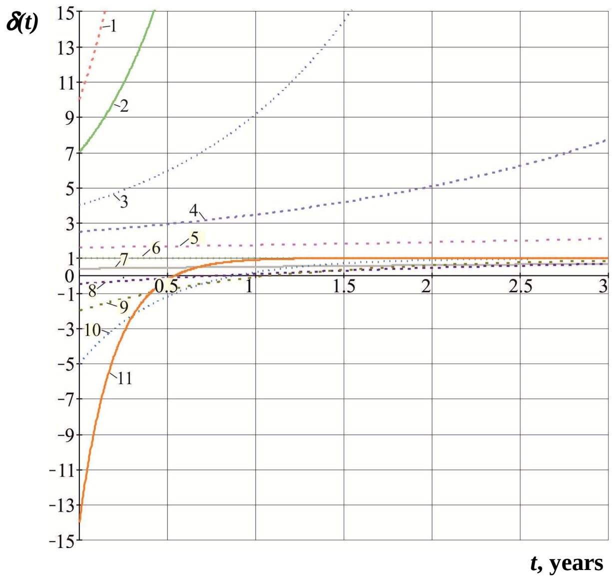
1 – 3; 2 – 2; 3 – 1; 4 – 0.5; 5 – 0.2;
6 – 0 (accordingly δ(t) = α = 1);
7 – -0.2; 8 – -0.5; 9 – -1; 10 – -2; 11 – -5

Figure 1 – Failure function δ(t) with β = 0,5 and different γ



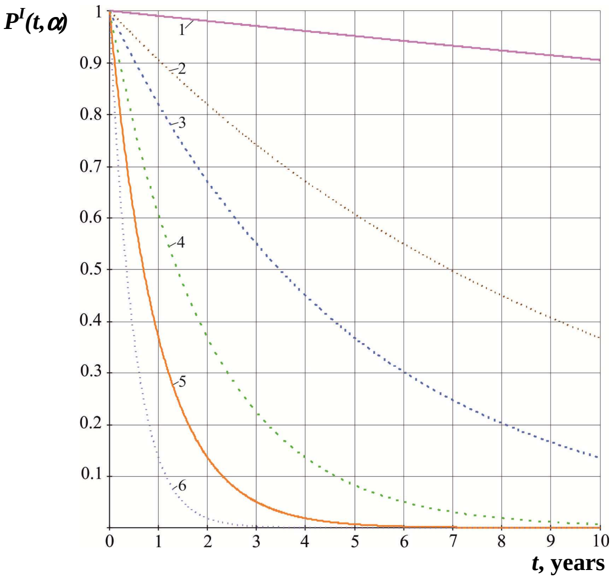
1 – 3; 2 – 2; 3 – 1; 4 – 0.5; 5 – 0.2;
6 – 0 (accordingly δ(t) = α = 1);
7 – -0.2; 8 – -0.5; 9 – -1; 10 – -2; 11 – -5

Figure 2 – Failure function δ(t) with β = 1 and different γ



1 – 3; 2 – 2; 3 – 1; 4 – 0.5; 5 – 0.2;
6 – 0 (accordingly δ(*t*) = α = 1);
7 – -0.2; 8 – -0.5; 9 – -1; 10 – -2; 11 – -5

Figure 3 – Failure function δ(*t*) with β = 3 and different γ



1 – 0.01; 2 – 0.1; 3 – 0.2; 4 – 0.5; 5 – 1; 6 – 2

Figure 4 – Reliability function $P^I(t, \gamma)$ (lack of colmatation) from Makeham's law when α = 0, β = 1 and different γ

one, not depending on ξ, and b) the second one, depending on space coordinate ξ, which is growing (if $\gamma > 0$) or falling (if $\gamma < 0$) exponentially with variable ξ (if $\gamma << 0$ value $\lambda_1(\xi)$ is growing again and approaching an asymptote $\alpha = idem$).

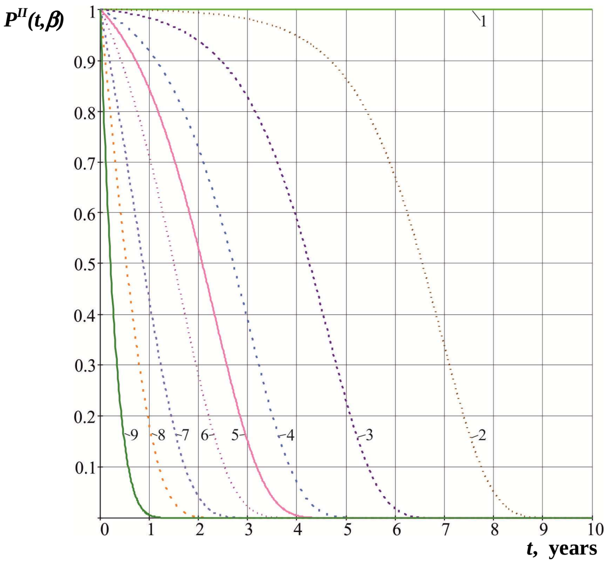
Temperature in the bottomhole zone is adequate to pressure according to choke effect formula (Joule-Thomson effect) [10]. It's known that if in case of established filtration of oil into well, transition from isobar to isobar is carried out in an arithmetic progression, then transition from radius to radius is done in geometric progression [12]. Clearly the same is for temperature; its change can be considered with the help of coefficient γ.

Then Makeham law as to time coordinate *t* can be written as:

$$P(t) = e^{-\alpha t - \beta\left(e^{\gamma t} - 1\right)},$$

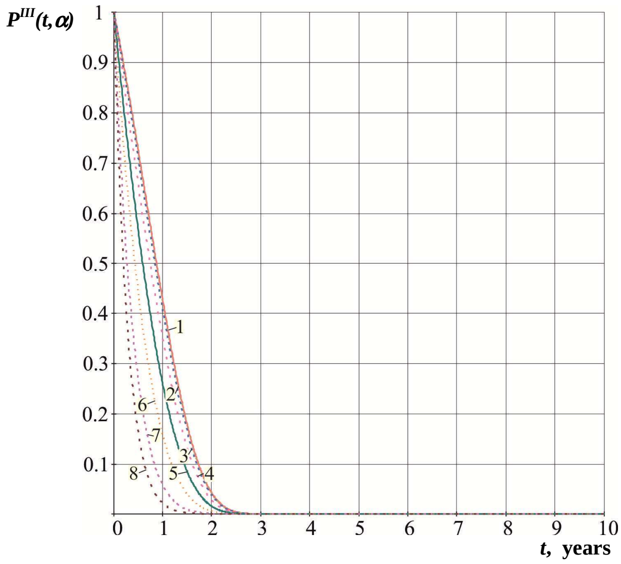
meanwhile exponential law is a specific case of this law if $\gamma = 0$.

Dependencies $P^{I}(t,\alpha)$, $P^{II}(t,\beta)$, $P^{III}(t,\alpha)$ and $P^{IV}(t,\alpha,\beta)$ are represented in a graphical form in Fig. 4–7. From this it follows that all lines emerge from point $t = 0$. With the increasing of α (if $\beta = 0$) and β



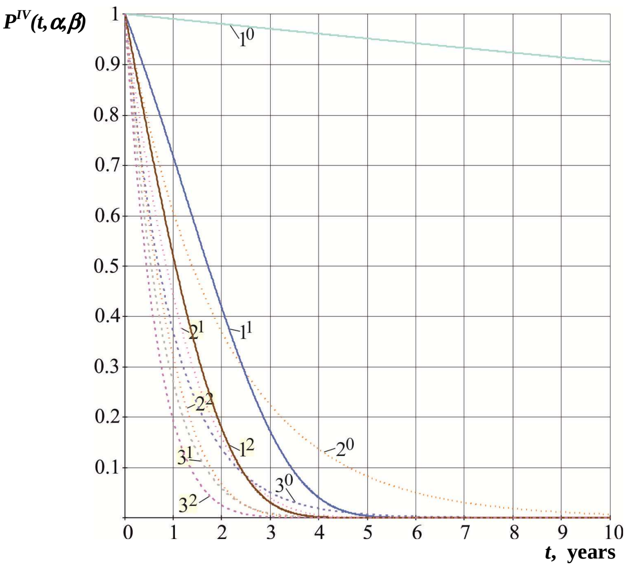
1 – 0; 2 – 0.001; 3 – 0.01; 4 – 0.05; 5 – 0.1;
6 – 0.2; 7 – 0.5; 8 – 1; 9 – 3

Figure 5 – Reliability function $P^{II}(t, \beta)$ from Makeham's law when $\alpha = 0$, $\gamma = 1$ and different β



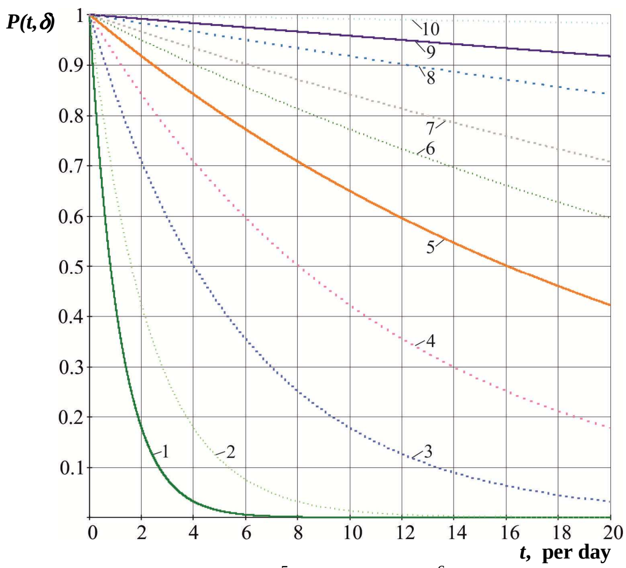
1 – 0; 2 – 0.01; 3 – 0.05; 4 – 0.2;
5 – 0.5; 6 – 1; 7 – 2; 8 – 3

Figure 6 – Reliability function $P^{III}(t, \alpha)$ from Makeham's law when $\beta = 0{,}5$, $\gamma = 1$ and different α



superscript 0 – β = 0,
line: 10 – α = 0.01; 20 – α = 0.5; 30 – α = 1;
superscript 1 – β = 0.5;
line: 11 – α = 0.01; 21 – α = 0.5; 31 – α = 1;
superscript 2 – β = 1,
line: 12 – α = 0.01; 22 – α = 0.5; 32 – α = 1

Figure 7 – Reliability function $P^{IV}(t, \alpha, \beta)$ from Makeham's law when $\gamma = 0.5$ and different α and β



1 – $\delta = 10^{-5}$; 2 – $\delta = 5\cdot10^{-6}$;
3 – $\delta = 2\cdot10^{-6}$; 4 – $\delta = 1\cdot10^{-6}$;
5 – $\delta = 5\cdot10^{-7}$; 6 – $\delta = 2\cdot10^{-7}$;
7 – $\delta = 1\cdot10^{-7}$; 8 – $\delta = 1\cdot10^{-7}$;
9 – $\delta = 5\cdot10^{-8}$; 10 – $\delta = 1\cdot10^{-8}$

Figure 8 – 3 Lack of colmatation probability dependence $P(t, \delta)$ on time t under different δ

(if $\alpha = 0$) reliability function is decreasing (see Fig. 4 and 5), but influence of β is greater (if $\gamma = 1$). If $\beta = 0.5$ at $\gamma = 1$ and different α, then reliability function is drastically decreasing with the increase of α (see Fig. 10). Common influence of α and β (if $\gamma = 0.5$) is of a different character (see Fig. 7).

Thus, functions of reliability and colmatation probability in space ξ and time t coordinates correspondingly are expressed in accordance with extended Makeham exponential law:

$$P(\xi)=e^{-\alpha\xi-\beta\left(e^{\gamma\xi}-1\right)},\quad Q(\xi)=1-P(\xi),$$

$$P(t)=e^{-\alpha_1 t-\beta_1\left(e^{\gamma_1 t}-1\right)},\quad Q(t)=1-P(t),$$

where α, β, γ, α_1, β_1, γ_1 is the correspondent constant parameters of law as for ξ and t.

Then, liquid flow consumption for any cross section with coordinate ξ at an arbitrary point of time t is written as:

$$q(\xi,t)=q_0P(\xi)+[q_0P(t)+q_2Q(t)]Q(\xi)$$

or in an expanded form:

$$q(\xi,t)=q_0\left[e^{-\alpha\xi-\beta\left(e^{\gamma\xi}-1\right)}+e^{-\alpha_1 t-\beta_1\left(e^{\gamma_1 t}-1\right)}\left(1-e^{-\alpha\xi-\beta\left(e^{\gamma\xi}-1\right)}\right)\right]+$$
$$+q_2\left(1-e^{-\alpha_1 t-\beta_1\left(e^{\gamma_1 t}-1\right)}\right)\left(1-e^{-\alpha\xi-\beta\left(e^{\gamma\xi}-1\right)}\right).$$

Similarly probability of absence of capillar colmatation with time *t in* cross section with fixed space coordinate and colmatation probability:

$$P(T>t)=P(t)=e^{-\int_0^t \delta(t)dt},$$
$$P(T\le t)=Q(t)=1-P(t),$$

where δ is the value, analogical to λ, besides in this case distribution law may be also drawn in accordance with Makeham law Character of change $P(t, \delta)$ if δ = idem is presented in Fig. 8.

Herefrom we get well production rate formula (if $\xi = 0$) at any current time *t*:

$$q(t)=q_0 e^{-\alpha_2 t-\beta_2\left(e^{\gamma_2 t}-1\right)}+q_1\left(1-e^{-\alpha_2 t-\beta_2\left(e^{\gamma_2+t}-1\right)}\right) \text{ or}$$

$$q(t)=q_0\left[(1-\psi)e^{-\alpha t-\beta\left(e^{\gamma t}-1\right)}+\psi\right],$$

where $\psi = q_2/q_0$.

Let's assume that after colmatation of a part of flow cross section, its consumption becomes equal to zero, that is $q_2 = 0$ and then $\psi = 0$. Then there is new equation of change of well production rate over *t* according to Makeham law:

$$q(t)=q_0 e^{-\alpha t-\beta\left(e^{\gamma t}-1\right)}. \quad (7)$$

If it's assumed that parameter $\gamma = 0$, then from (7) there is a specific case known exponential (logarithmic) law of well production rate change (or law of equal percentage-based production rate decrease):

$$q(t)=q_0 e^{-\alpha t},$$

where $\alpha = D$ in formula (4).

If an exponent in the last expression is expanded into a series and the first two terms of a series are singled out then there is a function $(1+\alpha_1 t)^{-1}$ and known law of harmonic production rate change:

$$q(t)=q_0\frac{1}{1+\alpha t},$$

where $\alpha = b$ in formula (5).

The same expression can be obtained after expanding into a series of both exponents in an extended expression $q(t)$ in accordance with Makeham law, that is

$$e^{-\left[\alpha t-\beta\left(e^{\gamma t}-1\right)\right]}\cong 1-\left[\alpha t+\beta\left(e^{\gamma t}-1\right)\right]\cong$$
$$\cong 1-\alpha t-\beta(1+\gamma t-1)=1-(\alpha+\beta\gamma)\,t$$
$$\text{or}\quad e^{-\alpha t-\beta\left(e^{\gamma t}-1\right)}\cong\frac{1}{1+(\alpha+\beta\gamma)t},$$

where $\alpha+\beta\gamma=b$.

In case of hyperbolic law of production rate decrease reliability function must be written in the following way:

$$P(t)=e^{-(1+\tau\alpha' t)^{1/\tau}},$$

and then well-known law of hyperbolic production rate decrease (after exponent expanding into a series) is as follows:

$$q(t)=q_0\frac{1}{(1+\tau\alpha' t)^{1/\tau}}, \quad (8)$$

and if $\tau = 1$, it can be reduced to the law of harmonic production rate change t (if $\alpha' = b$), therefore to equal percentage-based (if $\alpha = D$), and to production rate change according to Makeham law, where r, α', τ are the constant parameters.

If we use Weibull law as a generalized exponential law

$$P(t)=e^{-\alpha t^{\zeta}},$$

then formula of well production rate decrease according to Weibull law is written as follows:

$$q=q_0 e^{-\alpha t^{\zeta}},$$

if parameter $\zeta = 1$, it is transformed into exponential law formula.

Here nominal speed of production rate decrease is expressed as:

$$D=-\alpha\zeta t^{\zeta-1}.$$

If an exponent is expanded into a series and only two terms of a series are singled out

$$e^{-\alpha t^{\zeta}}\cong 1-\alpha t^{\zeta},$$

then known parabolic law of change of production rate of sucker-rod pump well [2]:

$$q=q_0\left(1-\alpha t^{\zeta}\right),$$

where $\alpha_1 = T_{np}^{-\zeta}$; $T_{пр}$ is the theoretical duration of work of sucker-rod pumping unit before liquid binding (if the reason for binding-plunger and barrel assembly wear then $T_{пр}$ means full physically feasible service life of pump); ζ is the power exponent of parabola (values of ζ are in the range of 1–3, mostly $\zeta = 2$).

From this it follows that assuming the validity of the Weibull law if $\zeta = 1$ parabolic law of production rate decrease is transformed into exponential law or in harmonic law of production rate decrease (because $1/(1+\alpha t)\cong 1-\alpha t$).

Investigations of well work [11, 13] testify that parabolic dependence describes production rate change, caused by equipment work and parabolic and exponential dependencies describe production rate change connected with change of reservoir parameters.

If we combine Makeham and Weibull laws, then we'll get a new law, called Weibull-Makeham law:

$$P(t)=e^{-\alpha t^{\zeta}-\beta\left(e^{\gamma t}-1\right)}.$$

This formula contains four constant parameters. For the convenience of their determination, extrapolation of curve $q - t$ can be performed in semilogarithmic coordinates ($\ln q - t$) which corresponds to the law of equal percentage-based production rate decrease and then by selection of other parameters ensure coincidence of actual data (points) of production rate with calculating line especially flattening of a final segment of production rate decrease curve on a graph.

The following approaches to parameters selection can be used as the transitional ones:

$$\left(\ln\frac{q}{q_0}\right)^{1/\zeta} = -\alpha' t ,$$

$$\ln\frac{q}{q_0} = -(\alpha+\beta\gamma)t ,$$

$$\frac{q}{q_0} = 1-(\alpha+\beta\gamma)t ,$$

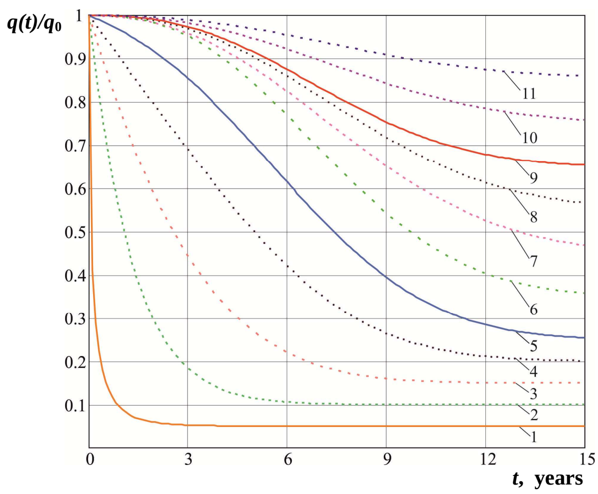
where

$\alpha' = (\alpha)^{1/\zeta}$; $\gamma_1 t \cong e^{\gamma_1 t} - 1$; $e^{-\left[\alpha+\beta\left(e^{\gamma t}-1\right)\right]} \cong 1-(\alpha+\beta\gamma)t$.

Taking this into account (8), at the end universal dependence of well production rate decrease in the course of time is written according to Weibull–Makeham law:

$$q(t) = q_0\left[(1-\psi)e^{-\alpha t^{\zeta} - \beta\left(e^{\gamma t}-1\right)} + \psi\right]. \qquad (9)$$

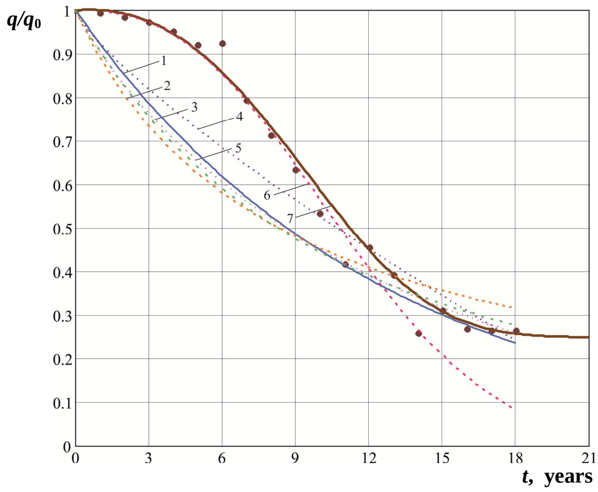
Character of change $q(t)/q_0$ over time t at different values ψ, α, ζ, β, γ is shown in Fig. 9. From here it follows that practically all possible changes of production rate can be described with the help of this dependence.



1 – ψ = 0.05, α = 3, ζ = 0.5, β = 2, γ = 0.1;
2 – ψ = 0.15, α = 0.005, ζ = 1, β = 3, γ = 0.1;
3 – ψ = 0.1, α = 0.5, ζ = 1, β = 2,5, γ = 0.1;
4 – ψ = 0.2, α = 0.005, ζ = 2.5, β = 2,5 γ = 0.1;
5 – ψ = 0.25, α = 0.005, ζ = 2.5, β = 3, γ = 0.015;
6 – ψ = 0.35, α = 0.005, ζ = 2.5, β = 1, γ = 0.0001;
7 – ψ = 0.45, α = 0.005, ζ = 2.4, β = 30, γ = 0.0001;
8 – ψ = 0.55, α = 0.005, ζ = 2.5, β = 10, γ = 0.0001;
9 – ψ = 0.65, α = 0.005, ζ = 2.5, β = 1, γ = 0.0001;
10 – ψ = 0.75, α = 0.005, ζ = 2.4, β = 10, γ = 0.0001;
11 – ψ = 0.855, α = 0.005, ζ = 2.4, β = 10, γ = 0.0001

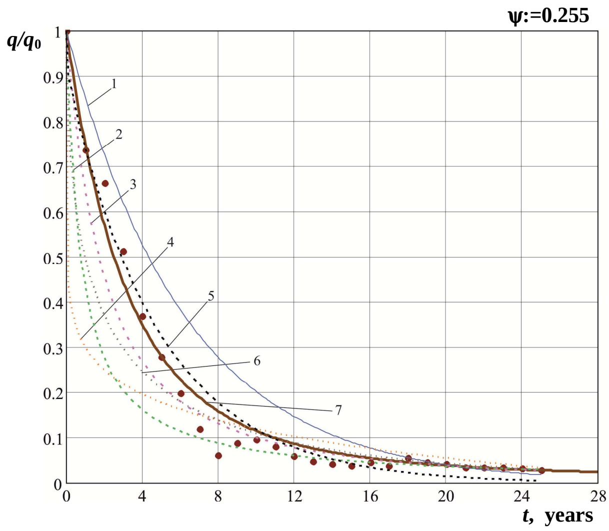
Figure 9 – Universal dependence of well rate decrease through time by Weibull-Makeham law under different ψ, α, ζ, β, γ

For the illustration of derived dependencies of well production rate decrease in the course of time for one well-day we have selected two different graphical dependencies, built upon actual data from Pnivske (Fig. 10) and Tanyavske (Fig. 11) oil fields.



1 – logarithmic law; 2 – harmonic law;
3 – hyperbolic law; 4 – parabolic law; 5 – Weilbull law;
6 – Makeham law; 7 – Weibull–Makeham law

Figure 10 – Actual relative data of Pnivske oilfield annual well rate decrease q/q0 in time t (dotted) and different relevant theoretical dependencies



1 – logarithmic law; 2 – harmonic law;
3 – hyperbolic law; 4 – parabolic law; 5 – Weilbull law;
6 – Makeham law; 7 – Weibull–Makeham law

Figure 11 – Actual relative data of Tanyavske oilfield annual well rate decrease q/q0 in time t (dotted) and different relevant theoretical dependencies

As for Pnivske field the best one was the law of production rate decrease (universal dependence) on the basis of Weibull–Makeham law. As for Tanyavske field, equation of production rate decrease according to Weibull–Makeham law (correlation relation σ = 0.991) was the most applicable. In the first case the following coefficients have been selected: α_1 = 0.079, β_1 = 0.052, γ_1 = 0.14, ψ = 0.25, and in the second case – α_1 = 0.005 β_1 = -4.3, γ_1 = -0.07, ψ = 0.5. High correlation relations have been expected because from other universal dependencies other formulas are derived (as specific cases).

Going beyond retrospective, production rate forecast in the course of time can be obtained.

Thus, Weibull–Makeham law to the fullest possible extent accounts for complicating factors, which cause decrease of well production rate in the course of time and gives the possibility to stipulate probabilistic universal law of well production rate change in the course of time, out of which as specific cases the most widely spread production rate decrease laws are derived as well as other dependencies in case of certain assumptions.

Conclusion

The main formulas for oil wells development parameters forecast are for the time being laws based on the fact that nominal speed of production rate decrease is correspondingly constant, proportional to the current production rate and power function of current production rate, logarithmic, parabolic and hyperbolic production rate decrease. But it's a well-known fact that production rate of wells is changed in the course of time. In this paper the authors have suggested derivation based on provisions of probability theory, universal law of production rate decrease in the course of time with the use of combined statistical Weibull–Makeham law. That provides the possibility for better description of well performance. To prove this, the examples of universal law application in Precarpathian fields have been provided.

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Ймовірнісний характер законів зменшення дебітів свердловин у часі. Універсальна залежність за законом Вейбулла–Макегама

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Для проектування розробки нафтових і газових родовищ у свій час ("старі" родовища) застосовували аналітичні методики, а в даний час гідне місце зайняли чисельні методи на основі комп'ютерних геолого-математичних моделей ("нові" родовища). Із застосуванням чисельних методів обов'язковим є етап адаптації моделі до фактичного стану з переходом на постійно діючу модель покладу, а із застосуванням аналітичних методів багато інформації не було одержано, та й не було потреби в ній. Сьогодні побудувати комп'ютерну геолого-математичну модель неможливо і економічно це збитково. Але в обох випадках як по "нових", так і по "старих" родовищах виникає потреба в додатковому вивченні покладів на завершальній стадії розробки та оцінки характеру поведінки їх у майбутньому за фактичними статистичними даними історії розробки.

Дебіт свердловин (нафтових, газових, нагнітальних, водозабірних) змінюється в часі з різних причин, зокрема внаслідок зміни пластових умов, пошкодження привибійної зони, а також і зниження ефективності роботи експлуатаційного обладнання. У даній роботі на основі положень теорії ймовірностей виведено залежності (закони) падіння дебіту в часі з використанням статистичних законів Макегама і Вейбулла, а також універсальну залежність на основі об'єднаного закону Вейбулла–Макегама, за якою падіння дебіту характеризується п'ятьма параметрами, що покращує опис роботи свердловин. Із цієї залежності, як окремі випадки, одержуємо відомі логарифмічний, гармонічний, гіперболічний та параболічний закони, а за певних припущень й інші залежності. Наведено приклади застосування цієї залежності.

Ключові слова: *підрахунок запасів, прогнозування розробки, тривалість розробки.*