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Propagation of Singularities for Large Solutions of Quasilinear Parabolic Equations

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The quasilinear parabolic equation with an absorption potential is considered:

$$\left(|u|^{q-1}u\right)_t - \Delta_p(u) = -b(t,x)|u|^{\lambda-1}u \quad (t,x) \in (0,T)\times\Omega, \quad \lambda > p > q > 0,$$

where Ω is a bounded smooth domain in R^n, $n \geqslant 1$, b is an absorption potential which is a continuous function such that $b(t,x) > 0$ in $[0,T)\times\Omega$ and $b(t,x) \equiv 0$ in $\{T\}\times\Omega$. In the paper, the conditions for $b(t,x)$ that guarantee the uniform boundedness of an arbitrary weak solution of the mentioned equation in an arbitrary subdomain $\Omega_0 : \overline{\Omega}_0 \subset \Omega$ are considered. Under the above conditions the sharp upper estimate for all weak solutions u is obtained. The estimate holds for the solutions of the equation with arbitrary initial and boundary data, including blow-up data (provided that such a solution exists), namely, $u = \infty$ on $\{0\}\times\Omega$, $u = \infty$ on $(0,T)\times\partial\Omega$.

Key words: partial differential equations, quasilinear parabolic equation, degenerate absorption potential, large solution.

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1. Introduction and formulation of main results

Let Ω be a bounded domain in R^n, $n \geqslant 1$, with C^2-smooth boundary $\partial\Omega$. We will consider the following quasilinear parabolic equation in the cylindrical domain $Q = (0,T)\times\Omega$, $0 < T < \infty$:

$$\left(|u|^{q-1}u\right)_t - \sum_{i=1}^{n}(a_i(t,x,u,\nabla u))_{x_i} = -b(t,x)|u|^{\lambda-1}u \quad \text{in } Q,\ \lambda > p > q > 0, \tag{1.1}$$

where $a_i(t,x,s,\xi)$, $i = 1,2,\dots,n$, are continuous functions satisfying the coercitivity and growth conditions:

$$d_0|\xi|^{p+1} \leqslant \sum_{i=1}^{n} a_i(t,x,s,\xi)\xi_i; \quad \forall (t,x,s,\xi) \in \bar{Q}\times R^1\times R^n; \tag{1.2}$$

$$|a_i(t,x,s,\xi)| \leqslant d_1|\xi|^p; \quad \forall (t,x,s,\xi) \in \bar{Q}\times R^1\times R^n;\ i = 1,\dots,n; \tag{1.3}$$

where $d_0 = \text{const} > 0$, $d_1 = \text{const} < \infty$. Thus, the model representative of equation (1.1) is the equation

$$\left(|u|^{q-1}u\right)_t - \Delta_p u = -b(t,x)|u|^{\lambda-1}u, \tag{1.4}$$

where $\Delta_p(u) = \sum_{i=1}^n \left(|\nabla_x u|^{p-1}u_{x_i}\right)_{x_i}$.

The function $b(t,x)$ (the absorption potential) is a continuous function in $[0,T]\times\overline{\Omega}$ satisfying the conditions:

$$b(t,x) > 0 \quad \text{in } [0,T)\times\overline{\Omega}, \qquad b(t,x) = 0 \quad \text{on } \{T\}\times\Omega. \tag{1.5}$$

In the paper, we consider weak solutions of equation (1.1). In this context let us introduce the following definition.

Definition 1.1. A function $u(t,x) \in C_{\text{loc}}((0,T); L^{q+1}(\Omega))$ is called *a weak (energy) solution* of equation (1.1) if:

$$u(t,x) \in L^{p+1}_{\text{loc}}\left((0,T); W^{1,p+1}_{loc}(\Omega)\right) \cap L^{\lambda+1}_{\text{loc}}\left((0,T)\times\Omega\right),$$
$$\left(|u|^{q-1}u\right)_t \in L^{\frac{p+1}{p}}_{\text{loc}}\left((0,T); (W^{1,p+1}_c(\Omega))^*\right) + L^{\frac{\lambda+1}{\lambda}}_{\text{loc}}\left((0,T); (L^{\lambda+1}_c(\Omega))^*\right),$$

and the integral identity

$$\int_a^b \left\langle \left(|u|^{q-1}u\right)_t, \eta \right\rangle dt + \int_a^b \int_\Omega \left[\sum_{i=1}^n a_i(t,x,u,\nabla u)\eta_{x_i} + b(t,x)|u|^{\lambda-1}u\eta\right] dxdt = 0 \tag{1.6}$$

holds for an arbitrary $0 < a < b < T$ and an arbitrary

$$\eta(t,x) \in L^{p+1}_{\text{loc}}\left((0,T); W^{1,p+1}_c(\Omega)\right) \cap L^{\lambda+1}_{\text{loc}}\left((0,T); L^{\lambda+1}_c(\Omega)\right),$$

where $W^{1,p+1}_c(\Omega)$, $L^{\lambda+1}_c(\Omega)$ are subspaces of $W^{1,p+1}(\Omega)$, $L^{\lambda+1}(\Omega)$ of the functions with a compact support in Ω, and $\langle\cdot,\cdot\rangle$ is the pairing of elements from $W^{1,p+1}_c(\Omega)\cap L^{\lambda+1}_c(\Omega)$ and $\left(W^{1,p+1}_c(\Omega)\cap L^{\lambda+1}_c(\Omega)\right)^*$.

Definition 1.2. A function $u(t,x)$ is called *a large solution* of equation (1.1) if it is a weak solution of equation (1.1) and satisfies the following singular initial and boundary values:

$$u = \infty \text{ on } \{0\}\times\Omega, \quad \text{i.e. } u \to \infty \text{ as } t \to 0 \quad \text{uniformly } \forall\, x \in \Omega, \tag{1.7}$$
$$u = \infty \text{ on } (0,T)\times\partial\Omega, \quad \text{i.e. } u \to \infty \text{ as } d(x,\partial\Omega) \to 0 \quad \text{uniformly } \forall\, t \in (0,T). \tag{1.8}$$

The existence of *weak solutions* of such equations as (1.1) with arbitrary finite initial and boundary values was proven in the papers published in the 1980s and 1990s, see, for example, [1,6,7].

The existence and properties of *large solutions* were studied by many authors, namely L. Veron, W. Al Sayed, C. Bandle, G. Diaz, J.I. Diaz, Y. Du, R. Peng, P. Polaĉik and others (see [2, 3, 9, 10, 16] and references therein). There were considered linear ($p = q = 1$) or semilinear ($q = 1$) equations. For the general case of equation (1.1) with two nonlinearities ($p \neq 1$ and the $q \neq 1$), the existence of a large solution has not been proven yet. In the current paper, the question of the existence of large solutions of general equation (1.1) is not affected. However, the main result is obtained for a whole class of weak solutions including large solutions (if any), see also Remark 1.5.

Now let us consider the most interesting for us results. For the case of $p = q = 1$, equation (1.4), namely

$$u_t - \Delta u = -b(t,x)|u|^{\lambda-1}u, \quad (t,x) \in Q, \quad \lambda > 1, \tag{1.9}$$

with conditions (1.5), (1.7), and (1.8) was studied in [3]. The following condition for the absorption potential $b(t,x)$ was considered:

$$a_1(t)d(x)^\beta \leqslant b(t,x) \leqslant a_2(t)d(x)^\beta, \quad \forall (t,x) \in [0,T] \times \Omega, \ \beta > -2, \tag{1.10}$$

where $a_1(t)$, $a_2(t)$ are positive continuous on $[0,T)$ functions, $d(x) := \mathrm{dist}\{x, \partial\Omega\}$. Under condition (1.10), the existence of maximal $\overline{u}$ and minimal $\underline{u}$ positive solutions of the problem under consideration was proved. Moreover, the main result of [3] says that under the following additional condition on the degeneration of $a_1(t)$ near $t = T$:

$$a_1(t) \geqslant c_0(T-t)^\theta \quad \text{ in } [0,T), \ c_0 = \mathrm{const} > 0, \ \theta = \mathrm{const} > 0, \tag{1.11}$$

for any $t_0 \in (0,T)$, there exists $C = C(t_0) < \infty$ such that

$$\overline{u}(t,x) \leqslant C \min\left\{(T-t)^{-\frac{\theta}{\lambda-1}}, d(x)^{-\frac{2\theta}{\lambda-1}}\right\} d(x)^{-\frac{2+\beta}{\lambda-1}}, \ \forall (t,x) \in [t_0,T)\times\Omega. \tag{1.12}$$

At the same time, an open question was whether condition (1.11) was a sharp condition of boundedness of solution u.

The answer to this question was found in [11]. Problem (1.7)–(1.9) was considered there. Conditions for the absorption potential $b(t,x)$ have the following form, which is similar to (1.10), but more general:

$$a_1(t)g_1(d(x)) \leqslant b(t,x) \leqslant a_2(t)g_2(d(x)), \quad \forall\, (t,x) \in [0,T) \times \Omega, \tag{1.13}$$

where $g_1(s) \leqslant g_2(s)$ are arbitrary nondecreasing positive for all $s > 0$ functions. In [11], it was proven that under the mentioned conditions and the addition condition for the minorant $a_1(t)$,

$$a_1(t) \geqslant c_0 \exp\left(-\frac{\omega_0}{(T-t)}\right) \quad \text{ in } [0,T), \ c_0 = \mathrm{const} > 0, \ \omega_0 = \mathrm{const} > 0, \tag{1.14}$$

there exists a constant $k > 0$ that does not depend on ω_0 such that

$$\limsup_{t\to T} u(t,x) \leqslant C < \infty, \quad \forall x \in \Omega_0 := \{x \in \Omega : d(x) > k\omega_0^{\frac{1}{2}}\}. \tag{1.15}$$

As we can see, this result proves that condition (1.11) is not sharp. So the most appropriate condition is (1.14). The proof of the sharpness of (1.14) will be given in a subsequent paper.

In [11], there was also studied a more general equation (1.4) with condition (1.5) for the case when $\lambda > p > q > 0$. For this problem, the precise conditions of the boundedness of the solution u was also found. Namely, there was proven that under the condition

$$a_1(t) \geqslant \omega_0^{-1}(T-t)^{\frac{\lambda-p}{p-q}}, \quad \forall\, t < T, \quad \omega_0 = \text{const} > 0, \tag{1.16}$$

an arbitrary *weak solution* (even *large solution* if any) u of the mentioned problem remains bounded as $t \to T$ for arbitrary $x \in \Omega_\varepsilon := \{x \in \Omega : d(x) > \varepsilon\}$, where $\varepsilon = \varepsilon(\omega_0) \to 0$ as $\omega_0 \to 0$.

It is also interesting to estimate the solution profile when the conditions of the boundedness hold. For the case of $p = q > 0$, this estimate was obtained in [14]. Problem (1.1), (1.7), (1.8) with the condition $\lambda > p = q > 0$ was considered in [14]. Under the conditions (1.5), (1.13) for the absorption potential $b(t,x)$ and the condition for the minorant $a_1(t)$,

$$a_1(t) \geqslant c_0 \exp\left(-\omega_0 (T-t)^{-\frac{1}{p+\mu}}\right), \quad \forall\, t < T, \quad c_0, \omega_0, \mu = \text{const} > 0, \tag{1.17}$$

the following estimate for an arbitrary *weak solution* (even for *large solution* if any) $u(t,x)$ of the problem under consideration was proven:

$$\begin{aligned} h(t,s) + E(t,s) &:= \int_{\Omega(s)} |u(t,x)|^{p+1} dx + \int_{\frac{T}{2}}^{t} \int_{\Omega(s)} |\nabla_x u(\tau,x)|^{p+1} dx d\tau \\ &\leqslant K_1 \min_{0<\bar{s}<s} \left\{ \exp\left(K_2 \omega^{\frac{p+\mu}{\mu}} (s-\bar{s})^{-\frac{p+1}{\mu}}\right) \right. \\ &\qquad \left. \times \left(\int_0^{\bar{s}} g_1(h)^{\frac{p+1}{1+p(\lambda+2)}} dh\right)^{-\frac{1+p(\lambda+2)}{\lambda-p}} \right\}, \quad \forall\, s \in (0, s_0'), \end{aligned} \tag{1.18}$$

where the constants $K_1, K_2 < \infty$, $s_0' > 0$ depend only on the known parameters of the problem, $h(t,s)$, $E(t,s)$ are energy functions connected with the solution u and describing the behavior of the solution profile. The domain $\Omega(s)$ from (1.18) is defined in the following way:

$$\Omega(s) := \{x \in \Omega : d(x) > s\}, \quad s > 0. \tag{1.19}$$

The purpose of the paper is to study the behavior of any weak solution u of equation (1.1) with arbitrary initial and boundary values (including singular values). We try to obtain the estimate of the solution profile and find the dependence of this estimate from the minorant a_1. In this context, the main result is the following one.

Theorem 1.3. *Let u be an arbitrary weak (energy) solution (see Definition* 1.1*) of equation* (1.1)*, where the absorption potential $b(t,x)$ satisfies conditions* (1.5)*,* (1.13) *and the following condition for the function $a_1(t)$ holds:*

$$a_1(t) \geqslant \kappa^{-1}(T-t)^{\mu}, \quad \forall\, t<T, \quad \kappa = \text{const} > 0, \ \frac{\lambda - p}{p} < \mu < \frac{\lambda - p}{p-q}. \qquad (1.20)$$

Then the following estimate holds for all $\frac{T}{2} < t < T$:

$$\begin{aligned} h(t,s) + E(t,s) &:= \int_{\Omega(s)} |u(t,x)|^{q+1} dx + \int_{\frac{T}{2}}^{t} \int_{\Omega(s)} |\nabla_x u(\tau,x)|^{p+1} dx d\tau \\ &\leqslant K \kappa^{\frac{q+1}{\lambda - p - \mu(p-q)}} \min_{0<\bar{s}<s} \left\{ (s-\bar{s})^{-\theta} G_1(\bar{s}) \right\}, \quad \forall\, s \in (0,\tilde{s}), \end{aligned} \qquad (1.21)$$

where

$$G_1(s) := \left(\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}} dh \right)^{-\frac{(q+1)(1+p(\lambda+2))}{(p+1)(\lambda - p - \mu(p-q))}},$$

$$\theta = \frac{(n(p-q) + (q+1)(p+1))(\mu(p+1) - (\lambda - p))}{(p+1)(\lambda - p - \mu(p-q))},$$

the constant $K < \infty$ depends only on the known parameters of the problem under consideration, $\tilde{s} > 0$, and the domain $\Omega(s)$ is defined by (1.19)*.*

To make the result more understandable, we introduce the following corollary which describes one of the particular cases.

Corollary 1.4. *Let $g_1(s) = as^{\varrho}$, $a = \text{const} > 0$, $\varrho = \text{const} \geqslant 0$. Then,*

$$G_1(s) = a^{-\frac{q+1}{\lambda - p - \mu(p-q)}} \left(1 + \frac{\varrho(p+1)}{\lambda p + 2p + 1} \right)^{\frac{(q+1)(1+p(\lambda+2))}{(p+1)(\lambda - p - \mu(p-q))}} s^{-\eta}, \qquad (1.22)$$

where $\eta = \eta(\varrho) = \frac{(q+1)((\varrho+1)(p+1)+p(\lambda+1))}{(p+1)(\lambda-p-\mu(p-q))}$. In this case, (1.21) *yields:*

$$h(t,s) + E(t,s) \leqslant \overline{K}\, \kappa^{\frac{q+1}{\lambda - p - \mu(p-q)}} s^{-(\theta+\eta)} \quad \forall\, t \in \left(\frac{T}{2}, T \right), \quad \forall\, s \in (0,\tilde{s}), \qquad (1.23)$$

where

$$\overline{K} = \overline{K}(a,\varrho) = K_1 a^{-\frac{q+1}{\lambda - p - \mu(p-q)}} \left(1 + \frac{\varrho(p+1)}{\lambda p + 2p + 1} \right)^{\frac{(q+1)(1+p(\lambda+2))}{(p+1)(\lambda - p - \mu(p-q))}} \frac{(\theta+\eta)^{\theta+\eta}}{\theta^{\theta}\eta^{\eta}},$$

K_1, θ are from (1.21)*, η is from* (1.22)*.*

Remark 1.5. As mentioned above, in this paper we do not prove the existence of a large solution of equation under consideration. Estimate (1.21) is obtained for all weak solutions of equation (1.1) regardless of initial and boundary data. Particularly, this result can be applied for the problem with singular data (1.7), (1.8) (under the condition that the solution of this problem exists). Of course, this case is the most interesting. Therefore the question about the existence of a large solution for general equation (1.1) with two nonlinearities remains open.

Remark 1.6. We will prove (1.21) by using a method of local energy estimates. It was proposed and developed in [4, 5, 8, 12, 17].

2. Auxiliary results

Let us introduce the following energy functions which are associated with the weak solution $u(t,x)$ of equation (1.1):

$$E(t) = E^{(u)}(t) := \int_0^t \int_\Omega |\nabla_x u(\tau,x)|^{p+1} dx d\tau, \tag{2.1}$$

$$h(t) = h^{(u)}(t) := \sup_{0<\tau<t} \int_\Omega |u(\tau,x)|^{q+1} dx, \quad \forall\, t < T. \tag{2.2}$$

These functions define the behavior of an arbitrary solution u. To study the behavior in the neighborhood of the boundary of the domain Ω, we parameterize the energy function by the parameter s which defines the distance to the boundary $\partial\Omega$. Namely, we consider the family of subdomains $\Omega(s)$ which is defined by (1.19) and the corresponding functions $E(t,s)$ and $h(t,s)$ which are defined in (1.21).

Notice that there exists a value s_Ω which defines the "radius" of the domain $\Omega(s)$. It is such a constant that function $d(\cdot) \in C^2(\Omega \setminus \Omega(s))$, $\forall s \leqslant s_\Omega$, and correspondingly $\partial\Omega(s)$, is a C^2-smooth manifold for all $0 < s \leqslant s_\Omega$. As is well known, the existence of this constant follows from the prescribed smoothness of $\partial\Omega$. Thus, the parameterized functions $E(t,s)$, $h(t,s)$ are defined by the following relations:

$$E(t,s) := \int_{\frac{T}{2}}^t \int_{\Omega(s)} |\nabla_x u(\tau,x)|^{p+1} dx d\tau, \tag{2.3}$$

$$h(t,s) := \int_{\Omega(s)} |u(t,x)|^{q+1} dx, \qquad \forall\, s \in (0, s_\Omega),\ \forall\, t \in (0,T). \tag{2.4}$$

To prove Theorem 1.3 we need to study the functions $E(t,s)$, $h(t,s)$. For this purpose, assume that the interval $[0,T)$ is splitted by some increasing sequence of points $\{t_j\}$ ($j = 1, 2, \ldots, j_0 \leqslant \infty$, $t_0 = 0$, $t_{j_0} = T$). Thus we get the intervals $[t_{j-1}, t_j)$ with the length $\Delta_j := t_j - t_{j-1} > 0$. Now consider the following layered energy functions:

$$E_j(s) := \int_{t_{j-1}}^{t_j} \int_{\Omega(s)} |\nabla_x u(t,x)|^{p+1} dx dt, \tag{2.5}$$

$$h_j(s) := \sup_{t_{j-1} \leqslant t < t_j} \int_{\Omega(s)} |u(t,x)|^{q+1} dx, \qquad \forall\, j \leqslant j_0, \forall\, s \in (0, s_\Omega). \tag{2.6}$$

Now we have the following lemma for these functions.

Lemma 2.1 (see [8,13]). *Let $u(t,x)$ be an arbitrary weak solution of equation (1.1) with conditions (1.5) for the absorption potential $b(t,x)$. Then, for almost all $s \in (0, s_\Omega)$, the following system for the layered energy functions holds:*

$$E_j(s) + h_j(s) \leqslant C_1 h_{j-1}(s) + C_2 \Delta_j^{\nu_1} \left(-E_j'(s)\right)^{1+\mu_1} + C_3 \Delta_j^{\nu_2} \left(-E_j'(s)\right)^{1+\mu_2},$$

$$h_j(s) \leqslant (1+\gamma) h_{j-1}(s) + C_4 \gamma^{-(\nu_1+\mu_1)} \Delta_j^{\nu_1} \left(-E_j'(s)\right)^{1+\mu_1} +$$

$$+ C_5\gamma^{-\frac{1}{q}}\Delta_j^{\nu_2}\left(-E_j'(s)\right)^{1+\mu_2}, \quad j = 1, 2, \ldots, j_0, \tag{2.7}$$

where $\gamma = \text{const} > 0$*,* $C_i = \text{const} > 0$*,* $\forall\, i = \overline{1,5}$*, depend only on the known parameters and do not depend on* γ*,*

$$\nu_1 = \frac{(1-\theta)(q+1)}{q(p+1)+\theta(p-q)}, \qquad \mu_1 = \frac{(1-\theta)(p-q)}{q(p+1)+\theta(p-q)},$$
$$\nu_2 = \frac{(q+1)}{q(p+1)}, \qquad \mu_2 = \frac{(p-q)}{q(p+1)},$$
$$\theta = \frac{n(p-q)+q+1}{n(p-q)+(q+1)(p+1)} < 1,$$

the functions $E_j(s)$ *and* $h_j(s)$ *are defined in* (2.5)*,* (2.6)*.*

Since the absorption potential $b(t,x) \geqslant 0$, $\forall\,(t,x) \in \overline{Q}$, the proof of Lemma 2.1 is analogous to that of Lemma 6.2.3 in [8] or Lemma 1 in [13].

The next step of the proof is analysis of asymptotic properties of solutions of ordinary differential inequality (ODI) system (2.7). The following result can be obtained by repeating all the steps of the proof of Theorem 1 in [13].

Lemma 2.2. *Let* u *be an arbitrary weak (energy) solution of equation* (1.1) *and let the layered energy functions* $E_j(s)$*,* $h_j(s)$ *satisfy the system of ODI* (2.7)*. Let also the "initial" conditions for energy functions hold,*

$$E(t) + h(t) \leqslant \omega_0(T-t)^{-\alpha}, \quad \forall\, t < T, \quad \frac{1}{p} < \alpha < \frac{q+1}{p-q}, \tag{2.8}$$

where $\omega_0 = \text{const} > 0$*, the functions* $E(t)$*,* $h(t)$ *are defined in* (2.1)*,* (2.2)*.*

Then there exists the constant $G > 0$ *and the value* $\hat{s}_0 > 0$*, which depend on known parameters of the problem only such that the following uniform by* $t \leqslant T$ *estimate holds:*

$$E(t,s) + \sup_{0<\tau<t} h(\tau,s) \leqslant G\omega_0^{\frac{q+1}{q+1-\alpha(p-q)}} s^{-\nu}, \quad \forall\, t \leqslant T,\ \forall\, s \in (0,\hat{s}_0), \tag{2.9}$$

where $\nu = \frac{\alpha(n(p-q)+(q+1)(p+1))}{q+1-\alpha(p-q)}$*,* $\Omega(s)$ *is a family of subdomains from* (1.19)*, the functions* $E(t,s)$*,* $h(t,s)$ *are defined in* (2.3)*,* (2.4)*.*

Thus the last step of the proof of Theorem 1.3 is to obtain condition (2.8) for the energy functions.

3. Proof of Theorem 1.3

Let $\Omega(s)$ be a family of subdomains from (1.19). Let us introduce an additional family of cylindrical subdomains of Q:

$$Q_\tau(s) := (s^r, \tau) \times \Omega(s), \quad \forall s \in (0, s_\Omega),\ \forall \tau < T,$$
$$1 < r < 1 + \frac{(\lambda+1)(p(\lambda+1)-q(p+1))}{(q+1)(\lambda-p)}. \tag{3.1}$$

Now we define the energy functions connected with a solution u of equation (1.1) under consideration:

$$\overline{h}_\tau(s) = \overline{h}_\tau^{(u)}(s) := \int_{\Omega(s)} |u(\tau,x)|^{q+1}dx, \quad 0<\tau<T,\, 0<s<s_\Omega, \tag{3.2}$$

$$\overline{E}_\tau(s) = \overline{E}_\tau^{(u)}(s) := \int_{s^r}^{\tau}\int_{\Omega(s)} (|\nabla_x u(t,x)|^{p+1} + a_1(t)g_1(d(x))|u|^{\lambda+1})\,dx\,dt. \tag{3.3}$$

Lemma 3.1. *Let u be a solution of equation* (1.1)*. And let the absorption potential $b(t,x)$ satisfy conditions* (1.5), (1.13). *Then energy functions* (3.2), (3.3) *satisfy the following relations:*

$$B_\tau(s) := \overline{h}_\tau(s) + \overline{E}_\tau(s) \leqslant \widehat{C}\,\Phi(\tau)G_1(s) \quad \forall\, s\in(0,\hat{s}), \tag{3.4}$$

where

$$\Phi(\tau) = \int_0^\tau a_1(t)^{-\frac{p+1}{\lambda-p}}dt, \quad G_1(s) = \left(\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}}dh\right)^{-\frac{1+p(\lambda+2)}{\lambda-p}},$$

$\widehat{C} = \mathrm{const} > 0$, $\hat{s}\in(0,s_\Omega)$ *depend only on the known parameters.*

Proof. Let us fix $s>0$, $\delta>0$ and introduce a Lipschitz cut-off function $\xi_{s,\delta}(h) : \xi_{s,\delta}(h) = 0$ for $h\leqslant s$, $\xi_{s,\delta}(h)=1$ for $h>s+\delta$, $\xi_{s,\delta}(h) = \frac{h-s}{\delta}$ for $h : s<h<s+\delta$. Now we substitute the test function $\eta(t,x) = u(t,x)\xi_{s,\delta}(d(x))$ into integral identity (1.6). Then, using the formula of integration by parts (see, for example, [1]), we get

$$\begin{aligned}\frac{q}{q+1}&\int_{\Omega(s)} |u(b,x)|^{q+1}\xi_{s,\delta}(d(x))dx\\ &+\int_a^b\int_{\Omega(s)}\Big(\sum_{i=1}^n a_i(\ldots,\nabla_x u)u_{x_i} + b(t,x)|u|^{\lambda+1}\Big)\xi_{s,\delta}(d(x))dxdt\\ &\quad= \frac{q}{q+1}\int_{\Omega(s)}|u(a,x)|^{q+1}\xi_{s,\delta}(d(x))dx\\ &\qquad-\int_a^b\int_{\Omega(s)\setminus\Omega(s+\delta)}\sum_{i=1}^n a_i(\ldots,\nabla_x u)u\xi_{s,\delta}(d(x))_{x_i}dxdt.\end{aligned} \tag{3.5}$$

Let us take $b=\tau<T$, $a=s^r$ in (3.5). Then, passing to the limit $\delta\to 0$ and using conditions (1.2), (1.3), by standard computations we derive the inequality

$$\overline{h}_\tau(s) + \overline{E}_\tau(s) \leqslant c_1\int_{s^r}^{\tau}\int_{\partial\Omega(s)}|\nabla_x u|^p|u|\,d\sigma dt + c_2\overline{h}_{s^r}(s), \quad \forall\, s\in(0,s_\Omega), \tag{3.6}$$

where $c_1<\infty$, $c_2<\infty$ depend only on d_0, d_1, p, n. Let us estimate the terms in the right-hand side from above. Using Hölder's inequality, we get

$$\int_{\partial\Omega(s)}|\nabla_x u|^p|u|\,d\sigma = \int_{\partial\Omega(s)}|u|g_1(s)^{\frac{1}{\lambda+1}}a_1(t)^{\frac{1}{\lambda+1}}|\nabla_x u|^p a_1(t)^{-\frac{1}{\lambda+1}}g_1(s)^{-\frac{1}{\lambda+1}}\,d\sigma$$

$$\leqslant c_3\left(\int_{\partial\Omega(s)}|u|^{\lambda+1}a_1(t)g_1(s)\,d\sigma\right)^{\frac{1}{\lambda+1}}$$
$$\times\left(\int_{\partial\Omega(s)}|\nabla_x u|^{p+1}d\sigma\right)^{\frac{p}{p+1}}a_1(t)^{-\frac{1}{\lambda+1}}g_1(s)^{-\frac{1}{\lambda+1}},$$

where $c_3=(\operatorname{meas}\partial\Omega)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}$. Integrating the last inequality with respect to t and using the Hölder and Young inequalities, we get

$$\int_{s^r}^{\tau}\int_{\partial\Omega(s)}|\nabla_x u|^p|u|\,d\sigma dt\leqslant c_4 g_1(s)^{-\frac{1}{\lambda+1}}\left(\int_{s^r}^{\tau}a_1(t)^{-\frac{p+1}{\lambda-p}}\,dt\right)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}$$
$$\times\left(\int_{s^r}^{\tau}\int_{\partial\Omega(s)}\left(|\nabla_x u|^{p+1}+a_1(t)g_1(s)|u|^{\lambda+1}\right)d\sigma dt\right)^{\frac{1+p(\lambda+2)}{(\lambda+1)(p+1)}}. \quad (3.7)$$

We estimate the second term of the right-hand side of (3.6) using the monotonicity of the function $g_1(\cdot)$ and Hölder's inequality

$$\overline{h}_{s^r}(s)=\int_{\Omega(s)}|u(s^r,x)|^{q+1}a_1(s^r)^{\frac{q+1}{\lambda+1}}g_1(d(x))^{\frac{q+1}{\lambda+1}}a_1(s^r)^{-\frac{q+1}{\lambda+1}}g_1(d(x))^{-\frac{q+1}{\lambda+1}}dx$$
$$\leqslant c_5\left(\int_{\Omega(s)}|u(s^r,x)|^{\lambda+1}a_1(s^r)g_1(d(x))dx\right)^{\frac{q+1}{\lambda+1}}a_1(s^r)^{-\frac{q+1}{\lambda+1}}g_1(s)^{-\frac{q+1}{\lambda+1}}, \quad (3.8)$$

where $c_5=(\operatorname{meas}\Omega)^{\frac{\lambda-q}{\lambda+1}}$. It is easy to check that the inequality

$$-\frac{d}{ds}\overline{E}_\tau(s)\geqslant\int_{s^r}^{\tau}\int_{\partial\Omega(s)}\left(|\nabla_x u|^{p+1}+a_1(t)g_1(s)|u|^{\lambda+1}\right)d\sigma dt$$
$$+rs^{r-1}\int_{\Omega(s)}\left(|\nabla_x u(s^r,x)|^{p+1}+a_1(s^r)g_1(d(x))|u(s^r,x)|^{\lambda+1}\right)dx \quad (3.9)$$

holds for almost all $s\in(0,s_\Omega)$. Using estimates (3.7), (3.8) and relation (3.9), from (3.6) we deduce the following inequality:

$$B_\tau(s):=\overline{h}_\tau(s)+\overline{E}_\tau(s)\leqslant C_1 g_1(s)^{-\frac{1}{\lambda+1}}\Phi(\tau)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}\left(-\frac{d}{ds}\overline{E}_\tau(s)\right)^{\frac{1+p(\lambda+2)}{(\lambda+1)(p+1)}}$$
$$+C_2 g_1(s)^{-\frac{q+1}{\lambda+1}}\left(-\frac{d}{ds}\overline{E}_\tau(s)\right)^{\frac{q+1}{\lambda+1}}s^{-\frac{(r-1)(q+1)}{\lambda+1}}\quad\text{for a.a. } s\in(0,s_\Omega), \quad (3.10)$$

where

$$\Phi(\tau)=\int_0^{\tau}a_1(t)^{-\frac{p+1}{\lambda-p}}dt,\quad C_1=c_1c_4,\quad C_2=c_2c_5\left(\min_{0\leqslant s\leqslant s_0}a_1(s^r)\right)^{-\frac{q+1}{\lambda+1}}.$$

Now, using the monotonic decreasing of the function $\overline{h}_\tau(s)$, by a simple computation we derive from (3.10) the following inequality:

$$-\frac{d}{ds}B_\tau(s)\geqslant H(s,B_\tau(s)):=\min\left\{H_\tau^{(1)}(s,B_\tau(s)),H_\tau^{(2)}(s,B_\tau(s))\right\}$$

$$\text{for a.a. } s \in (0, s_\Omega), \ \forall \tau < T, \quad (3.11)$$

$$H_\tau^{(1)}(s, B_\tau(s)) := \left(\frac{g_1(s)^{\frac{1}{\lambda+1}} B_\tau(s)}{2C_1 \Phi(\tau)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}} \right)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}},$$

$$H_\tau^{(2)}(s, B_\tau(s)) := \left(\frac{g_1(s)^{\frac{q+1}{\lambda+1}} B_\tau(s)}{2C_2 s^{-\frac{(r-1)(q+1)}{\lambda+1}}} \right)^{\frac{\lambda+1}{q+1}}.$$

Now we will solve ODI (3.11) and get the estimate for $B_\tau(s)$. For this purpose we consider a domain $D = D_\tau \subset \mathbb{R}^2$ as a set of points $(s, B) : 0 < s < s_\Omega$, $B > 0$, where the function $H(s, B)$ from (3.11) is defined by the first term in the right-hand side of (3.11). It means that

$$D_\tau = \left\{ (s, B) : \left(\frac{g_1(s)^{\frac{1}{\lambda+1}} B}{2C_1 \Phi(\tau)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}} \right)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}} < \left(\frac{g_1(s)^{\frac{q+1}{\lambda+1}} B}{2C_2 s^{-\frac{(r-1)(q+1)}{\lambda+1}}} \right)^{\frac{\lambda+1}{q+1}} \right\}.$$

After simple transformation, we can rewrite the last definition as

$$D_\tau = \{(s, B) : B > B_\tau^{(0)}(s)\}, \quad (3.12)$$

where

$$B_\tau^{(0)}(s) = C_3 s^{-\frac{(r-1)(q+1)(1+p(\lambda+2))}{(\lambda+1)(p(\lambda+1)-q(p+1))}} g_1(s)^{-\frac{p(q+1)}{p(\lambda+1)-q(p+1)}}, \Phi(\tau)^{-\frac{(\lambda-p)(q+1)}{(\lambda+1)(p(\lambda+1)-q(p+1))}},$$

$$C_3 = 2C_1^{-\frac{(p+1)(q+1)}{p(\lambda+1)-q(p+1)}} C_2^{\frac{1+p(\lambda+2)}{p(\lambda+1)-q(p+1)}}.$$

Let us consider now all possible cases for the solution $B_\tau(s)$ with respect to the domain D_τ. The first possibility is

$$(s, B_\tau(s)) \in \overline{D}_\tau \quad \text{for all } s \in (0, s_\Omega). \quad (3.13)$$

In this situation ODI (3.11) has the form

$$-\frac{d}{ds} B_\tau(s) \geqslant H_\tau^{(1)}(s, B_\tau(s)) = \left(\frac{g_1(s)^{\frac{1}{\lambda+1}} B_\tau(s)}{2C_1 \Phi(\tau)^{\frac{\lambda-p}{(\lambda+1)(p+1)}}} \right)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}},$$

$$\forall \tau < T, \quad \forall\, s \in (0, s_\Omega). \quad (3.14)$$

From assumption (3.13) it follows that $B_\tau(0) = \infty$. Solving now ODI (3.14) with this initial condition, we get simply

$$B_\tau(s) \leqslant B_\tau^{(1)}(s) := C_4 \Phi(\tau) \left(\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}} dh \right)^{-\frac{1+p(\lambda+2)}{\lambda-p}},$$

$$\forall\, s \in (0, s_\Omega), \forall\, \tau < T, \quad (3.15)$$

where $C_4 = \left(\frac{1+p(\lambda+2)}{\lambda-p} \right)^{\frac{1+p(\lambda+2)}{\lambda-p}} (2C_1)^{\frac{(\lambda+1)(p+1)}{\lambda-p}}$.

Let us check that (3.15) does not contradict the assumption that $B_\tau(s) \in \overline{D}_\tau$. For this we have to guarantee the inequality

$$B_\tau^{(1)}(s) > B_\tau^{(0)}(s), \quad \forall\, s \in (0, s_\Omega), \tag{3.16}$$

or, due to definitions (3.12), (3.15):

$$C_4\Phi(\tau)\left(\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}}\,dh\right)^{-\frac{1+p(\lambda+2)}{\lambda-p}} > C_3 s^{-\frac{(r-1)(q+1)(1+p(\lambda+2))}{(\lambda+1)(p(\lambda+1)-q(p+1))}} g_1(s)^{-\frac{p(q+1)}{p(\lambda+1)-q(p+1)}} \times \Phi(\tau)^{-\frac{(\lambda-p)(q+1)}{(\lambda+1)(p(\lambda+1)-q(p+1))}}, \quad \forall\, s \in (0, s_\Omega).$$

Due to the monotonicity of the function $g_1(\cdot)$, we have

$$\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}}\,dh \leqslant s g_1(s)^{\frac{p+1}{1+p(\lambda+2)}},$$

and therefore,

$$\left(\int_0^s g_1(h)^{\frac{p+1}{1+p(\lambda+2)}}\,dh\right)^{-\frac{1+p(\lambda+2)}{\lambda-p}} \geqslant s^{-\frac{1+p(\lambda+2)}{\lambda-p}} g_1(s)^{-\frac{p+1}{\lambda-p}}, \quad \forall\, s \in (0, s_\Omega). \tag{3.17}$$

As a consequence, we have that for validity of (3.16) it is sufficient to guarantee that

$$C_4\Phi(\tau) > C_3 s^{\frac{1+p(\lambda+2)}{\lambda-p}-\frac{(r-1)(q+1)(1+p(\lambda+2))}{(\lambda+1)(p(\lambda+1)-q(p+1))}} \times g_1(s)^{\frac{(p-q)(1+p(\lambda+2))}{(\lambda-p)(p(\lambda+1)-q(p+1))}} \Phi(\tau)^{-\frac{(\lambda-p)(q+1)}{(\lambda+1)(p(\lambda+1)-q(p+1))}},$$

which is equivalent to

$$\overline{\Phi}(\tau) \geqslant C_3 C_4^{-1} \overline{g}_1(s), \tag{3.18}$$

$$\overline{\Phi}(\tau) := \Phi(\tau)^{1+\frac{(\lambda-p)(q+1)}{(\lambda+1)(p(\lambda+1)-q(p+1))}},$$

$$\overline{g}_1(s) := s^{\frac{1+p(\lambda+2)}{\lambda-p}-\frac{(r-1)(q+1)(1+p(\lambda+2))}{(\lambda+1)(p(\lambda+1)-q(p+1))}} g_1(s)^{\frac{(p-q)(1+p(\lambda+2))}{(\lambda-p)(p(\lambda+1)-q(p+1))}}.$$

Since the parameter r satisfies the assumption from (3.1), the function $\overline{g}_1(s)$ is a monotonically increasing function. Using the monotonicity of the functions $\overline{g}_1(s)$ and $\Phi(\tau)$, we can get that inequality (3.17) holds for the arbitrary $\tau \in [\frac{T}{2}, T)$ and arbitrary s, satisfying the condition

$$0 < s < s_0, \tag{3.19}$$

where $s_0 := \min\{s_\Omega, \bar{s}_0\}$, $\bar{s}_0$ is determined by $\overline{g}_1(\bar{s}_0) = C_3^{-1} C_4 \overline{\Phi}\left(\frac{T}{2}\right)$. Thus, relationship (3.16) is true for all $\tau > \frac{T}{2}$ and s from (3.19). As a consequence, (3.15) holds if condition (3.13) is satisfied.

Let us suppose that estimate (3.15) is not true for some $s \in (0, s_0)$. So, there exists $s_1 \in (0, s_0)$ such that

$$B_\tau(s_1) > B_\tau^{(1)}(s_1). \tag{3.20}$$

If we suppose that $B_\tau(s) > B_\tau^{(1)}(s)$, $\forall\, s \in (0, s_1)$, then it also satisfies condition (3.13). Therefore, due to the previous consideration estimate (3.15) holds for all $s \in (0, s_1)$, which contradicts assumption (3.20). Thus, it remains the following possibility only: there exists a point $s_2 \in (0, s_1)$ such that

$$B_\tau(s_2) = B_\tau^{(1)}(s_2), \quad B_\tau(s) > B_\tau^{(1)}(s), \quad \forall\, s \in (s_2, s_1). \tag{3.21}$$

From (3.21) it follows easily that there exists a point $\bar{s}_2 \in (s_2, s_1)$ such that

$$\frac{d}{ds} B_\tau(\bar{s}_2) > \frac{d}{ds} B_\tau^{(1)}(\bar{s}_2), \quad B_\tau(\bar{s}_2) > B_\tau^{(1)}(\bar{s}_2). \tag{3.22}$$

But on the other side, due to (3.14), (3.15) and (3.22), we have

$$\begin{aligned} -\frac{d}{ds} B_\tau(\bar{s}_2) &\geqslant \frac{g_1(\bar{s}_2)^{\frac{p+1}{1+p(\lambda+2)}}}{C_5 \Phi(\tau)^{\frac{\lambda-p}{1+p(\lambda+2)}}} B_\tau(\bar{s}_2)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}} \\ &> \frac{g_1(\bar{s}_2)^{\frac{p+1}{1+p(\lambda+2)}}}{C_5 \Phi(\tau)^{\frac{\lambda-p}{1+p(\lambda+2)}}} B_\tau^{(1)}(\bar{s}_2)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}} = -\frac{d}{ds} B_\tau^{(1)}(\bar{s}_2), \end{aligned}$$

where $C_5 := (2C_1)^{\frac{(\lambda+1)(p+1)}{1+p(\lambda+2)}}$, which contradicts (3.22) and, consequently, (3.21). Thus, estimate (3.4) is true with $\widehat{C} = C_4$, $\hat{s} = \min\{s_\Omega, s_0\}$ for all $\tau \in \left(\frac{T}{2}, T\right)$. $\square$

Proof of Theorem 1.3. Due to condition (1.20), we can get the following estimate for the function $\Phi(\cdot)$ from (3.10):

$$\Phi(t) \leqslant \Phi_0(t) := K_1\, \kappa^{\frac{p+1}{\lambda-p}} (T-t)^{-\left(\mu \frac{p+1}{\lambda-p} - 1\right)}, \quad \forall t < T. \tag{3.23}$$

Now inequality (3.4) yields the estimate

$$h(t,s) + E(t,s) \leqslant \widehat{C} \Phi(t) G_1(s), \quad \forall t \in (t_0, T),\ \forall s \in (0, \bar{s}_\Omega), \tag{3.24}$$

where $t_0 = \frac{T}{2}$, $\bar{s}_\Omega := \min\left(s_\Omega, t_0^{\frac{1}{r}}\right)$, r is from (3.1), the functions $E(t,s)$, $h(t,s)$ are from (2.3), (2.4), respectively. Now we fix some value $\bar{s} \in (0, \bar{s}_\Omega)$ and deduce from (3.24) the following "initial" energy estimate:

$$h(t,\bar{s}) + E(t,\bar{s}) \leqslant \widehat{C} G_1(\bar{s}) \Phi(t) \quad \forall\, t \in (t_0, T). \tag{3.25}$$

We will consider $u(t,x)$ as a solution of equation (1.1) in the domain $(t_0, T) \times \Omega(\bar{s})$. Using (3.25) and (3.23), we get

$$h(t,\bar{s}) + E(t,\bar{s}) \leqslant K_2 \kappa^{\frac{p+1}{\lambda-p}} G_1(\bar{s}) (T-t)^{-\beta}, \quad \forall\, t \in (t_0, T), \tag{3.26}$$

where $\beta = \mu\frac{p+1}{\lambda-p} - 1$, $K_2 = \widehat{C}K_1$ and condition from (1.20) for μ gives us the condition for $\beta : \frac{1}{p} < \beta < \frac{q+1}{p-q}$.

Now we have system (2.7) for the layered functions $E_j(s)$, $h_j(s)$ and "initial" condition (3.26). Applying Lemma 2.2, we get the estimate

$$h(t,s) + E(t,s) \leqslant GK_2^{\frac{(q+1)(\lambda-p)}{(p+1)(\lambda-p-\mu(p-q))}} \kappa^{\frac{q+1}{\lambda-p-\mu(p-q)}} (s-\bar{s})^{-\theta} G_1(\bar{s})^{\frac{(q+1)(\lambda-p)}{(p+1)(\lambda-p-\mu(p-q))}}$$
$$\forall\, t \in (t_0, T),\ \forall\, s, \bar{s} \text{ such that } 0 < \bar{s} < s < \tilde{s}, \quad (3.27)$$

where $\tilde{s} := \min\{\bar{s}_\Omega, \hat{s}_0\}$, G, $\hat{s}_0$ are from (2.9), θ is from (1.21). Optimizing the last estimate with respect to a free parameter $\bar{s}$, $0 < \bar{s} < s < \tilde{s}$, we get (1.21) with $K = GK_2^{\frac{(q+1)(\lambda-p)}{(p+1)(\lambda-p-\mu(p-q))}}$. □

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Поширення сингулярностей великих розв'язків квазілінійних параболічних рівнянь

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Розглянуто квазілінійне параболічне рівняння з потенціалом абсорбції:

$$(|u|^{q-1}u)_t - \Delta_p(u) = -b(t,x)|u|^{\lambda-1}u \quad (t,x) \in (0,T) \times \Omega, \quad \lambda > p > q > 0,$$

де Ω є обмеженою гладкою областю в R^n, $n \geqslant 1$, b є потенціалом абсорбції, який є неперервною функцією такою, що $b(t,x) > 0$ в $[0,T) \times \Omega$ та $b(t,x) \equiv 0$ в $\{T\} \times \Omega$. У роботі розглянуто умови на $b(t,x)$, які гарантують рівномірну обмеженість довільного слабкого розв'язку зазначеного рівняння в будь-якій підобласті $\Omega_0 : \overline{\Omega}_0 \subset \Omega$. За цих умов одержано точну оцінку зверху для всіх слабких розв'язків u. Цю оцінку виконано для розв'язків цього рівняння з довільними початковими та граничними даними, включаючи сингулярні дані (якщо такі розв'язки існують), а саме $u = \infty$ на $\{0\} \times \Omega$, $u = \infty$ на $(0,T) \times \partial\Omega$.

Ключові слова: диференціальні рівняння з частинними похідними, квазілінійне параболічне рівняння, вироджений потенціал абсорбції, великі розв'язки.