JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. 11 No 1, 01024(10pp) (2019)
ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том 11 № 1, 01024(10cc) (2019)

A Nonrelativistic Subatomic Model to Describe a Behavior of 2D Anharmonic Sextic Potential for Atomic Nucleus in the Symmetries of Extended Quantum Mechanics

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(Received 12 September 2018; revised manuscript received 01 February 2019; published online 25 February 2019)

In the present work, a nonrelativistic analytical model is presented and used to modify the Schrödinger equation for subatomic scales with two-dimensional new anharmonic sextic potential (2DNASP) for atomic nuclei in noncommutative two-dimensional real space-phase (NC: 2D-RSP). We applied the generalized Bopp's shift method to obtain 2DNASP, which is composed of ordinary anharmonic sextic potential and new additive part proportional to the infinitesimal parameter θ. We have also observed the kinetic term containing a new additive part proportional to the infinitesimal parameter $\bar{\theta}$. Thus, the global potential proportional to two infinitesimal parameters allowed us to consider the perturbation terms due to (space-phase) noncommutativity. Furthermore, the standard perturbation theory allowed us to find the corrections $E_{(u-d)-as}(n,m,l,b,c)$ and $E_{\text{mag-}as}(n,m,b,c)$ corresponding to spin-orbital interaction and modified Zeeman effect, respectively. These corrections allowed us to find global energy levels $E_{\text{nc-(u-d)clh}}(n,b,c,j,l,s,m)$, which depend on the discrete atomic quantum numbers (*j*, *l*, *s*, *m*) and the parameters of the studied potential (*a*, *b*, *c*), and we constricted the corresponding Hamiltonian operator $H_{nc-as}(\hat{p}_i,\hat{x}_i)$. We have also generalized our obtained results to include the other nuclei's atoms with spin $\vec{S}$ differs from $\vec{1/2}$. This study has allowed us to obtain the energy spectrum in the extended quantum mechanics, which interpreted three physical phenomena. The first one is an ordinary two-dimensional new anharmonic sextic potential, while the second and third are the automatic appearance of spin-orbit interaction and modified magnetic influence, these phenomena were generated from the property of noncommutativity of space and phase. The obtained analytical results (the energy eigenvalues and the corresponding Hamiltonian operator) are in good agreement with the existing results. The previous results, in ordinary quantum mechanics, become special cases when we make simultaneously the limits $(\theta,\bar{\theta})\to(0,0)$.

Keywords: Schrödinger equation, Anharmonic sextic potential, Noncommutative space-phase, Star product and generalized Bopp's shift method.

DOI: 10.21272/jnep.11(1).01024

PACS numbers: 11.10.Nx, 32.30-r, 03.65-w, 03.65.Ca, 03.65.Ge

1. INTRODUCTION

The interaction between the nucleons (protons and neutrons) in nuclei is produced by the coupling between Bohr Hamiltonian in conjunction with anharmonic sextic potential, which consists in solving a Schrodinger like equation in deformation parameter space; the motion of nucleons within the nucleus changes its shape because of the effects of this potential [1-3]. The aim of our work is to extend the study of S B Bhardwaj et *al*. [1] to the case of extended quantum mechanics (EQM), which is known by noncommutativity of space-time, introduced firstly by Heisenberg, and formalized by Snyder in 1947 [4-5], to find other applications and more profound interpretations in the subatomic scales. On the other hand, we extended our study in ref. [6] from noncommutative two-dimensional real spaces (NC: 2D-RS) to the case of two-dimensional real space-phase (NC: 2D-RSP). We are based on some previous studies of other authors and some of our related works in this context. The nonrelativistic energy levels for nuclei atoms, which interact with anharmonic sextic potential in the context of NC space, have not been obtained yet. In last few years, many efforts have been produced to study some potentials using the notions of noncommutativity of space and phase based essentially on Seiberg-Witten map and generalized Bopp's shift method and the star product, defined by the two antisymmetric first-order infinitesimal parameters

$$2\left(\theta^{\mu\nu},\bar{\theta}^{\mu\nu}\right)\equiv\varepsilon^{\mu\nu\rho}\left(\theta_{\rho},\bar{\theta}_{\rho}\right) \text{ as [6-12]:}$$

$$(f*g)(x,p)=(fg)(x,p)-\frac{i}{2}\left(\theta^{\mu\nu}\partial_{\mu}^{x}f\,\partial_{\nu}^{x}g+\bar{\theta}^{\mu\nu}\partial_{\mu}^{p}f\partial_{\nu}^{p}g\right)(x,p). \quad (1)$$

As direct results for the above two modes of star product due to the space-space and phase-phase noncommutativity, allow us to find new none null commutators [13-18]:

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$$\begin{cases}\left[x^{\mu},x^{\nu}\right]=0\\ \left[x^{\mu}(t),x^{\nu}(t)\right]=0\\ \left[p^{\mu},p^{\nu}\right]=0\\ \left[p^{\mu}(t),p^{\nu}(t)\right]=0\end{cases}\Rightarrow\begin{cases}\left[\hat{x}^{\mu},\hat{x}^{\nu}\right]_{*}=i\theta^{\mu\nu}\\ \left[\hat{x}^{\mu}(t),\hat{x}^{\nu}(t)\right]_{*}=i\theta^{\mu\nu}\\ \left[\hat{p}^{\mu},\hat{p}^{\nu}\right]_{*}=i\bar{\theta}^{\mu\nu}\\ \left[\hat{p}^{\mu}(t),\hat{p}^{\nu}(t)\right]_{*}=i\bar{\theta}^{\mu\nu}.\end{cases} \tag{2}$$

The rest of present paper is arranged as follows. Section 2 is a brief outline of the 2DSE with anharmonic sextic potential based on the main ref. [1]. The Section 3 is devoted to studying the 2DMSE by applying the generalized Bopp's shift method and standard perturbation theory. We find the quantum spectrum of ground states, first excited states and n^{th} excited sates, for modified spin-orbit interaction, in the framework of the global quantum group (NC-2D: RSP), then we end this section by deriving the modified magnetic spectrum for 2DNASP. In the fourth section, we resume the global spectrum and corresponding NC Hamiltonian operator for 2DNASP of nuclei atoms. In next section, we generalize our obtained results to include the other atomic nuclei with spin $\vec{S}\neq\vec{1}/2$. The conclusions of the present study are given in the last section.

2. REVIEW OF THE EIGENVALUES OF THE SCHRÖDINGER EQUATION WITH 2DASP

In this section, let us provide a brief review of the eigenvalues and eigenfunctions for spherically symmetric 3DNASP $V(r)$, and we convert the different results to the case of 2D space [1]:

$$V(r)=ar^{2}-br^{4}+cr^{6} \tag{3}$$

The parameters a, b and c are the constants, the radial part $R(r)$ satisfies two following equations in 3D and 2D, respectively [1]:

$$\frac{d^{2}R(r)}{dr^{2}}+\frac{2}{r}\frac{dR(r)}{dr}+\left(\varepsilon-a_{1}r^{2}+b_{1}r^{4}-c_{1}r^{6}-\frac{l(l+1)}{r^{2}}\right)R(r)=0,$$
$$\rightarrow\frac{d^{2}R(r)}{dr^{2}}+\frac{1}{r}\frac{dR(r)}{dr}+\left(\varepsilon-a_{1}r^{2}+b_{1}r^{4}-c_{1}r^{6}-\frac{l^{2}}{r^{2}}\right)R(r)=0 \tag{4}$$

with $\varepsilon=2\mu E$, $a_1=2\mu a$, $b_1=2\mu b$ and $c_1=2\mu c$. The explicit form of the radial part $R(r)$ of eigenfunction is given by [1]

$$R(r)=\exp\left(-\alpha r^{2}-\beta r^{4}\right)\sum_{n=0}^{\infty}a_{n}r^{n+l}$$
$$\rightarrow R(r)=\exp\left(-\alpha r^{2}-\beta r^{4}\right)\sum_{n=0}^{\infty}a_{n}r^{n+m-1/2}. \tag{5}$$

Here $\alpha=\frac{b_{1}}{4\sqrt{c_{1}}}$, $\beta=\frac{\sqrt{c_{1}}}{4}$ and $a_1=4\beta(2n+2l+5)+4\alpha^2$, while the parameter a of anharmonic sextic potential satisfies the following restriction [1]:

$$a=\sqrt{\frac{c}{2\mu}}\left(2n+2m+4\right)+\frac{b^{2}}{4c}. \tag{6}$$

The energy eigenvalues and corresponding eigenfunctions from the same values in three dimensional case after changing $l=m-1/2$ are written as [1, 19]:

$$E_{nl}(3D)=\frac{b}{2}\sqrt{\frac{1}{2\mu c}}\left(2n+2l+3\right)\rightarrow E_{nm}(2D)=\frac{b}{2}\sqrt{\frac{1}{2\mu c}}\left(2n+2m+2\right),$$
$$\Psi_{nlm}(r,\theta,\phi)\rightarrow\Psi_{nlm}(r,\phi)=\frac{1}{\sqrt{2\Pi}}\exp\left(-\alpha r^{2}-\beta r^{4}\right)\left(\sum_{n=0}^{\infty}a_{n}r^{n+m-1/2}\right)\exp(im\phi) \tag{7}$$

with the magnetic quantum number $m=0,\pm1,\pm2,.....$. The purpose of the present paper is to attempt to study the 2DMSE with 2DNASP in (NC: 2D-RSP) symmetries, using the generalized Bopp's shift method which depends on the concepts that we present below in the third section to discover the new symmetries and a possibility to obtain another applications to this potential in different fields.

3. METHOD AND THEORETICAL APPROACH

In this section, we shall give an overview or a brief preliminary for 2DNASP in (NC: 2D-RSP) symmetries. To perform this task, in the physical form of modified Schrödinger equation (MSE), it is necessary to replace ordinary two-dimensional Hamiltonian operators $\hat{H}(p_i,x_i)$, ordinary complex wave function $\Psi\left(\vec{r}\right)$ and ordinary energy E_{nm} by new 2D-Hamiltonian operators $\hat{H}_{nc-as}(\hat{p}_i,\hat{x}_i)$, new complex wave function $\hat{\Psi}\left(\vec{\hat{r}}\right)$ and new values E_{nc-as}, respectively. In addition, we will replace the ordinary old product by star product (*), which allows us to construct the MSE in (NC-2D: RSP) symmetries as [19-24]:

$$\hat{H}_{as}(p_i,x_i)\Psi\left(\vec{r}\right)=E_{nl}\Psi\left(\vec{r}\right)\Rightarrow\hat{H}_{nc-as}(\hat{p}_i,\hat{x}_i)*\Psi\left(\vec{\hat{r}}\right)=$$
$$=E_{nc-as}\Psi\left(\vec{\hat{r}}\right). \tag{8}$$

The Bopp's shift method employed in the solutions enables us to explore an effective way of obtaining 2DNASP in EQM, it is based on the following new commutators: [20-22]:

$$\begin{cases}\left[\hat{x}^{\mu},\hat{x}^{\nu}\right]_{*}=i\theta^{\mu\nu}\\ \left[\hat{x}^{\mu}(t),\hat{x}^{\nu}(t)\right]_{*}=i\theta^{\mu\nu}\\ \left[\hat{p}^{\mu},\hat{p}^{\nu}\right]_{*}=i\bar{\theta}^{\mu\nu}\\ \left[\hat{p}^{\mu}(t),\hat{p}^{\nu}(t)\right]_{*}=i\bar{\theta}^{\mu\nu}\end{cases}\Rightarrow\begin{cases}\left[\hat{x}^{\mu},\hat{x}^{\nu}\right]=i\theta^{\mu\nu}\\ \left[\hat{x}^{\mu}(t),\hat{x}^{\nu}(t)\right]=i\theta^{\mu\nu}\\ \left[\hat{p}^{\mu},\hat{p}^{\nu}\right]=i\bar{\theta}^{\mu\nu}\\ \left[\hat{p}^{\mu}(t),\hat{p}^{\nu}(t)\right]=i\bar{\theta}^{\mu\nu}\end{cases} \tag{9}$$

The generalized positions and momentum coordinates $(\hat{x}_\mu, \hat{p}_\nu)$ in (NC: 2D-RSP) symmetries depend on the corresponding usual generalized positions and momentum coordinates (x_μ, p_ν) in ordinary quantum mechanics as follows, respectively [23-24]:

$$(x_\mu, p_\nu) \Rightarrow (\hat{x}_\mu, \hat{p}_\nu) = \left(x_\mu - \frac{\theta^{\mu\nu}}{2} p_\nu, p_\mu + \frac{\bar{\theta}^{\mu\nu}}{2} x_\nu \right). \quad (10)$$

The above equation allows us to obtain the two operators ($\hat{r}^2$ and $\hat{p}^2$) in (NC-2D: RSP), respectively [20-22]:

$$(r^2, p^2) \Rightarrow (\hat{r}^2, \hat{p}^2) = (r^2 - \theta L_z \ , p^2 + \bar{\theta} L_z) \quad (11)$$

with $\theta \equiv \theta_{12}$ and $\bar{\theta} \equiv \bar{\theta}_{12}$, thus, the reduced Schrödinger equation (without star product) can be written as:

$$\begin{aligned} &\hat{H}(\hat{p}_i, \hat{x}_i) * \Psi\left(\vec{\hat{r}}\right) = E_{nc-as} \Psi\left(\vec{\hat{r}}\right) \Rightarrow \\ &\Rightarrow H_{nc-as}(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc-as} \psi(\vec{r}). \end{aligned} \quad (12)$$

This equation is equivalent to ordinary SE with two simultaneous translations which are given by eq. (10). The first translation is applied to the kinetic energy, while the second one is applied to the interaction potential. Thus, the Hamiltonian operator $H_{nc-as}(\hat{p}_i, \hat{x}_i)$ can be expressed as:

$$H_{nc-as}(\hat{p}_i, \hat{x}_i) \equiv \frac{\hat{p}^2}{2\mu} + V_{as}(\hat{r}). \quad (13)$$

The 2DNASP $V_{as}(\hat{r})$ is given by:

$$V_{as}(\hat{r}) = a\hat{r}^2 - b\hat{r}^4 + c\hat{r}^6. \quad (14)$$

After straightforward calculations, we can obtain the important terms ($a\hat{r}^2$, $-b\hat{r}^4$ and $c\hat{r}^6$), which will be used to determine the 2DNASP in (NC: 2D- RSP) symmetries as:

$$\begin{aligned} ar^2 &\to a\hat{r}^2 = ar^2 - a\theta L_z + O(\theta), \\ br^4 &\to b\hat{r}^4 = br^4 - 2br^2\theta L_z + O(\theta), \\ cr^6 &\to c\hat{r}^6 = cr^6 - 3cr^4\theta L_z + O(\theta). \end{aligned} \quad (15)$$

We further substitute the Eqs. (15) and (11) into eq. (13) and obtain the global our working new Hamiltonian operator $H_{\text{nc-as}}(\hat{r})$ for 2DNASP, which satisfies the equation for (NC: 2D-RSP) symmetries:

$$H_{nc-as}\ (\hat{r}\hat{p}\ ) = H_{as}(p_i, x_i) + \left(2br^2 - 3cr^4 - a\right)\theta L_z + \frac{\bar{\theta} L_z}{2\mu}, \quad (16)$$

where the operator $H_{as}(p_i, x_i)$ is just the ordinary Hamiltonian operator for anharmonic sextic potential in commutative space:

$$H_{as}(p_i, x_i) = \frac{p^2}{2\mu} + ar^2 - br^4 + cr^6, \quad (17)$$

while the rest five terms are proportional to two infinitesimals parameters (θ and $\bar{\theta}$), and then we can consider perturbations terms $H_{\text{per-}as}(r)$ in (NC: 2D-RSP) symmetries for 2DNASP as:

$$H_{\text{per-}as}(r) = \left(2br^2 - 3cr^4 - a\right)\theta L_z + \frac{\bar{\theta} L_z}{2\mu}. \quad (18)$$

3.1 The Exact Modified Spin-orbital Spectrum for 2DNASP in global (NC: 2D- RSP) Symmetries

It is well known that, in two-dimensional space, the only non-null component of the operator of the moment $\vec{L}$ is the vertical at the movement; in our study we have $L_x = L_y = 0$ and $L_z \neq 0$, thus we introduce the gauge condition of Maireche, we have oriented the two arbitraries vectors $\vec{\theta}$ and the spin direction $\vec{S}$ [22-24]:

$$\begin{cases} \vec{\theta}\vec{L} = \theta_x L_x + \theta_y L_y + \theta L_z \ \text{and} \ \vec{\bar{\theta}}\vec{L} = \bar{\theta}_x L_x + \bar{\theta}_y L_y + \bar{\theta} L_z, \\ \vec{\theta} = \theta_x e_x + \theta_y e_y + \theta e_z \quad \text{and} \quad \vec{S} = S_x e_x + S_y e_y + S_z e_z. \end{cases} \quad (19)$$

Now, we orient the spin moment $\vec{S}$ with the directions of two arbitrary vectors $\vec{\theta}$ and $\vec{\bar{\theta}}$, this allows us to apply our gauge condition:

$$\vec{\theta}\vec{L} = \gamma\theta\vec{S}\vec{L} \ \text{and} \ \vec{\bar{\theta}}\vec{L} = \gamma\bar{\theta}\vec{S}\vec{L}. \quad (20)$$

Here $\gamma \approx \frac{1}{137}$ is a constant, which plays the role of fine structure constant. This allows us to obtain the forms of $H_{\text{so-as}}(r, \theta, \bar{\theta})$ for 2DNASP as follows:

$$\begin{aligned} &H_{\text{so-as}}(r, \theta, \bar{\theta}) = -\gamma \left\{ \left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu} \right\} G^2 \\ &\text{with } G^2 \equiv \vec{L}\vec{S} = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2 \right). \end{aligned} \quad (21)$$

This operator traduces the coupling between spin $\vec{S}$ and orbital momentum $\vec{L}\vec{S}$. The set $H_{so-as}(r, \theta, \bar{\theta})$, J^2, L^2, and J_z) forms a complete of conserved physics quantities and for $\vec{S} = \vec{1/2}$, the eigenvalues of the spin orbital coupling operator are:

$$k_{\pm} \equiv \tfrac{1}{2}\left\{\left(l \pm \frac{1}{2}\right)(l \pm \frac{1}{2}+1) - l(l+1) - \frac{3}{4}\right\} = \\ = \begin{cases} +\dfrac{l}{2} & \text{for} \quad j = l+1/2, \\ -\dfrac{l+1}{2} & \text{for} \quad j = l+1/2. \end{cases} \tag{22}$$

Corresponding: $j = l+1/2$ (spin up) and $j = l-1/2$ (spin down), respectively, then, one can form a diagonal (2×2) matrix, with diagonal elements are $(H_{so-as})_{11}$ and $(H_{so-as})_{22}$ for 2DNASP in (NC: 2D-RSP) symmetries, as:

$$\begin{aligned} (H_{so-as})_{11} &= -\gamma k_{+}\left\{\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right\} \quad \text{if } j = l+1/2 \\ (H_{so-as})_{22} &= -\gamma k_{-}\left\{\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right\} \quad \text{if } j = l-1/2 \end{aligned} \tag{23}$$

Let us close this subsection by writing the new radial function $R(r)$ for 2DNASP in the symmetries of (NC: 2D-RSP):

$$\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} - \frac{l^2}{r^2}R(r) + (\varepsilon - a_1 r^2 + b_1 r^4 \\ -c_1 r^6 - \left(2br^2 - 3cr^4 - a\right)\vec{\theta}\vec{L} - \vec{\bar{\theta}}\mathbf{L})R(r) = 0. \tag{24}$$

The two terms which composed the expression of $H_{\text{per-}as}(r)$ are proportional to two infinitesimals parameters (θ and $\bar{\theta}$), thus, in what follows, we proceed to solve the modified radial part of the 2DMSE that is, equation (21) by applying standard perturbation theory for their exact solutions at first order of two parameters θ and $\bar{\theta}$.

The modified spin-orbital spectrum of atom's nuclei under 2DNASP interactions

In the present subsection, by means of standard perturbation theory, we want to give a complete prescription to determine the energy level of ground state and first excited states, of nuclei atoms with 2DNASP which is produced by the effect of modified spin-orbital interactions. The general form of the corrections obtained by applying the standard perturbation theory is given by:

$$\begin{aligned} E_{u-as}(n,m,l,b,c) &= -\gamma k_{+}(l)\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right)\left(\sum_{n=0}^{\infty} a_n r^{n+m-1/2}\right)\left(\sum_{n'=0}^{\infty} a_{n'} r^{n'+m-1/2}\right)\left(\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right) r dr, \\ E_{d-as}(n,m,l,b,c) &= -\gamma k_{-}(l)\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right)\left(\sum_{n=0}^{\infty} a_n r^{n+m-1/2}\right)\left(\sum_{n'=0}^{\infty} a_{n'} r^{n'+m-1/2}\right)\left(\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right) r dr. \end{aligned} \tag{25}$$

For ground state, we have the following corrections:

$$\begin{aligned} E_{u-as}(n=0,l=0,b,c) &= -\gamma a_0{}^2 k_{+}(l=0)\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right)\left(\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right) dr, \\ E_{d-as}(n=0,l=0,b,c) &= -\gamma a_0{}^2 k_{-}(l=0)\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right)\left(\left(2br^2 - 3cr^4 - a\right)\theta - \frac{\bar{\theta}}{2\mu}\right) dr. \end{aligned} \tag{26}$$

Now, we can write the above two equations in the new form:

$$\begin{aligned} E_{u-as}(n=0,l=0,b,c) &= -\gamma k_{+}(l=0)\left\{\theta\sum_{i=1}^{3} T_i(n=0,,b,c) + \frac{\bar{\theta}}{2\mu}T_4(n=0,l,b,c)\right\}, \\ E_{d-as}(n=0,l=0,b,c) &= -\gamma k_{-}(l=0)\left\{\theta\sum_{i=1}^{3} T_i(n=0,b,c) + \frac{\bar{\theta}}{2\mu}T_4(n=0,l,b,c)\right\}. \end{aligned} \tag{27}$$

Moreover, the expressions of the four factors $T_i(n=0,m,b,c)$ ($i=\overline{1,4}$) are given by:

$$\begin{aligned} T_1(n=0,b,c) &= 2b\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right) r^2\, dr \to b\int_0^{+\infty} X^{3/2-1}\exp\left(-\lambda X^2 - \gamma X\right) dX \\ T_2(n=0,b,c) &= -3c\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right) r^4\, dr \to -\frac{3c}{2}\int_0^{+\infty} X^{5/2-1}\exp\left(-\lambda X^2 - \gamma X\right) dX \\ T_3(n=0,b,c) &= aT_4(n=0,l,b,c) = -a\int_0^{+\infty} \exp\left(-2\alpha r^2 - 2\beta r^4\right) dr \to -\frac{a}{2}\int_0^{+\infty} X^{1/2-1}\exp\left(-\lambda X^2 - \gamma X\right) dX \end{aligned} \tag{28}$$

We have make a transformation of the variable $r^2 = X$ and we set $\lambda = 2\beta$ and $\gamma = 2\alpha$. Now, we apply the following special integration [25]:

$$\int_0^{+\infty} x^{\nu-1} \exp\left(-\lambda x^2-\gamma x\right)dx=\left(2\lambda\right)^{-\frac{\nu}{2}}\Gamma\left(\nu\right)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-\nu}\left(\frac{\gamma}{\sqrt{2\lambda}}\right). \quad (29)$$

Here $D_{-\nu}\left(\frac{\gamma}{\sqrt{2\lambda}}\right)$ denotes the parabolic cylinder functions function, $\Gamma\left(\nu\right)$ is the Gamma function, $\mathrm{Re}\,l(\lambda)\rangle 0$ and $\mathrm{Re}\,l(\nu\rangle 0)$. After straightforward calculations, we can obtain the results:

$$\begin{aligned}
&T_1\left(n=0,b,c\right)=\frac{b}{2}\sqrt{\Pi}\left(2\lambda\right)^{-\frac{3}{4}}\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-3/2}\left(\frac{\gamma}{\sqrt{2\lambda}}\right)\\
&T_2\left(n=0,b,c\right)=-\frac{3c}{16}\sqrt{\Pi}\left(2\lambda\right)^{-\frac{1}{2}}\Gamma\left(5/2\right)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-5/2}\left(\frac{\gamma}{\sqrt{2\lambda}}\right)\\
&T_3\left(n=0,b,c\right)\ =aT_4\left(n=0,l,b,c\right)=-a\sqrt{\Pi}\left(2\lambda\right)^{-\frac{1}{4}}\Gamma\left(1/2\right)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-1/2}\left(\frac{\gamma}{\sqrt{2\lambda}}\right)
\end{aligned} \quad (30)$$

As mentioned in ***Wikipedia***, we have used the formula $\Gamma\left(1/2+n\right)=\frac{2n!}{4^{\mathrm{n}}n!}\sqrt{\Pi}$, allowing us to obtain the exact modifications $E_{u-as}\left(n=0,l,b,c\right)$ and $E_{d-as}\left(n=0,l,b,c\right)$ of ground states for nuclei atoms with 2DNASP, which is produced by modified spin-orbital effect $H_{so-as}\left(r,\theta,\bar{\theta}\right)$ as:

$$\begin{aligned}
&E_{u-as}\left(n=0,l=0,b,c\right)=0,\\
&E_{d-as}\left(n=0,l=0,b,c\right)=\frac{\gamma}{2}\left\{\theta T_s\left(n=0,b,c\right)+\frac{\bar{\theta}}{2\mu}T_4\left(n=0,b,c\right)\right\}.
\end{aligned} \quad (31)$$

We have set ($k_+\left(l=0\right)=0$ and $k_+\left(l=0\right)=-1/2$) and $T\left(n=0,b,c\right)=\sum_{i=1}^{3}T_i\left(n=0,b,c\right)$. For the first exited states, we have the following corrections:

$$\begin{aligned}
&E_{u-as}\left(n=1,l,b,c\right)=-\gamma k_+\left(l=1\right)\Big)\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)\left(a_0r^{m-1/2}+a_1r^{m+1/2}\right)^2\left(\left(2br^2-3cr^4-a\right)\theta-\frac{\bar{\theta}}{2\mu}\right)rdr,\\
&E_{d-as}\left(n=1,l,b,c\right)=-\gamma k_-\left(l=1\right)\Big)\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)\left(a_0r^{m-1/2}+a_1r^{m+1/2}\right)^2\left(\left(2br^2-3cr^4-a\right)\theta-\frac{\bar{\theta}}{2\mu}\right)rdr.
\end{aligned} \quad (32)$$

Now, we can write the above two equations in the new form:

$$\begin{aligned}
&E_{u-as}\left(n=1,m,b,c\right)=-\gamma k_+\left(l=1\right)\Big)\left\{\theta\sum_{i=1}^{9}T_i\left(n=1,m,b,c\right)+\frac{\bar{\theta}}{2\mu}\sum_{i=10}^{12}T_i\left(n,=1,m,b,c\right)\right\}\\
&E_{d-as}\left(n=1,m,b,c\right)=-\gamma k_-\left(l=1\right)\Big)\left\{\theta\sum_{i=1}^{9}T_i\left(n=1,m,b,c\right)+\frac{\bar{\theta}}{2\mu}\sum_{i=10}^{12}T_i\left(n=1,m,b,c\right)\right\}
\end{aligned} \quad (33)$$

Moreover, the expressions of the four factors $T_i\left(n,m,b,c\right)$ ($i=\overline{1,4}$) are given by:

$$\begin{aligned}
&T_1\left(n=1,m,b,c\right)=2ba_0{}^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+2}\,dr\to ba_0{}^2\int_0^{+\infty}X^{m+3/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_2\left(n=1,m,b,c\right)=-3ca_0{}^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+4}\,dr\to-\frac{3}{2}ca_0{}^2\int_0^{+\infty}X^{m+5/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_3\left(n=1,m,b,c\right)\ =-aa_0{}^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m}dr\to-\frac{aa_0{}^2}{2}\int_0^{+\infty}X^{m+1/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_{10}\left(n=1,m,b,c\right)=-a_0{}^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m}dr\to-\frac{a_0{}^2}{2}\int_0^{+\infty}X^{m+1/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX.
\end{aligned} \quad (34)$$

$$\begin{aligned}
&T_4(n=1,m,b,c)=2ba_1^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+4}\,dr\to ba_1^2\int_0^{+\infty}X^{m+5/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_5(n=1,m,b,c)=-3ca_1^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2l+8}\,dr\to-\frac{3}{2}ca_1^2\int_0^{+\infty}X^{m+7/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_6(n=1,m,b,c)\ =-aa_1^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+2}dr\to-\frac{aa_1^2}{2}\int_0^{+\infty}X^{m+3/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_{11}(n=1,m,b,c)=-a_1^2\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+2}dr\to-\frac{a_1^2}{2}\int_0^{+\infty}X^{m+3/2-1}\exp\left(-\lambda X^2-\gamma X\right)dX.
\end{aligned}\tag{35}$$

$$\begin{aligned}
&T_7(n=1,m,b,c)=4ba_1a_0\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+3}\,dr\to2ba_1a_0\int_0^{+\infty}X^{m+2-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_8(n=1,m,b,c)=-6ca_1a_0\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+5}\,dr\to-3ca_1a_0\int_0^{+\infty}X^{m+3-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_9(n=1,m,b,c)\ =-2aa_1a_0\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+1}dr\to-aa_1a_0\int_0^{+\infty}X^{m+1-1}\exp\left(-\lambda X^2-\gamma X\right)dX,\\
&T_{12}(n=1,m,b,c)=-2a_1a_0\int_0^{+\infty}\exp\left(-2\alpha r^2-2\beta r^4\right)r^{2m+1}dr\to-a_1a_0\int_0^{+\infty}X^{m+1-1}\exp\left(-\lambda X^2-\gamma X\right)dX.
\end{aligned}\tag{36}$$

Now, we apply the previous special integration to evaluate the above factors and after straightforward calculations we can obtain the explicit results

$$\begin{aligned}
&T_1(n=1,m,b,c)=ba_0^2(2\lambda)^{-\frac{m+3/2}{2}}\Gamma(m+3/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+3/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_2(n=1,m,b,c)=-\frac{3}{2}ca_0^2(2\lambda)^{-\frac{m+5/2}{2}}\Gamma(m+5/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+5/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_3(n=1,m,b,c)\ =aT_{10}(n=1,m,b,c)=-\frac{aa_0^2}{2}(2\lambda)^{-\frac{m+1/2}{2}}\Gamma(m+1/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+1/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right).
\end{aligned}\tag{37}$$

$$\begin{aligned}
&T_4(n=1,m,b,c)=ba_1^2(2\lambda)^{-\frac{m+5/2}{2}}\Gamma(m+5/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+5/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_5(n=1,m,b,c)=-\frac{3}{2}ca_1^2(2\lambda)^{-\frac{m+7/2}{2}}\Gamma(m+7/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+7/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_6(n=1,m,b,c)\ =aT_{11}(n=1,m,b,c)=-\frac{aa_1^2}{2}(2\lambda)^{-\frac{m+3/2}{2}}\Gamma(m+3/2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+3/2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right).
\end{aligned}\tag{38}$$

$$\begin{aligned}
&T_7(n=1,m,b,c)=2ba_1a_0(2\lambda)^{-\frac{m+2}{2}}\Gamma(m+2)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+2)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_8(n=1,m,b,c)=-3ca_1a_0(2\lambda)^{-\frac{m+3}{2}}\Gamma(m+3)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+3)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right),\\
&T_9(n=1,m,b,c)\ =aT_{12}(n=1,m,b,c)=-aa_1a_0(2\lambda)^{-\frac{m+1}{2}}\Gamma(m+1)\exp\left(\frac{\gamma^2}{8\lambda}\right)D_{-(m+1)}\left(\frac{\gamma}{\sqrt{2\lambda}}\right).
\end{aligned}\tag{39}$$

This allows us to obtain the exact modifications $E_{u-as}(n=1,m,b,c)$ and $E_{d-as}(n=1,m,b,c)$ of the first excited states for nuclei atoms with 2DNASP, which is produced by modified spin-orbital effect $H_{so-as}(r,\theta,\bar{\theta})$ as:

$$E_{u-as}(n=1,m,b,c) = -\gamma k_{+}(l)\left\{\theta T_s(n=1,m,b,c) + \frac{\bar{\theta}}{2\mu}T_p(n=1,m,b,c)\right\},$$
$$E_{d-as}(n=1,m,b,c) = -\gamma k_{-}(l)\left\{\theta T_s(n=1,m,b,c) + \frac{\bar{\theta}}{2\mu}T_p(n=1,m,b,c)\right\} \quad (40)$$

with $T_s(n=1,m,b,c) = \sum_{i=1}^{9} T_i(n=1,m,b,c)$ and $T_p(n=1,m,b,c) = \sum_{i=10}^{12} T_i(n,=1,m,b,c)$.

3.2 The Modified Magnetic Spectrum of Atomic Nuclei under 2DNASP Interactions

Further to the important previously obtained results, now, we consider another physically meaningful phenomena produced by the effect of 2DNASP related to the influence of an external uniform magnetic field $\vec{B}$, to avoid the repetition in the theoretical calculations, it's sufficient to apply the prescription of Maireche:

$$\vec{\Theta} \to \chi\vec{B} \Rightarrow \left(2br^2 - 3cr^4 - a\right)\vec{\mathbf{L}}\vec{\Theta} \to$$
$$\to \chi\left(2br^2 - 3cr^4 - a\right)\vec{B}\vec{L}, \quad (41)$$
$$\vec{\theta} \to \bar{\sigma}\vec{B} \equiv \bar{\sigma}B \Rightarrow \frac{\vec{\theta}\vec{L}}{2} \to \frac{\bar{\sigma}}{2}\vec{B}\vec{L}.$$

Here χ and $\bar{\sigma}$ are two infinitesimal real proportional constants, and we choose the arbitrary external magnetic field $\vec{B} = B\vec{k}$ parallel to the (Oz) axis, which allows us to introduce the modified magnetic Hamiltonian H_{m-as} in (NC: 2D-RSP) symmetries as:

$$H_{m-as} = \left\{\left(2br^2 - 3cr^4 - a\right)\chi + \frac{\bar{\sigma}}{2\mu}\right\}\aleph_{\text{mod}-z}. \quad (42)$$

Here $\aleph_{\text{mod}-z} \equiv \vec{B}\vec{J} - \aleph_z$ denotes the modified Zeeman effect, while $\aleph_z \equiv -\vec{S}\vec{B}$ is the ordinary Hamiltonian operator of Zeeman Effect. To obtain the exact noncommutative magnetic modifications of energy $E_{\text{mag-}as}(n=0,m,b,c)$ and $E_{\text{mag-}as}(n=1,m,b,c)$ for ground states and first exited states, we just replace k_+ and θ into the eqs. (31) and (40) by the following parameters m and χ, respectively:

$$E_{\text{mag-}as}(n=0,m,b,c) = -\gamma B\left\{\chi T(n=0,b,c) + \frac{\bar{\sigma}}{2\mu}T_4(n=0,b,c)\right\}m,$$
$$E_{\text{mag-}as}(n=1,m,b,c) = -\gamma B\left\{\chi T_s(n=1,m,b,c) + \frac{\bar{\sigma}}{2\mu}T_p(n=1,m,b,c)\right\}m. \quad (43)$$

We have $-l \le m \le +l$, which allows us to fix $(2l+1)$ values for discreet number m.

4. GLOBAL SPECTRUM OF NUCLEI ATOMS UNDER 2DNASP

We are now in a position to attack the main objective of our study, let us resume the nonrelativistic modified eigenenergies $E_{\text{nc -}as\text{(u,d)}}(n=0,j,l=0,s,m,b,c) \equiv E_{\text{nc -}as0\text{(u,d)}}$ and $E_{\text{nc -}as\text{(u,d)}}(n=1,j,l,s,m,b,c) \equiv E_{\text{nc -}as1\text{(u,d)}}$ of nuclei atoms under 2DNASP obtained from solving 2DMSE for ground states and first excited states in (NC: 2D-RSP) symmetries. Based on our original results presented in Eqs. (31), (40) and (43), in addition to the ordinary energy E_{nm} for anharmonic sextic potential, which is presented in the Eq. (7):

$$E_{\text{nc -}as0\text{u}} = \frac{3b}{2}\sqrt{\frac{1}{2\mu c}},$$
$$E_{\text{nc -}as0\text{d}} = \frac{3b}{2}\sqrt{\frac{1}{2\mu c}} + \frac{1}{2}\gamma\left\{\theta T_s(n=0,b,c) + \frac{\bar{\theta}}{2\mu}T_4(n=0,b,c)\right\} \quad (44)$$

and

$$E_{\text{nc -}as1\text{u}} = \frac{b}{2}\sqrt{\frac{1}{2\mu c}}(2m+4) - \frac{\gamma}{2}k_{+}\left\{\theta T_s(n=1,m,b,c) + \frac{\bar{\theta}}{2\mu}T_p(n=1,m,b,c)\right\} - \gamma B\left\{\chi T_s(n=1,m,b,c) + \frac{\bar{\sigma}}{2\mu}T_p(n=0,m,b,c)\right\}m,$$
$$E_{\text{nc -}as1\text{d}} = \frac{b}{2}\sqrt{\frac{1}{2\mu c}}(2m+4) + \gamma\left\{\theta T_s(n=1,m,b,c) + \frac{\bar{\theta}}{2\mu}T_p(n=1,m,b,c)\right\} - \gamma B\left\{\chi T_s(n=1,m,b,c) + \frac{\bar{\sigma}}{2\mu}T_p(n=0,m,b,c)\right\}m \quad (45)$$

with $m = 0, \pm 1$. In what follows, to obtain the generalized energy $\left(E_{\text{nc -}as\text{nu}}, E_{\text{nc -}as\text{nd}}\right)$ for the case of the n^{th} excited states, we need to replace both $\left(T_s(n=1,m,b,c), T_p(n=1,m,b,c)\right)$ by new two factors $\left(T_s(n,m,b,c), T_p(n,m,b,c)\right)$ and E_{1m} by E_{nm}, thus we have:

$$E_{\text{nc -asnu}}=\frac{b}{2}\sqrt{\frac{1}{2\mu c}}(2n+2m+2)-\frac{\gamma l}{2}\left\{\theta T_s(n,m,,b,c)+\frac{\bar{\theta}}{2\mu}T_p(n,m,b,c)\right\}-\gamma B\left\{\chi T(n,m,b,c)+\frac{\bar{\sigma}}{2\mu}T_p(n,m,b,c)\right\}m,$$
$$E_{\text{nc -asnd}}=\frac{b}{2}\sqrt{\frac{1}{2\mu c}}(2n+2m+2)+\frac{\gamma(l+1)}{2}\left\{\theta T_s(n,m,b,c)+\frac{\bar{\theta}}{2\mu}T_p(n,m,b,c)\right\}-\gamma B\left\{\chi T_s(n,m,b,c)+\frac{\bar{\sigma}}{2\mu}T_p(n,m,b,c)\right\}m \quad (46)$$

with $l=\overline{0,n-1}$ and $m=\overline{0,\pm l}$, this is the main goal of this work. It's clearly, that the obtained eigenvalues of energies are real and then the NC diagonal Hamiltonian H_{nc-as} is Hermitian, furthermore it is possible to write the two elements $(H_{nc-as})_{11}$ and $(H_{nc-as})_{22}$ of NC nonrelativistic Hamiltonian describing nuclei atoms with 2DNASP as follows:

$$(H_{nc-as})_{11}=-\frac{\Delta_{nc}}{2\mu}+H_{\text{int}-asu},$$
$$(H_{nc-as})_{22}=-\frac{\Delta_{nc}}{2\mu}+H_{\text{int}-asd}, \quad (47)$$

where the kinetic energy $\frac{\Delta_{nc}}{2\mu}$ modified by the operator $(-\frac{\bar{\theta}L_z+\bar{\sigma}L_z}{2\mu})$ and the two modified interactions $H_{\text{int}-asu}$ and $H_{\text{int}-asd}$ are determined by the following equations, respectively:

$$\frac{\Delta_{nc}}{2\mu}=\frac{1}{2\mu}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)-\frac{\bar{\theta}L_z+\bar{\sigma}L_z}{2\mu},$$
$$H_{\text{int}-uas}=ar^2-br^4+cr^6-\gamma\left(k_+(l)\theta+\chi\aleph_{\text{mod}-z}\right)\left(2br^2-3cr^4-a\right),$$
$$H_{\text{int}-das}=ar^2-br^4+cr^6-\gamma\left(k_-(l)\theta+\chi\aleph_{\text{mod}-z}\right)\left(2br^2-3cr^4-a\right). \quad (48)$$

Thus, the ordinary kinetic term for anharmonic sextic $\left(-\frac{\Delta}{2\mu}\right)$ and ordinary interaction described by Eq. (3) are replaced by modified form of kinetic term $\left(-\frac{\Delta_{nc}}{2\mu}\right)$ and modified interactions ($H_{\text{int}-asu}$ and $H_{\text{int}-asd}$), respectively. On the other hand, it is evident to consider that the quantum number m takes $(2l+1)$ values and we have also two values for $j=l\pm\frac{1}{2}$, thus every state in usual 2D space of energy for 2DNASP will be $2(2l+1)$ sub-states. To obtain the total complete degeneracy of energy level of 2DNASP in 2D-NC spaces-phases, we need to sum all allowed values of l. Total degeneracy is thus

$$\sum_{i=0}^{n-1}(2l+1)\equiv n^2\rightarrow 2\sum_{i=0}^{n-1}(2l+1)\equiv 2n^2. \quad (49)$$

Note that the obtained energy eigenvalues $E_{\text{nc -asn(u,d)}}(n,j,l,s,m,b,c)$ depend on the discrete atomic quantum numbers (n,j,l,s) and m in addition to the parameters (a,b,c) of the 2DNASP.

5. GENERALIZATION OF THE OBTAINED RESULTS TO THE ENERGY SPECTRA FOR THE NUCLEI ATOMS WITH $\vec{S}\neq 1/\vec{2}$

For nuclei atoms with spin $\vec{S}\neq 1/\vec{2}$ in (NC: 2D-RSP), the atomic quantum numbers m and j can take $(2l+1)$ values and $\underbrace{j=|l-s|,|l-s|+1,\ldots,\ldots j=|l+s| \text{ and } j=|l+s|}_{\text{N-possible values for j}}$ values, respectively, thus every old state in usual 2D space will be $N(2l+1)$ sub-states for 2DNASP. Furthermore, we can deduce the global noncommutative Hamiltonian H_{nc-as} in (NC: 2D-RSP) for studied potential based on the previously obtained results:

$$H_{nc-as}=-\frac{1}{2\mu}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)+ar^2-br^4+cr^6+\gamma\left(\theta\left(2br^2-3cr^4-a\right)+\frac{\bar{\theta}}{2\mu}\right)\vec{L}\vec{S}+\left\{\chi\left(2br^2-3cr^4-a\right)+\frac{\bar{\sigma}}{2\mu}\right\}\left(\vec{B}\vec{J}-\vec{B}\vec{S}\right). \quad (50)$$

The corresponding energy spectra for nuclei's atoms:

$$E_{\text{nc -asnu}}=\frac{b}{2}\sqrt{\frac{1}{2\mu c}}(2n+2m+2)-\gamma k(j,l,s)\left\{\theta T_s(n,m,b,c)+\frac{\bar{\theta}}{2\mu}T_p(n,m,b,c)\right\}-\gamma B\left\{\chi T_p(n,m,b,c)+\frac{\bar{\sigma}}{2\mu}T_4(n,m,b,c)\right\}m. \quad (51)$$

We have replaced the factor $k_\pm(j=l\pm 1/2,l,s=1/2)$ by the values $k(j,l,s)\equiv\frac{1}{2}\{j(j+1)+l(l+1)-s(s+1)\}$. Paying attention to the behavior of the spectrums (44), (45), (46) and (51), it is possible to recover the results of commutative space when we consider $(\theta,\bar{\theta})\rightarrow(0,0)$.

6. CONCLUSIONS

In this paper, 2DMSE for 2DNASP has been solved via Bopp's shift method and time-independent standard perturbation theory in (NC: 2D-RSP) symmetries. We resume the main obtained results:

- The energy eigenvalues E_{nm} were replaced by new degenerated energy eigenvalues $E_{\text{nc -(u-d)clh}}(n,b,c,j,l,s,m)$,
- Ordinary interactions $V(r)$ were replaced by 2DNASP ($H_{\text{int}-uas}$ and $H_{\text{int}-das}$) for nuclei atoms,
- The ordinary kinetic term $-\frac{\Delta}{2\mu}$ is modified to the form $\frac{\Delta_{nc}}{2\mu}$ for 2DNASP,
- It has been shown that the 2DMSE presents useful rich spectrums for improved understanding of nuclei atoms influenced by the 2DNASP, and we have also seen that the modified spin-orbital interaction and modified Zeeman effect appeared due to the presence of the two infinitesimal parameters (θ,χ), which are induced by position-position noncommutativity property of space.

ACKNOWLEDGEMENTS

This work was supported with search laboratory of Physics and Material Chemistry, in Physics department, Sciences faculty, University of M'sila, Algeria. The author would like to thank the kind's referees for the positive comments and suggestions, which led to the improved standard of this article.

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Нерелятивістська Субатомна Модель для Опису Поведінки Двомірного Ангармонічного Потенціалу Шостого Порядка для Атомного Ядра в Симетріях Розширеної Квантової Механіки

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У роботі представлена нерелятивістська аналітична модель, яка використовується для модифікації рівняння Шредінгера для субатомних масштабів з двовимірним новим ангармонічним потенціалом шостого порядку для атомних ядер в некомутативній двомірній реальній просторовій фазі. Ми застосували узагальнений метод зсуву Боппа, щоб отримати цей потенціал, який складається зі звичайного ангармонічного потенціалу шостого порядка і нової адитивної частини, пропорційної нескінченно малому параметру θ. Ми також спостерігали кінетичний доданок, що містить нову адитивну частину, пропорційну параметру $\bar{\theta}$. Таким чином, запропонований потенціал, пропорційний двом нескінченно малим параметрам, дозволив розглянути доданки збурення завдяки некомутативності.

Крім того, стандартна теорія збурень дозволила знайти поправки $E_{(u-d)-as}(n,m,l,b,c)$ і $E_{\text{mag-}as}(n,m,b,c)$, що відповідають спін-орбітальній взаємодії та модифікованому ефекту Зеемана відповідно. Ці поправки дозволили знайти глобальні енергетичні рівні $E_{\text{nc -(u-d)clh}}(n,b,c,j,l,s,m)$, які залежать від дискретних атомних квантових чисел (*j*, *l*, *s*, *m*) та параметрів досліджуваного потенціалу (*a*, *b*, *c*), і спростили відповідний оператор Гамільтона $H_{nc-as}(\hat{p}_i,\hat{x}_i)$. Ми також узагальнили отримані результати, щоб включити атоми інших ядер із спіном $\vec{S}$ відмінним від $\overrightarrow{1/2}$. Це дослідження дозволило отримати енергетичний спектр у розширеній квантовій механіці, яка інтерпретувала три фізичні явища. Перше явище – це звичайний двовимірний новий ангармонічний потенціал шостого порядку, а друге та третє – це автоматична поява спін-орбітальної взаємодії і модифікованого магнітного впливу. Ці явища породжені з властивості некомутативності простору і фази. Отримані аналітичні результати (власні значення енергії та відповідний оператор Гамільтона) добре узгоджуються з відомими результатами. Попередні результати, у звичайній квантовій механіці, стають особливими випадками, коли $(\theta,\bar{\theta})\to(0,0)$.

Ключові слова: Рівняння Шрьодінгера, Ангармонічний потенціал шостого порядку, Некомутативна просторова фаза, Продукт зірки, Узагальнений метод зсуву Боппа.