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REGULARIZATION OF ILL-POSED PROBLEMS IN HILBERT SPACE BY MEANS OF THE IMPLICIT ITERATION PROCESS

O. V. MATYSIK

Резюме. В роботі доведена збіжність методу з апостеріорним вибором числа ітерацій у вихідній нормі гільбертового простору в разі самоспряженого оператора, в припущенні, що похибки вносяться у праву частину рівняння. Отримано оцінку похибки методу і оцінку для апостеріорного моменту зупинки. Отримані результати можуть бути використані в теоретичних дослідженнях при розв'язуванні лінійних операторних рівнянь, а також при вирішенні прикладних некоректних задач, які зустрічаються в динаміці і кінетиці, математичній економіці, геофізиці, спектроскопії, системах повної автоматичної обробки та інтерпретації експериментів, діагностиці плазми, сейсмології, медицині.

Abstract. The article substantiates the convergence of the method with a posteriori choice of the number of iterations in the original norm of Hilbert space in case of a self-adjoint operator on the assumption of existing errors in the equation right-hand member. There has been secured error estimate of the method and the estimate of a posteriori stopping moment. The results obtained can be used in theoretic research while solving linear operator equations as well as in solving applied incorrect problems which occur in dynamics and kinetics, mathematical economics, geophysics, spectroscopy, systems of full automatic procession and interpretation of experiments, plasma diagnostics, seismology, medicine.

1. Introduction

The article calls attention to the implicit iteration method of solving ill-posed problems, described by iteration equations of type I in Hilbert space. The method represents a family of iterative schemes depending on parameter k.

The comparison of the suggested implicit method with the well-known explicit iteration method $x_{n+1,\delta} = x_{n,\delta} + \alpha\,(y_\delta - Ax_{n,\delta})$, $x_{0,\delta} = 0$ [1–8] demonstrates that the degrees of their optimum estimates coincide. The advantage of explicit methods lies in the fact that explicit methods do not require any operator inversion. They require only the calculation of the operator value on progressive approximation. In this sense the explicit method of [1–8] is preferred to the suggested implicit method. However, the recommended implicit method has a very important advantage. In the explicit method of [1–8] step α is constrained from above by the in equation $0 < \alpha \le \dfrac{5}{4\,\|A\|}$, which may

Key words. Regularization, iteration method, incorrect problem, Hilbert space, self–conjugated and non self–conjugated approximately operator.

actually necessitate a great number of iterations. In the implicit method under consideration there are no restraints from above on the iteration parameter $b > 0$. It follows from this that the optimum estimate of the implicit method under consideration can be obtained as early as at the first iteration steps.

2. Problem statement

One deals with solving the equation

$$Ax = y \tag{1}$$

with the unbounded linear self-adjoint operator A operating in Hilbert space, on the assumption that zero belongs to the spectrum of this operator, though, generally speaking, it is not its characteristic value. According to the suggested hypotheses the problem of solving the equation (1) is incorrect. If the solution of the equation (1) really exists, then a new implicit iteration method is proposed for its finding:

$$\left(A^{2k} + B\right) x_{n+1} = Bx_n + A^{2k-1}y, x_0 = 0, k \in N, \tag{2}$$

where E is a unit operator, while B is a bounded auxiliary self-adjoint operator which is chosen for enhancing conditionality. Let's take operator $B = bE,\ b > 0$ as B. Usually the right-hand member of the equation is known with a certain accuracy δ, i.e. we know y_δ, for which $\|y - y_\delta\| \leq \delta$. That is why instead of (2) it is necessary to consider the approximation

$$\left(A^{2k} + B\right) x_{n+1,\delta} = Bx_{n,\delta} + A^{2k-1}y_\delta, x_{0,\delta} = 0, k \in N. \tag{3}$$

In what follows, the convergence of the method is understood as the statement that approximations (3) fit arbitrarily close the exact solution of the operator equation in case of the suitable choice of n and sufficiently small δ. In other words, method (3) is convergent if

$$\lim_{\delta \to 0} \left(\inf_n \|x - x_{n,\delta}\| \right) = 0.$$

If $b > 0$, the convergence for method (3) is proved in case of an accurate and approximate right-hand member of the equation, and on the assumption that the accurate solution of the equation is sourcewise representable, that is $x = A^{2s}z,\ s > 0$, there has been obtained a priori error estimate

$$\|x - x_{n,\delta}\| \leq \|x - x_n\| + \|x_n - x_{n,\delta}\| \leq \left(\frac{bs}{2kn}\right)^{\frac{s}{k}} \|z\| + 2k \left(\frac{n}{b}\right)^{\frac{1}{2k}} \delta,$$

$n \geq 1$ [9]. This error estimate has been optimized:

$$\|x - x_{n,\delta}\|_{opt} \leq (1 + 2s) \left(\frac{s}{k}\right)^{\frac{s(1-2k)}{k(1+2s)}} 2^{-\frac{s}{k(2s+1)}} \|z\|^{\frac{1}{2s+1}} \delta^{\frac{2s}{2s+1}}$$

and a priori stopping moment has been found

$$n_{opt} = 2^{-\frac{2s}{2s+1}} \left(\frac{s}{k}\right)^{\frac{2(s+k)}{2s+1}} b\|z\|^{\frac{2k}{2s+1}} \delta^{-\frac{2k}{2s+1}}.$$

It is evident that the optimum estimate does not depend on iteration parameter b, but n_{opt} does depend on b. Consequently, for reducing the calculating procedure one should take b satisfying the condition $b > 0$ and proceed from the assumption that $n_{opt} = 1$. For that purpose it is enough to choose

$$b_{opt} = 2^{\frac{2s}{2s+1}} \left(\frac{s}{k}\right)^{-\frac{2(s+k)}{2s+1}} \|z\|^{-\frac{2k}{2s+1}} \delta^{\frac{2k}{2s+1}}.$$

The article [10] proves that provided $b > 0$, the iteration method (3) converges in the energy norm of Hilbert space $\|x\|_A = \sqrt{(Ax, x)}$, when one chooses the number of iterations n from the condition $\sqrt[4k]{n}\delta \to 0$ at $n \to \infty$, $\delta \to 0$. Without knowing the sourcewise representability of the exact solution, it is in the energy norm that there has been found a priori stopping moment $n_{opt} = b2^{-\frac{3+2k}{2}} k^{-\frac{1+2k}{2}} \|x\|^{2k}\delta^{-2k}$ and the conditions when the convergence in the energy norm results in the convergence in the original norm of Hilbert space H. In case of non-unique solution of the equation (1) the article [10] also proves that process (2) comes to the normal solution, i.e. the solution with a minimum norm.

3. Rule of stopping due to infinitesimal residual

When there is no information about the sourcewise representability of the exact solution, method (3) becomes ineffective, as it is impossible to get the error estimate and find the a priori stopping epoch in the original norm of Hilbert space. Nevertheless, one can make method (3) quite effective if one uses the following rule due to infinitesimal residual $[3-4]$. Here and in what follows, we shall consider that A is a bounded linear self-adjoint operator.

Let us set the stopping moment level $\varepsilon > 0$, $\varepsilon = b_1\delta$, $b_1 > 1$ and the moment m of stopping the iteration process (3) by condition

$$\|Ax_{n,\delta} - y_\delta\| > \varepsilon, (n < m), \|Ax_{m,\delta} - y_\delta\| \le \varepsilon. \qquad (4)$$

Let us suppose that at initial approximation $x_{0,\delta}$ the residual is large enough, that is, larger than stopping level, i.e. $\|Ax_{0,\delta} - y_\delta\| > \varepsilon$. In what follows method (3) with stopping rule (4) is convergent provided

$$\lim_{\delta\to 0}\left(\inf_m \|x - x_{m,\delta}\|\right) = 0$$

Let us show the possible application of rule (4) to method (3). Consider the collection of functions $g_n(\lambda) = \frac{1}{\lambda}\left[1 - \frac{b^n}{(\lambda^{2k}+b)^n}\right] \ge 0$. By using the results of [9] it is easy to show that at $b > 0$ for $g_n(\lambda)$ the following conditions hold

$$\sup_{-M\le\lambda\le M} |g_n(\lambda)| \le 2k\left(\frac{n}{b}\right)^{1/(2k)}, n > 0, M = \|A\|, \qquad (5)$$

$$\sup_{-M\le\lambda\le M} |1 - \lambda g_n(\lambda)| \le 1, n > 0, \qquad (6)$$

$$1 - \lambda g_n(\lambda) \to 0, n \to \infty, \forall\lambda \in [-M, M], \qquad (7)$$

$$\sup_{-M\le\lambda\le M} \left|\lambda^{2s}(1 - \lambda g_n(\lambda))\right| \le \left(\frac{bs}{2kn}\right)^{s/k}, kn > s, 0 \le s < \infty. \qquad (8)$$

One finds valid

Lemma 1. *Let A be a bounded operator, $A = A^*$. Then for any $\omega \in H$ $(E - Ag_n(A))\,\omega \to 0, n \to \infty$.*

Proof. By using the integral expression of operator $A = \int\limits_{-M}^{M} \lambda dE_\lambda$, where $M = \|A\|$ and E_λ is the spectral function of operator A, we get

$$(E - Ag_n\,(A))\,\omega = \int\limits_{-M}^{M} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega =$$

$$= \int\limits_{0}^{M} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega + \int\limits_{-M}^{0} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega = I_1 + I_2.$$

Let us break up the first of the integrals obtained into two integrals

$$I_1 = \int\limits_{0}^{\varepsilon_0} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega + \int\limits_{\varepsilon_0}^{M} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega.$$

Since $1 - \lambda g_n(\lambda) = \dfrac{b^n}{(\lambda^{2k} + b)^n} \le q^n(\varepsilon_0) < 1$ for all $\lambda \in [\varepsilon_0, M]$, we get

$$\left\| \int\limits_{\varepsilon_0}^{M} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega \right\| \le q^n(\varepsilon_0) \left\| \int\limits_{\varepsilon_0}^{M} dE_\lambda\omega \right\| \le q^n(\varepsilon_0)\|\omega\| \to 0, n \to \infty.$$

On the basis of condition (6) we have

$$\left\| \int\limits_{0}^{\varepsilon_0} (1 - \lambda g_n(\lambda))\,dE_\lambda\omega \right\| \le \left\| \int\limits_{0}^{\varepsilon_0} dE_\lambda\omega \right\| \le \|E_{\varepsilon_0}\omega\| \to 0, \quad \varepsilon_0 \to 0,$$

because of the properties of spectral function [11]. Similarly to that, $I_2 \to 0$, $n \to \infty$. Consequently, $(E - Ag_n(A))\omega \to 0$, $n \to \infty$. Lemma 2.1 is proved.

There occurs

Lemma 2. *Let A be a bounded operator, $A = A^*$. Then for any $\vartheta \in \overline{R(A)}$ there exists correlation $n^{s/k} \left\|A^{2s}(E - Ag_n(A))\vartheta\right\| \to 0$ at $n \to \infty$, $0 \le s < \infty$.*

Proof. Since (8) is true, then

$$n^{s/k} \left\|A^{2s}(E - Ag_n(A))\right\| \le n^{s/k} \sup_{-M\le\lambda\le M} \left|\lambda^{2s}(1 - \lambda g_n(\lambda))\right| \le$$
$$\le n^{s/k}\gamma_s n^{-s/k} = \gamma_s,$$

where $\gamma_s = \left(\dfrac{bs}{2k}\right)^{s/k}$. Let us use Banach-Steingaus theorem [11, p. 151], according to which convergence $B_n u \to Bu$ at $n \to \infty$ for all $u \in H$ is realized only when this convergence occurs in some compact subset in H and $\|B_n\|$, $n = 1, 2, \ldots$, are limited by the constant independent from n.

Let us take subset $R(A)$ as a compact one in $\overline{R(A)} = H$. We suppose that $s_1 = s + \frac{1}{2}$. Then for every $\vartheta = A\omega \in R(A)$ we have

$$n^{s/k} \left\| A^{2s}(E - Ag_n(A))\vartheta \right\| = n^{s/k} \left\| A^{2s+1}(E - Ag_n(A))\omega \right\| =$$

$$= n^{s/k} \left\| A^{2s_1}(E - Ag_n(A))\omega \right\| \leq \gamma_{s_1} n^{\frac{-(s_1-s)}{k}} \|\omega\| = \gamma_{s_1} \|\omega\| n^{-1/(2k)} \to 0,$$

$n \to \infty$, as $s_1 < \infty$. Lemma 2.2 is proved.

There is validity in

Lemma 3. *Let A be a bounded operator, $A = A^*$. Provided for some sequence $n_p < \overline{n} = const$ and $\vartheta_0 \in \overline{R(A)}$ at $p \to \infty$ we get $\omega_p = A\left(E - Ag_{n_p}(A)\right)\vartheta_0 \to 0$, then $\vartheta_p = \left(E - Ag_{n_p}(A)\right)\vartheta_0 \to 0$.*

Proof. Due to (6) sequence ϑ_p is bounded $\|\vartheta_p\| \leq 1, p \in N$. That is why out of this sequence in Hilbert space we can extract a weakly convergent subsequence $\vartheta_p - \to \vartheta, (p \in N' \subseteq N)$, then $A\vartheta_p - \to A\vartheta, (p \in N')$.

But by the data $\omega_p = A\vartheta_p \to 0, p \to \infty$, consequently, $A\vartheta = 0$. Since zero is not the characteristic value of operator A, then $\vartheta = 0$. Hence,

$$\|\vartheta_p\|^2 = \left(\vartheta_p, \left(E - Ag_{n_p}(A)\right)\vartheta_0\right) = (\vartheta_p, \vartheta_0) - \left(\vartheta_p, Ag_{n_p}(A)\vartheta_0\right) =$$

$$= (\vartheta_p, \vartheta_0) - \left(A\vartheta_p, g_{n_p}(A)\vartheta_0\right) =$$

$$= (\vartheta_p, \vartheta_0) - \left(\omega_p, g_{n_p}(A)\vartheta_0\right) \to (\vartheta, \vartheta_0) = 0, \quad \left(p \in N'\right),$$

since $\vartheta = 0, \omega_p \to 0, p \to \infty$ and by the data (5)

$$\left\| g_{n_p}(A) \right\| \leq 2k \left(\frac{n_p}{b}\right)^{1/(2k)} \leq 2k \left(\frac{\overline{n}}{b}\right)^{1/(2k)}$$

Thus, every weakly convergent subsequence of the bounded sequence ϑ_p mentioned above tends to zero according to the norm. Consequently, the whole sequence $\vartheta_p \to 0, p \to \infty$. Lemma 2.3 is proved.

If A is a bounded non self-adjoint operator, lemma 2.3 which is analogous to lemma 4 proves its validity.

Lemma 4. *Let A be a bounded non self-adjoint operator. If for some sequence $n_p < \overline{n} = const$ and $\vartheta_0 \in \overline{R(A)}$ at $p \to \infty$ we have*

$$\omega_p = A^*A\left(E - A^*Ag_{n_p}\left(A^*A\right)\right)\vartheta_0 \to 0,$$

then $\vartheta_p = \left(E - A^*Ag_{n_p}\left(A^*A\right)\right)\vartheta_0 \to 0$.

For proving lemma 2.4 it is necessary to go over to operator $A = A^*A$ and use lemma 2.3.

Let us use the proved lemmas for proving the following theorem.

Theorem 1. *Let A be a bounded operator, $A = A^*$, and let the stopping moment $m = m(\delta)$ in method (3) be chosen according to rule (4). Then $x_{m(\delta),\delta} \to x$ at $\delta \to 0$.*

Proof. In [9] we find that $x_{n,\delta} = A^{-1}\left[E-(CB)^n\right]y_\delta$, where

$$C = \left(A^{2k}+B\right)^{-1}.$$

That is why

$$\begin{aligned} x_{n,\delta}-x &= A^{-1}\left[E-(CB)^n\right]y_\delta - x = \\ &= A^{-1}\left[E-(CB)^n\right](y_\delta-y) + A^{-1}\left[E-(CB)^n\right]y - A^{-1}y = \\ &= A^{-1}\left[E-(CB)^n\right](y_\delta-y) - (CB)^n x = \\ &= g_n(A)(y_\delta-y) - (E-Ag_n(A))x, \end{aligned} \tag{9}$$

consequently,

$$Ax_{n,\delta}-y = Ax_{n,\delta}-Ax = -A(E-Ag_n(A))x + Ag_n(A)(y_\delta-y).$$

Let us consider

$$\begin{aligned} Ax_{n,\delta}-y_\delta &= -A(E-Ag_n(A))x + (y-y_\delta) + Ag_n(y_\delta-y) = \\ &= -A(E-Ag_n(A))x - (E-Ag_n(A))(y_\delta-y). \end{aligned} \tag{10}$$

On the strength of lemmas 2.1 and 2.2 we have

$$\|(E-Ag_n(A))x\| \to 0, n\to\infty, \tag{11}$$

$$\sigma_n = n^{1/(2k)}\|A(E-Ag_n(A))x\| \to 0, n\to\infty. \tag{12}$$

What is more, it follows from (5) and (6) that

$$\|g_n(A)(y_\delta-y)\| \le 2k\left(\frac{n}{b}\right)^{1/(2k)}\delta, \tag{13}$$

$$\|E-Ag_n(A)\| \le 1. \tag{14}$$

Let us use stopping rule (4). Then

$$\|Ax_{m,\delta}-y_\delta\| \le b_1\delta, \quad b_1>1$$

and from (10) and (14) we get

$$\begin{aligned} \|A(E-Ag_m(A))x\| &\le \|Ax_{m,\delta}-y_\delta\| + \|(E-Ag_m(A))(y_\delta-y)\| \le \\ &\le (b_1+1)\delta. \end{aligned} \tag{15}$$

For any $n<m$ $\|Ax_{n,\delta}-y_\delta\| > \varepsilon$, that is why

$$\|A(E-Ag_n(A))x\| \ge \|Ax_{n,\delta}-y_\delta\| - \|(E-Ag_n(A))(y-y_\delta)\| \ge (b_1-1)\,\delta.$$

Thus, for $\forall n<m$

$$\|A(E-Ag_n(A))x\| \ge (b_1-1)\,\delta. \tag{16}$$

From (12) and (16) at $n=m-1$ we have

$$\frac{\sigma_{m-1}}{(m-1)^{1/(2k)}} = \|A(E-Ag_{m-1}(A))x\| \ge (b_1-1)\,\delta$$

or $(m-1)^{1/(2k)}\delta \le \dfrac{\sigma_{m-1}}{b-1} \to 0, \delta\to 0$ (because from (12) $\sigma_m\to 0, m\to\infty$). If in this case $m\to\infty$ at $\delta\to 0$, then using (9), we get

$$\|x_{m,\delta}-x\| \le \|(E-Ag_m(A))\,x\| + \|g_m(A)(y_\delta-y)\| \le$$

$$\leq \|(E - Ag_m(A))\, x\| + 2k \left(\frac{m}{b}\right)^{1/(2k)} \delta \to 0$$

at $m \to \infty, \delta \to 0$, since from (11)

$$\|(E - Ag_m(A))\, x\| \to 0, \quad m \to \infty.$$

Provided for some $\delta \to 0$ the sequence $m(\delta_n)$ turns out to be bounded, $x_{m(\delta_n),\delta_n} \to x, \delta_n \to 0$ is relevant in this case as well. Actually, from (15) we have

$$\left\|A \left(E - Ag_{m(\delta_n)}(A)\right) x\right\| \leq (b_1 + 1)\, \delta_n \to 0, \delta_n \to 0.$$

Hence, according to lemma 2.3 we get that

$$\left(E - Ag_{m(\delta_n)}(A)\right) x \to 0, \delta_n \to 0.$$

As a result

$$\left\|x_{m(\delta_n),\delta_n} - x\right\| \leq \left\|\left(E - Ag_{m(\delta_n)}(A)\right) x\right\| + 2k \left(\frac{m(\delta_n)}{b}\right)^{1/(2k)} \delta_n \to 0, \delta_n \to 0.$$

This proves theorem 2.5.

4. Error estimate

We have

Theorem 2. *Suppose the conditions of theorem 2.5 are fulfilled, operator A is positive and $x = A^{2s}z, s > 0$. Then the following estimates hold*

$$m \leq 1 + \frac{(2s+1)b}{4k} \left[\frac{\|z\|}{(b_1 - 1)\delta}\right]^{\frac{2k}{2s+1}},$$

$$\|x_{m,\delta} - x\| \leq \left[(b_1 + 1)\, \delta\right]^{\frac{2s}{2s+1}} \|z\|^{\frac{1}{2s+1}} +$$
$$+\frac{2k}{b^{1/(2k)}} \left\{1 + \frac{(2s+1)b}{4k} \left[\frac{\|z\|}{(b_1 - 1)\delta}\right]^{\frac{2k}{2s+1}}\right\}^{\frac{1}{2k}} \delta. \tag{17}$$

Proof. Since $x = A^{2s}z$, then

$$\|A(E - Ag_{m-1}(A))x\| = \left\|A^{2s+1}(E - Ag_{m-1}(A))z\right\| =$$

$$= \left\|\int_0^M \frac{\lambda^{2s+1} b^{m-1}}{\left(\lambda^{2k} + b\right)^{m-1}} dE_\lambda z\right\| \leq \left[\frac{(2s+1)b}{4k(m-1)}\right]^{\frac{2s+1}{2k}} \|z\|.$$

By using (16), we get

$$(b_1 - 1)\delta \leq \left[\frac{(2s+1)b}{4k(m-1)}\right]^{\frac{2s+1}{2k}} \|z\|.$$

Hence we have

$$m \leq 1 + \frac{(2s+1)b}{4k} \left[\frac{\|z\|}{(b_1 - 1)\delta}\right]^{\frac{2k}{2s+1}}.$$

With the help of moment inequality let us estimate

$$\|(E - Ag_m(A))x\| = \left\|A^{2s}(E - Ag_m(A))z\right\| \leq$$

$$\leq \left\|A^{2s+1}(E - Ag_m(A))z\right\|^{\frac{2s}{2s+1}} \|(E - Ag_m(A))z\|^{\frac{1}{2s+1}} \leq$$

$$\leq \|A(E - Ag_m(A))x\|^{\frac{2s}{2s+1}} \|z\|^{\frac{1}{2s+1}} \leq [(b_1+1)\delta]^{\frac{2s}{2s+1}} \|z\|^{\frac{1}{2s+1}}.$$

Then

$$\|x_{m,\delta} - x\| \leq \|(E - Ag_m(A))x\| + \|g_m(A)(y_\delta - y)\| \leq$$

$$\leq [(b_1+1)\delta]^{\frac{2s}{2s+1}} \|z\|^{\frac{1}{2s+1}} + 2k\left(\frac{m}{b}\right)^{\frac{1}{2k}} \delta \leq$$

$$\leq [(b_1+1)\delta]^{\frac{2s}{2s+1}} \|z\|^{\frac{1}{2s+1}} + \frac{2k}{b^{1/(2k)}} \left\{1 + \frac{(2s+1)b}{4k}\left[\frac{\|z\|}{(b_1-1)\delta}\right]^{\frac{2k}{2s+1}}\right\}^{\frac{1}{2k}} \delta.$$

This proves theorem 3.1.

Note 1. *The estimate procedure* (17) *is* $O\left(\delta^{\frac{2s}{2s+1}}\right)$ *and, as it follows from* [3], *it is optimal in the class of problems with sourcewise representable solutions.*

Note 2. *The knowledge of order* $2s > 0$ *of sourcewise representability of exact solution, which is used in theorem 2, is not required in practice as it does not hold for the rule of stopping due to infinitesimal residual. Theorem 2 states that the number of iterations m, supporting the optimum error order. But even if the sourcewise representability of the exact solution is missing, stopping due to residual provides the convergence of the method, as it is shown in theorem 1.*

Conclusion. The paper studies some properties of the suggested implicit iteration method of solving ill-posed problems: it proves the convergence of the method with the a posteriori choice of the iteration number in the original norm of Hilbert space. It also presents the obtained error estimate of the method and the estimate of a posteriori stopping moment.

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O. V. Matysik,
Applied Mathematics and Programming Technologies Department,
Brest State University named after A. S. Pushkin,
21 Kosmonavtov boulevard, Brest, 224016, Belarus

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