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THERMODYNAMICS OF SMALL ELECTROMAGNETIC GENERATORS: AN EXPERIMENTAL PERSPECTIVE

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The fabrication of relatively small electromagnetic generators has been reported recently in the literature by a number of research groups. Their characteristic sizes are of the order of one millimeter. With proper tune up, these devices have been used to convert waste ambient vibration noise into useful electric power. We would like to analyze, including experimental considerations, the possibility of reducing the size of the generator to nanometer scale allowing, in principle, the exploitation of more subtle sources of vibrations. We propose a technique for fabricating small coils using nanowires and derive an approximate expression for evaluating the mechanical stress on the nanowire. For an ideal model of a micrometric sized electromagnetic generator we estimate the strength of the induced electromotive force (EMF) and compare it with the intrinsic electrical noise present in the nanowire. We found the strength of the induced EMF to be larger than the electrical noise by two orders of magnitude which is an encouraging result.

Key words: electromagnetic generator, MEMS, nanowires, electrodeposition, Brownian motion, microcoils.

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I. INTRODUCTION

Although the quest for alternative, clean, energy resources poses enormous challenges to modern science and technology, recent advances on nano-scale fabrication and manipulation represent a most promising source for potential solutions. Definitive answers, however, are still to be achieved and the road is open for creative new ideas. Small scale devices open the possibility of considering less conventional sources of energy. Energy *recycling*, for instance, has been proposed as a mean to utilize the energy dissipated to the ambient by inefficient appliances. This ambient noise has been used to power small electronic devices with the use of transducers converting it into electricity [1–4]. An interesting application has been developed by fabricating a small electromagnetic generator [5]. A set of small permanent magnets were mounted on a cantilever resonator beam. As the magnets move, product of the mechanical coupling to the environmental vibrations, an electromotive force (EMF) is induced on a static set of four high density winded coils (600 turns each) producing usable power. Characteristic sizes of the components are of the order of a few millimeters. The coils were made by winding a commercially available enameled copper wire of 25 microns diameter on a core 300 microns wide which is about the smallest size that can be manually winded, in practice, using the given wire. The design of the generator was carefully tuned up to resonate at the environmental noise dominant frequency for best efficiency. The power obtained was enough to operate a radio-frequency emitter thus making an autonomous sensor device. These encouraging results motivate the present study.

Further size reduction would allow other more subtle sources of vibrations to provide the energy source. Small devices experimenting thermal noise induced vibrations, for instance, have already been fabricated. A well known example is found in Scanning Probe Microscopy (SPM), where the cantilever tip undergoes such (undesired) motion which limits the resolution of the probe microscopes. The vibration of scanning tips has been characterized by experiments and computer simulations [6,7]. Depending on tip design, characteristic amplitudes due to thermal noise are of the order of a few Ångström. It is also possible to fabricate a less rigid cantilever to allow amplitudes of the order of a few nanometers. In the case of a magnetized cantilever, there would be a time varying magnetic field. In order to produce a measurable EMF by means of a coil on the proximity of the cantilever, such a coil needs to be small enough to *see* the time changes of the magnetic flux through it. Such a small electromagnetic generator would have very interesting properties. When a load is connected to the coil, a current will flow and heat will be dissipated on the load. At the same time, the coil will induce back a damping magnetic field on the cantilever thus cooling it down. Thus, the system will move heat from the cantilever to the load as a microscopic heat pump.

Experimentally, the construction of an electromagnetic generator capable of sensing the small amplitude Brownian fluctuations faces several challenges, the most relevant being the fabrication of nano and micro metric sized resonant cantilevers, permanent magnets and coils. The first two are readily achievable by standard techniques: cantilever making, for instance, is available through Electron Beam Lithography followed by plasma etching on Si, GaAs, Si_3N_4 and other crystal materials; permanent magnetic moments can be electrochemically grown in-situ. What remains to be developed is a technique to fabricate a high density winded coil of size on the scale of nanometers.

In what follows we propose a technique for mechanically winding a nanowire around a core of small dimensions. We also give some considerations on the experimental setup along with an estimation of the stress applied to the nanowires during the winding process. The expected strength of the induced EMF is compared to the intensity of intrinsic electrical noise present in the coil.

II. MICRO-COIL FABRICATION

We give a brief description of a new fabrication technique for making small sized coils by mechanically winding a metallic nanowire around a core. The fabrication of suitable nanowires have been reported recently [8] under the name of *Lithographically Patterned Nanowire Electrodeposition* (LPNE). We propose a four-step scheme for fabricating the coils, illustrated in Figure 1. A more detailed description is in preparation and will appear elsewhere.

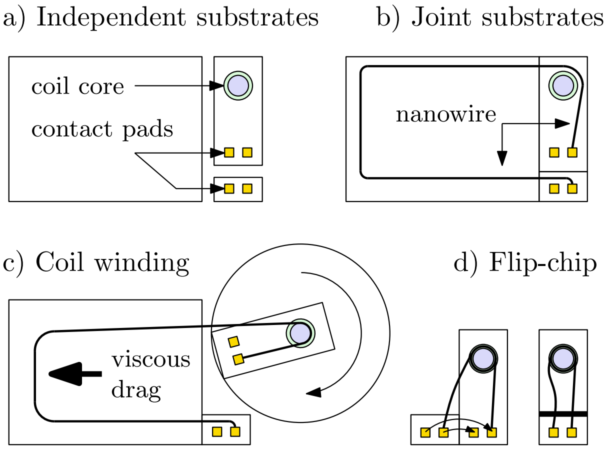

Fig. 1. Four steps scheme for coil fabrication. See the text for details.

a) Independent substrates. Three independent substrates are prepared from a single wafer by slicing, edge cleaning and reassembling. Careful handling is necessary to minimize the mismatch. Lithography is used for making contact pads and the coil core before reassembling the substrate. Deposition of a relatively thick photoresisting layer is necessary to create a smooth surface over which the nanowires will be grown. Previous tests on silicon wafers have been performed and suggest that a layer thickness of 1 micron or thicker will cover the separation between the reassembled pieces.

b) Joint substrates. LPNE [8], a technique recently developed by the group of Penner and which is capable of producing nanowires as small as 20×50 nanometers section and length on the order of centimeters, is applied to create a free standing nanowire attached to the contact pads and surrounding the coil core. After freeing the nanowire from the substrate, an insulating coat is produced by electrodepositing a thin layer of metal on the nanowire which is then oxidized. The technique can produce nanowires of several metals including gold and platinum.

c) Coil winding. A piece of the substrate containing the coil core is laterally displaced to allow in-plane rotation around the coil core axis. Alignment of the rotation axis and the core axis should be close but this is not critical because the wire will follow the core movement. Angular and displacement errors are inevitable even though the present scheme can tolerate small errors. To provide a tight winding, the nanowire is immersed in a viscous liquid which will cause a viscous drag force. There are several parameters to calibrate including the fluid viscosity and the speed of the winding.

d) Flip-chip finishing. The contact pads are flipped onto each other to provide a compact package and to protect the nanowire against mechanical stress. Indium bumps, which have been previously deposited on the contact pads, form external electrical contacts and glue the pieces together after heating to Indium melting point.

III. VISCOUS DRAG FORCES ON NANOWIRES

Coil winding using nanowires requires careful manipulation due to the extreme fragility of the wires. We present an estimation of the viscous drag forces involved on the process which should serve as guidelines for the design of a winding device. We assume the nanowires are of uniform circular section having a diameter of the order of 50 nanometers immersed in a regular, that is, Newtonian viscous fluid. At this length scale it is still approximately correct to treat the fluid as a continuum. Such would not be the case, for instance, at length scales comparable to the fluid molecular size where macroscopic quantities like viscosity would not be properly defined. We here derive an approximate expression for the critical stress on the nanowire σ_c assuming low Reynold's number regime. By using this expression one can estimate the maximum winding speed, for a specific viscous fluid, to safely handle the nanowire.

We consider an infinitely long circular cylinder immersed in a Newtonian viscous fluid with no other boundaries (see Figure 2). The cylinder moves with respect to the fluid at constant speed perpendicular to its axis. The force on the cylinder due to viscosity is known as the *drag* and has been extensively studied in the past [9, 10] in terms of the dimensionless Reynold's number defined as

$$Re = \frac{\rho}{\mu} u_0 d, \tag{1}$$

where the properties of the fluid are given by the density ρ and the dynamical viscosity μ; u_0 is the relative velocity between the fluid and the cylinder and d is the diameter of the immersed cylinder. The drag D is defined as the force exerted by the fluid on the cylinder per unit length. A dimensionless quantity associated with the drag force is the *drag coefficient* C_D. They are related by

$$C_D = \frac{D}{\frac{1}{2}\rho u_0^2 d}. \tag{2}$$

For the low Reynolds number ($Re \lesssim 1$) an analytical expression for the drag coefficient is given by

$$C_D = \frac{8\pi}{Re\,(2.002 - \ln Re)}. \tag{3}$$

Using the given expressions, we have the viscous drag force per unit length as

$$D = \frac{4\pi\mu u_0}{2.002 - \ln\left(\frac{\rho}{\mu}u_0 d\right)} \tag{4}$$

which, in SI units, gives the force in N/m.

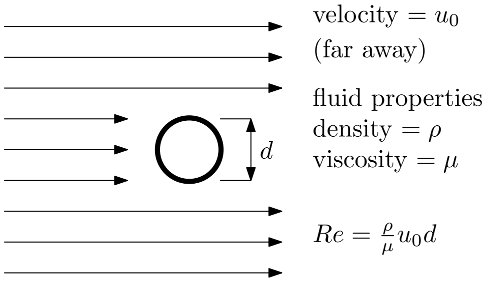

Fig. 2. An infinitely long cylindrical wire immersed on a fluid. The drag force per unit length is a function of the Reynolds number Re.

The winding device consists of a coil core of radius r_c, rotating at an angular speed ω with one end of the wire attached to it. The other end of the wire is attached to a fixed contact pad located a distance L apart. During the winding procedure, the wire will be pulled from one end at a speed of $u_0 = \omega r_c$, the other end remaining at rest (see Figure 3). Using the frame of reference shown in Figure 3 we consider the movement along the x axis only and thus represent the wire by a straight segment along the y axis having the length L.

The velocity of this segment changes linearly with the distance to the pad, as $u(y) = u_0 y/L$. This produces a distributed drag force over the wire acting in the direction of $-x$. The tension produced on the wire can be found considering a reference frame moving with it and using a static balance of forces. Equilibrium along the x axis requires, using the notation of Figure 3 that

$$T_A + T_B = \int_0^L dy D(u), \tag{5}$$

where $u = u(y) = u_0 y/L$. The torques should be balanced as well, thus (with respect to the point A)

$$T_B L = \int_0^L dy\, y\, D(u). \tag{6}$$

Although these integrals can be evaluated in terms of infinite series, we adopt here a simpler approach by noting that $D(u)$ is approximately a linear function of $u(y)$ (therefore also a linear function of y) for $Re \ll 1$ which is the limit we are considering. Thus

$$\int_0^L dy D(u) \approx \frac{1}{2} L D(u_0) \tag{7}$$

and

$$\int_0^L dy\, y\, D(u) \approx \frac{1}{3} L^2 D(u_0) \tag{8}$$

which overestimate the integrals by less than 2.5 and 1.9% respectively, as can be checked numerically. Using these expressions we find

$$\begin{aligned} T_A &\approx \frac{1}{6} L D(u_0) \\ T_B &\approx \frac{1}{3} L D(u_0), \end{aligned} \tag{9}$$

where

$$D(u_0) = \frac{4\pi\mu u_0}{2.002 - \ln(\frac{\rho}{\mu}u_0 d)} \tag{10}$$

and d is the diameter of the nanowire assumed to be approximately cylindrical. From relations (9), the maximum tension on the wire occurs at point B of Figure 3. We refer to this as the critical tension and it is found to be (in Newtons)

$$T_c = T_B = \frac{\frac{4}{3}\pi\mu u_0 L}{2.002 - \ln(\frac{\rho}{\mu}u_0 d)}. \tag{11}$$

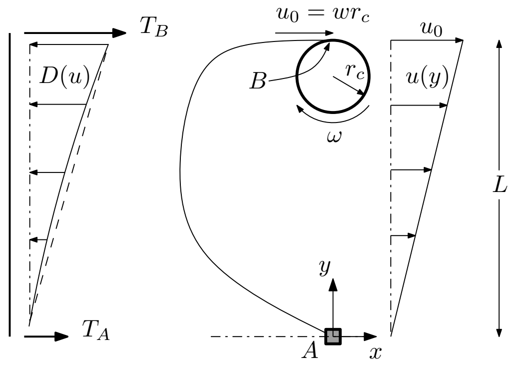

Fig. 3. Outline of the proposed winding device. A projection of the wire movement is considered to estimate the viscous drag force. A frame of reference is located at the fixed contact pad, point A. Point B is located where the wire makes contact with the coil core. The viscous force provides a tight winding. See the text for details.

This expression is valid for a wire far away from other obstacles and substrates. For design we calculate the critical stress (in Pascals), dividing by the cross section area of the nanowire

$$\sigma_c = \frac{16}{3d^2}\left\{\frac{\mu u_0 L}{2.002 - \ln\left(\dfrac{\rho}{\mu}u_0 d\right)}\right\}. \tag{12}$$

As an example we consider a gold nanowire submerged in glycerol. In this case $\rho = 1261.0$ kg/m^3, $\mu = 1.5$ Pa s. For a wire diameter $d = 50$ nm and a coil core of radius $r_c = 5.0\mu$ and a projected length $L = 2.0$ cm (worst case scenario), this gives a pulling speed of 0.1 mm/s (about 3 turns per second) for reaching the break point on the nanowire, which is about 300 MPa. The Reynolds number for this case is about 4×10^{-9}, much less than unity. Thus the wire should be handled at a lower speed, for instance at 2 turns per second. If the wire is immersed in a less dense fluid, the speed of winding can be made higher.

IV. SIGNAL STRENGTH

Here we give an estimation of the expected intensity of the generated EMF and the intrinsic noise due to the electrical resistance of the coil nanowire. For practical use, the intensity of the EMF should be significantly larger than that of the noise.

An estimation of the induced EMF can be obtained considering a silicon cantilever of dimensions $L_c \times W_c \times t_c$ (length, width and thickness, respectively) carrying a permanent magnetic dipole $\mathbf{m}$ (see Figure 4). The magnetic dipole strength is $m = 9.3\times10^{-7}$ J/T (or A m^2) equivalent to 10^{17} spins. The coil consists of one loop of radius $a = 5\mu$ normal to the direction of the magnetic dipole. The center of the loop is on the axis of the dipole and a distance h from its center.

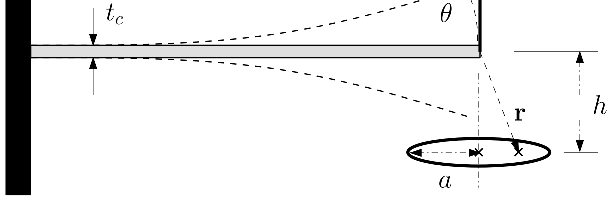

Fig. 4. Scheme of an electromagnetic generator. We consider a silicon cantilever of the dimensions $L_c \times W_c \times t_c$ (W_c is not shown here) carrying a permanent magnetic moment $\mathbf{m}$ located as shown. $\mathbf{r}$ denotes the position of a generic point on the surface enclosed by the loop and θ is the deflection angle of the vibrating cantilever.

Using the notation of Figure 4, the magnetic field at any position $\mathbf{r}$ on the surface enclosed by the loop is given by

$$\mathbf{B}(\mathbf{m},\mathbf{r}) = \frac{\mu_0}{4\pi r^3}\left[3(\mathbf{m}\cdot\hat{\mathbf{e}}_r)\hat{\mathbf{e}}_r - \mathbf{m}\right], \tag{13}$$

where μ_0 is the magnetic constant and $\hat{\mathbf{e}}_r$ is the unit vector along the direction of $\mathbf{r}$. The magnetic flux through the loop is, up to an integration constant,

$$\begin{aligned}\Phi_B &= \int_{\mathbf{A}} \mathbf{B}\cdot d\mathbf{A}\\ &= \frac{\mu_0 m a^2}{2L_c^2}\left[\frac{1}{h-\theta L_c} - \frac{h}{2(h-\theta L_c)^2}\right],\end{aligned} \tag{14}$$

where θ is the deflection angle of the cantilever and $\mathbf{A}$ is the area enclosed by the loop. We consider small oscillations of the cantilever on its fundamental mode, with the frequency ω_0, forced by thermal agitation at the temperature T (details of this calculation will appear elsewhere.) The induced EMF is obtained by differentiating Eq. (14) with respect to time. The result is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt}\\ &= \frac{\mu_0 m a^2}{2L_c}\left[\frac{h}{(h-\delta\theta\ L_c)^3} - \frac{1}{(h-\delta\theta\ L_c)^2}\right]\omega_0,\end{aligned} \tag{15}$$

where $\delta\theta$ is the deflection angle at resonance frequency. Using the values $a = 5.0\mu$, $L_c = 400.0\mu$, $W_c = 40.0\mu$, $t_c = 0.8\mu$ and $h = 1.0\mu$ we obtain $\delta\theta = 4.6\times10^{-7}$ rad. and $\omega_0 = 9.4\times10^4$ Hz. Then, substituting into Eq.(15), we finally obtain

$$|\varepsilon| \approx 6.3\times10^{-7}\text{V}. \tag{16}$$

For estimating the noise we consider the voltage fluctuations caused by thermal motion of the electrons which then relax back through resistance. This process is commonly modeled as Johnson noise [11], where the root mean square (RMS) of the noise voltage v_n is given by

$$v_n = \sqrt{4k_\mathrm{B}TR\Delta f}, \tag{17}$$

where we use a typical band width of the measuring system of $\Delta f = 1$ Hz. Here k_B is Boltzmann constant, T is the temperature and R is the electrical resistance of the wire. For room temperature and a loop of 5μ radius made of a gold wire 50 nm in diameter, this gives

$$v_n = 2.4\times10^{-9}\text{V}. \tag{18}$$

Thus, the estimated strength of the induced EMF should be about 2 orders of magnitude stronger than the intrinsic Johnson noise which is a very encouraging result.

V. APPLICATIONS

Successful fabrication of a 10 microns coil will give enough insight for further size reduction. Our next step will be to fabricate 1 micron size coils and beyond. Limitations are imposed by the mechanical strength of the coil core and the intensity of the viscous drag force, but the winding technique remains the same. Fabrication of micro and nano-metric sized high density coils, as proposed here, would be a major step towards the construction of small electromagnetic generators. There are numerous other applications as well. They can be used as sensors of small magnetic moments, heat sinks (coolers), they would be an important component in a spintronic circuit. A very interesting application would be the fabrication of a micrometric size transformer which could be used to amplify small voltage signals with very little loss. It could be fabricated with the technique we propose here with few modifications.

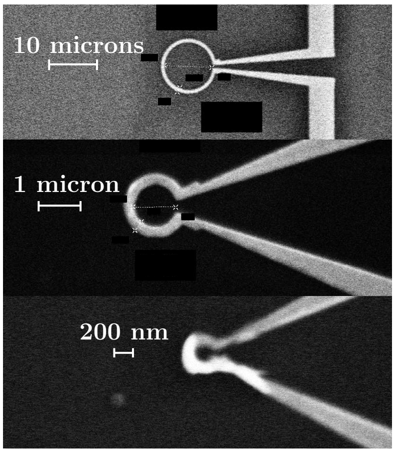

Fig. 5. SEM images of single loop coils made by EBL. The coils are fabricated over a silicon substrate by evaporating 10 nm of chromium followed by 50 nm of gold through a patterned PMMA photoresistor which is removed afterwards.

A preliminary experiment is currently under development. It consists of the fabrication of single loop coils of small sizes, i.e. 10, 5, 1 and 0.2 microns internal diameter, which has been accomplished by Electron Beam Lithography (Figure 5). A measurable EMF is expected to be induced on them by controlled vibrations of a magnetized Atomic Force Microscope (AFM) tip placed at the center with high precision. The results will be very useful for later design of small electromagnetic generators.

VI. CONCLUSIONS

A technique for fabricating micro-metric sized coils for electromagnetic applications is proposed. Coil internal diameters are targeted on the range $1 - 10$ microns and the technique is expected to achieve sub-micron sizes as well, with proper tune-up. The wire is proposed to be fabricated by using *Lithographically Patterned Nanowire Electrodeposition* (LPNE), reported recently, which can produce gold nanowires as small as 20×50 nanometers section and length of the order of centimeters. A procedure for mechanically winding the nanowires on a core of small dimensions using viscous drag forces is presented. We have also derived approximate expressions for the mechanical stress acting on the fragile nanowires that should serve as guides for proper handling. The preliminary estimations for the intensity of the induced EMF compares favorably to the intrinsic noise levels.

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ТЕРМОДИНАМІКА МАЛЕНЬКИХ ЕЛЕКТРОМАГНІТНИХ ГЕНЕРАТОРІВ, ЕКСПЕРИМЕНТАЛЬНА ПЕРСПЕКТИВА

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Недавно в літературі кілька дослідницьких груп повідомили про виготовлення відносно маленьких електромагнітних ґенераторів. Їхні характерні розміри близько одного міліметра. Після певних удосконалень такі прилади використовували для перетворення навколишнього вібраційного шуму в корисну електричну енерґію. Ми хочемо проаналізувати, включно з експериментальним розглядом, можливість зменшення розміру ґенератора до нанометрів, що дасть змогу, в принципі, використовувати слабші джерела вібрацій. Ми пропонуємо техніку для виготовлення маленьких котушок із застосуванням нанодротів та отримуємо наближений вираз для оцінки механічного напруження на нанодріт. Для ідеальної моделі мікророзмірного електромагнітного ґенератора оцінюємо силу індукованої електрорушійної сили (ЕРС) та порівнюємо її з внутрішнім електричним шумом, наявним у нанодроті. Ми виявили, що сила індукованої ЕРС більша на два порядки, ніж електричний шум, що є обнадійливим результатом.