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NEW CONCEPT OF STATISTICAL ENSEMBLES

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An extension of the standard concept of the statistical ensembles is suggested. Namely, the statistical ensembles with extensive quantities fluctuating according to an externally given distribution is introduced. Applications in the statistical models of multiple hadron production in high energy physics are discussed.

Key words: statistical ensembles, fluctuations, hadron production

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I. GENERAL CONCEPT OF STATISTICAL ENSEMBLES

A successful application of the statistical model to the description of mean hadron multiplicities in high energy collisions (see, e.g., recent papers [1] and references therein) has stimulated investigations of properties of statistical ensembles. Whenever possible, one prefers to use the grand canonical ensemble (GCE) due to its mathematical convenience. The canonical ensemble (CE) [2] should be applied when the number of carriers of conserved charges is small (of the order of 1), such as strange hadrons [3], anti-baryons [4], or charmed hadrons [5]. The micro-canonical ensemble (MCE) [6] has been used to describe small systems with fixed energy, e.g. mean hadron multiplicities in proton-antiproton annihilation at rest. In all these cases, calculations performed in different statistical ensembles yield different results. This happens because the systems are "small" and they are "far away" from the thermodynamic limit (TL). The mean multiplicity of hadrons in relativistic heavy ion collisions ranges from 10^2 to 10^4, and mean multiplicities (of light hadrons) obtained within GCE, CE, and MCE approach each other. One refers here to the thermodynamical equivalence of statistical ensembles and uses the GCE for calculating the hadron yields.

Measurements of a hadron multiplicity distribution $P(N)$ in interactions, including nucleus-nucleus collisions, open a new field of applications of the statistical models. The particle multiplicity fluctuations are usually quantified by the ratio of variance to mean value of a multiplicity distribution $P(N)$, the scaled variance, and are a subject of current experimental activities. In statistical models there is a qualitative difference in the properties of mean multiplicity and scaled variance of multiplicity distributions. It was recently found [7–12] that even in the TL corresponding results for the scaled variance are different in different ensembles. Hence the equivalence of ensembles holds for mean values in the TL, but does not extend to fluctuations. This fact has non-trivial consequences for relativistic gases, and was not discussed previously in the standard (non-relativistic) statistical mechanics.

A statistical system is characterized by the extensive quantities: volume V, energy E, and conserved charge(s)[1] Q. In non-relativistic statistical mechanics, the number of particles plays a role of the conserved charge. The MCE is defined by the postulate that all micro-states with given V, E, and Q have equal probabilities of being realized. This is the basic postulate of the statistical mechanics. The MCE partition function just calculates the number of microscopic states with given fixed (V, E, Q) values. In the CE the energy exchange between the considered system and "infinite thermal bath" is assumed. Consequently, a new parameter, temperature T is introduced. To define the GCE, one makes a similar construction for the conserved charge Q. An "infinite chemical bath" and the chemical potential μ are introduced. The CE introduces the energy fluctuations. In the GCE, there are additionally the charge fluctuations. The MCE, CE, and GCE are most familiar statistical ensembles. In the textbooks (see, e.g., Ref. [13, 14]), the pressure (or isobaric) canonical ensemble has been also discussed. The "infinite bath of the fixed external pressure" p_0 is then introduced. This leads to the volume fluctuations around the average value.

In general, there are 3 pairs[2] of variables — (V, p_0), (E, T), (Q, μ) — and, thus, the 8 statistical ensembles[3] can be constructed. Among these 8 ensembles there are 4 pressure ensembles: (p_0, E, Q), (p_0, T, Q), (p_0, E, μ), and (p_0, T, μ). In addition to the pressure canonical ensemble known from the literature, three other possibil-

[1] In statistical description of hadron or quark-gluon systems, these conserved charges are usually the net baryon number, strangeness, and electric charge.

[2] In the present study we do not discuss the role of the total 3-momentum. As shown in Ref. [15] the total momentum conservation is not important in the TL for thermodynamical functions and fluctuations in the full phase space. It may, however, influence the particle number fluctuations in the limited segments of the phase space.

[3] For several conserved charges $\{Q_i\}$ the number of possible ensembles is larger, as each charge can be treated either canonically or grand canonically.

ities — pressure micro-canonical, pressure grand micro-canonical, and pressure grand canonical ensembles — are constructed and studied in Ref. [16]. Note that the pressure grand canonical ensemble has a unique property. Among 8 possible ensembles this is the only one where the system description includes only intensive quantities: p_0, T and μ. This special ensemble has unusual features discussed in details in Ref. [16].

A more general concept of the statistical ensembles was suggested in Ref. [17]. The statistical ensemble is defined by an externally given distribution of extensive quantities, $P_\alpha(E, V, Q)$. The construction of distribution of any variable X in such an ensemble proceeds in two steps. Firstly, the MCE X-distribution, $P_{\rm mce}(X; \mathbf{A})$, is calculated at fixed values of the extensive quantities $\mathbf{A} = (V, E, Q)$. Secondly, this result is averaged over the external distribution $P_\alpha(\mathbf{A})$ [17],

$$P_\alpha(X) = \int d\mathbf{A}\, P_\alpha(\mathbf{A}) P_{\rm mce}(X; \mathbf{A}). \tag{1}$$

Fluctuations of extensive quantities $\mathbf{A}$ around their average values depend not on the system's physical properties, but rather on external conditions. One can imagine a huge variety of these conditions, thus, the standard statistical ensembles discussed above are only some special examples. Thermodynamics relates the average quantities of the statistical ensemble. Thus, it may work in these new ensembles. The ensembles defined by Eq. (1), the α-ensembles, include the standard statistical ensembles as particular cases.

II. STATISTICAL MODELS FOR HADRON PRODUCTION

In collisions at relativistic energies many new particles are produced. Their number, masses and charges as well as their momenta vary from event to event. Most of the experimental results concern single particle production properties averaged over many interactions. It is well established that some of these properties, namely, mean particle multiplicities and transverse momentum spectra, follow simple rules of statistical mechanics. In proton-proton (p+p) collisions the single particle momentum distribution has an approximately Boltzmann form in the local rest frame of produced matter: $dN/(p^2dp) \sim \exp\left(-\sqrt{p^2+m^2}/T\right)$, where T, p and m are the temperature parameter, the particle momentum and its mass, respectively. At large momentum, $p \gg m$, this gives: $dN/(p^2dp) \sim \exp(-p/T)$. Integration over momentum yields the mean particle multiplicity, $\langle N\rangle$, which is also governed by the Boltzmann factor for $m >> T$: $\langle N\rangle \sim (mT)^{3/2}\exp(-m/T)$. The approximate validity of the exponential distributions is confirmed by numerous experimental results on bulk particle production in high energy collisions. The agreement is limited to the low transverse momentum ($p_T \leq 2\,$GeV) and the low mass ($m \leq 2\,$GeV) domains. However, the temperature parameter T extracted from the data on p+p interactions is in the range 160–190 MeV. Thus, almost all particles are produced at low p_T and with low masses.

Along with evident successes there are obvious problems of the statistical approach. The probability $P(N)$ to create N particles in p+p collisions obeys the so called KNO scaling[4], namely:

$$P(N) = \langle N\rangle^{-1}\psi(z), \tag{2}$$

where $\langle N\rangle$ is the mean multiplicity and the KNO scaling function $\psi(z)$ only depends on $z \equiv N/\langle N\rangle$. The mean multiplicity increases with increasing collision energy, whereas the KNO scaling function remains unchanged. The latter implies that the scaled variance ω of the multiplicity distribution $P(N)$ grows linearly with the mean multiplicity:

$$\omega \equiv \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} \propto \langle N\rangle. \tag{3}$$

A qualitatively different behavior is predicted within the existing statistical models where the scaled variance is expected to be independent of the mean multiplicity: $\omega \approx$ const ≈ 1. This contradiction between the data and the statistical models constitutes the first problem which will be considered in this paper.

The second and the third problems which will be addressed here concern particle production at high (transverse) momenta and with high masses, respectively. In these regions the single particle energy distribution seems to obey a power law behavior [19]: $dN/(p^2dp) \sim \left(\sqrt{p^2+m^2}\right)^{-K}$. At $p \gg m$ this gives: $dN/(p^2dp) \sim p^{-K_p}$, with $K_p = K$. Integration over particle momentum yields the mean multiplicity which follows a power law dependence on the particle mass: $\langle N\rangle \sim m^{-K_m}$, with $K_m = K_p - 3$. The above power laws describe the data on spectra of light particles at large ($p \geq 3$ GeV) (transverse) momenta and on the mean multiplicity of heavy ($m \geq 3$ GeV) particles, respectively. The parameters fitted to the data are $K_p \cong 8$ and $K_m \cong 5$ [19]. One observes a growing disagreement between the exponential behavior and power law dependence with increasing (transverse) momentum and/or mass. At $p = 10$ GeV or $m = 10$ GeV the statistical models underestimate the data by more than 10 orders of magnitude.

In Ref. [20] we made an attempt to extend the statistical model to the hard domain of high transverse momenta and/or high hadron masses (hard domain). The proposal is inspired by statistical type regularities [19] in the high transverse mass region, as well as by the recent work on the statistical ensembles with fluctuating extensive quantities [17]. We postulate that the volume of the

[4]This scaling was suggested by **K**oba, **N**ielsen, and **O**lesen in Ref. [18] and triggered an intensive discussion in the literature.

system created in p+p collision changes from event to event[5]. The main assumptions of the proposed approach are the following.

1. Each final state created in p+p interactions is identified with a micro-state of a micro-canonical ensemble (MCE) defined by the volume V, energy E, and conserved charges of the system. By definition of the MCE, all its micro-states appear with the same probability.

2. The volume of micro-canonical ensembles represented in p+p interactions fluctuates from collision to collision. The volume probability density function, $P_\alpha(V)$, is given by the scaling function, $P_\alpha(V) = \overline{V}^{-1}\phi_\alpha(V/\overline{V})$, where $\overline{V}$ is the scaling parameter.

The model based on these assumptions will be referred as the **M**icro-**C**anonical **E**nsemble with **s**caling **V**olume **F**luctuations, the **MCE/sVF**.

III. MICRO-CANONICAL ENSEMBLE

The MCE partition function for the system with N Boltzmann massless neutral particles reads [8]:

$$W_N(E,V) = \frac{1}{N!}\left(\frac{gV}{2\pi^2}\right)^N \times \int_0^\infty p_1^2 dp_1 \ldots \int_0^\infty p_N^2 dp_N \;\delta(E - \sum_{i=1}^N p_i) = \frac{1}{E}\;\frac{A^N}{(3N-1)!N!}, \quad (4)$$

where E and V are the system energy and volume, respectively, g is the degeneracy factor, and $A \equiv gVE^3/\pi^2$. The MCE partition function (4) includes exact energy conservation, but neglects the momentum conservation. The MCE multiplicity distribution is given by $P_{\rm mce}(N;E,V) = W_N(E,V)/W(E,V)$, where $W(E,V)$ is the total MCE partition function [8]:

$$W(E,V) \equiv \sum_{N=1}^\infty W_N(E,V) = \frac{A}{2E}\;{}_0F_3\left(;\frac{4}{3},\frac{5}{3},2;\frac{A}{27}\right), \quad (5)$$

where ${}_0F_3$ is the generalized hyper-geometric function. For $A \gg 1$ the mean multiplicity equals to $\langle N\rangle_{\rm mce} \equiv \sum_{N=1}^\infty N P_{\rm mce}(N;E,V) \cong (A/27)^{1/4}$, where $P_{\rm mce}(N;E,V)$ was approximated by the normal distribution [8]:

$$P_{\rm mce}(N;E,V) \cong (2\pi\,\omega_{\rm mce}\cdot\langle N\rangle_{\rm mce})^{-1/2} \times \exp\left[-\frac{(N-\langle N\rangle_{\rm mce})^2}{2\,\omega_{\rm mce}\cdot\langle N\rangle_{\rm mce}}\right], \quad (6)$$

with $\omega_{\rm mce} \equiv (\langle N^2\rangle_{\rm mce} - \langle N\rangle^2_{\rm mce})/\langle N\rangle_{\rm mce} \cong 1/4$. Note that in the grand canonical ensemble (GCE) the multiplicity distribution is equal to the Poisson one: $P_{\rm gce}(N;T,V) = \overline{N}^N\exp(-\overline{N})/N!$, where $\overline{N}$ is the mean multiplicity in the GCE. It approaches the Gaussian for large $\overline{N}$:

$$P_{\rm gce}(N;T,V) \cong (2\pi\,\omega_{\rm gce}\cdot\overline{N})^{-1/2}\exp\left[-\frac{\left(N-\overline{N}\right)^2}{2\omega_{\rm gce}\cdot\overline{N}}\right],$$

with $\omega_{\rm gce} \equiv (\overline{N^2} - \overline{N}^2)/\overline{N} = 1$.

The numerical calculations presented in this paper will be performed for $g = 1$ and the energy density which corresponds to the temperature parameter $T = 160$ MeV. The latter relates the values of E and V via equation: $E = 3VT^4/\pi^2$. The mean multiplicity $\langle N\rangle_{\rm mce}$ in the MCE is then approximately equal to the GCE value: $\overline{N} = VT^3/\pi^2$. The approximation $\langle N\rangle_{\rm mce} \cong \overline{N}$ is valid for $\overline{N} \gg 1$ and reflects the thermodynamic equivalence of the MCE and the GCE. The scaled variance of the MCE distribution is $\omega_{\rm mce} = 1/4$ [8], and is approximately independent of $\langle N\rangle_{\rm mce}$ already for $\langle N\rangle_{\rm mce} > 5$. Thus, despite of thermodynamic equivalence of the MCE and GCE the value of $\omega_{\rm mce}$ is four times smaller than the scaled variance of the GCE (Poisson) distribution, $\omega_{\rm gce} = 1$.

The single particle momentum spectrum in the GCE reads:

$$F_{\rm gce}(p) \equiv \frac{1}{\overline{N}}\frac{dN}{p^2dp} = \frac{V}{2\pi^2\overline{N}}\exp\left(-\frac{p}{T}\right) = \frac{1}{2T^3}\exp\left(-\frac{p}{T}\right), \quad (7)$$

whereas the corresponding spectrum in the MCE is given by [20]:

$$F_{\rm mce}(p) \equiv \frac{1}{\langle N\rangle_{\rm mce}}\frac{dN}{p^2dp} = \frac{V}{2\pi^2\langle N\rangle_{\rm mce}}\sum_{N=2}^\infty\frac{W_{N-1}(E-p,V)}{W(E,V)} \equiv \frac{V}{2\pi^2\langle N\rangle_{\rm mce}}f(p;E,V) = \frac{1}{\langle N\rangle_{\rm mce}}\frac{1}{2E^3} \times \sum_{N=2}^\infty\frac{N\,(3N-1)!}{(3N-4)!}\left(1-\frac{p}{E}\right)^{3N-4}P_{\rm mce}(N;E,V) \quad (8)$$

where $f(p;E,V)$ is the MCE analogue of the Boltzmann factor, $\exp(-p/T)$. From Eq. (8) follows $f(p =$

[5]The volume fluctuations in hadron statistical physics were first introduced in the framework of the isobaric ensemble in Ref. [21].

$0; E, V) = 1$. Both (7) and (8) are normalized such that $\int_0^\infty p^2 dp\, F_{\rm gce}(p) = 1$ and $\int_0^E p^2 dp F_{\rm mce}(p) = 1$.

Figure 1a shows a comparison of the MCE and GCE results for the multiplicity distribution and Fig. 1b shows the momentum spectrum for $\overline{N} = 50$. The MCE spectrum is close to the Boltzmann distribution (7) at low momenta. This can be shown analytically using the asymptotic form of the generalized hyper-geometric function at $E \to \infty$ and $p/E \ll 1$. The MCE spectrum decreases faster than the GCE one at high momenta. Close to the threshold momentum, $p = E$, where the MCE spectrum goes to zero, large deviations from (7) are observed. In order to demonstrate this the MCE and GCE momentum spectra are shown in Fig. 1b over 90 orders of.

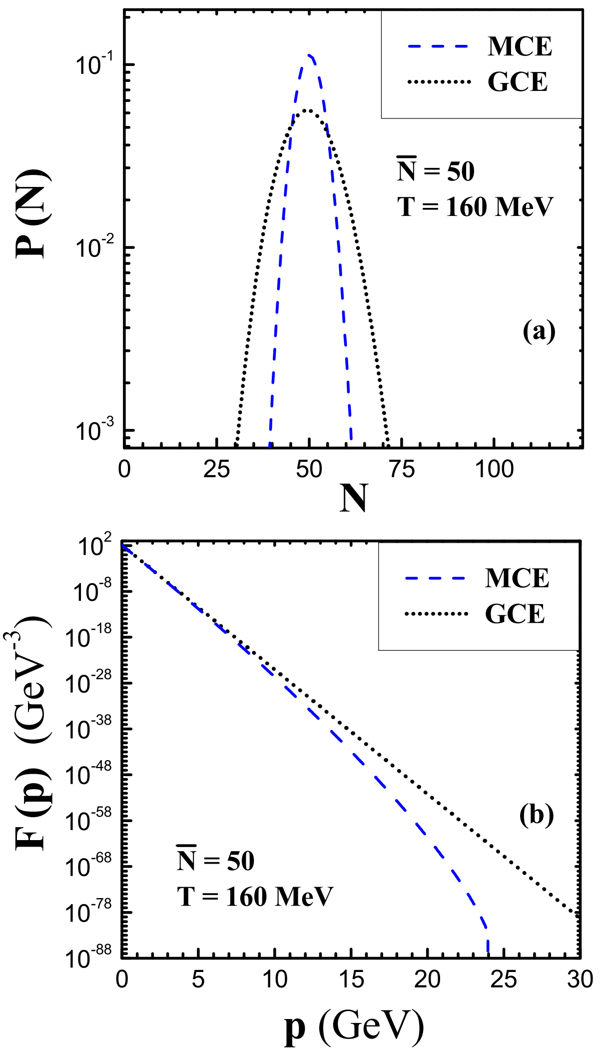

Fig. 1. (Color online) **(a):** The multiplicity distribution of massless neutral particles in the MCE, dashed line, and the GCE, dotted line. **(b):** The momentum spectrum of massless neutral particles calculated within the MCE (8), dashed line, and the GCE (7), dotted line. The system energy is $E = 3\overline{N}T = 24$ GeV for both plots.

IV. MICRO-CANONICAL ENSEMBLE WITH SCALING VOLUME FLUCTUATIONS

Let us consider a set of micro-canonical ensembles with the same energy E but different volumes V. The probability density which describes the volume fluctuations is denoted by $P_\alpha(V)$. The distribution of any quantity X can then be calculated as:

$$P_\alpha(X;E) = \int_0^\infty dV P_\alpha(V) P_{\rm mce}(X;E,V), \qquad (9)$$

where $P_{\rm mce}(X;E,V)$ is the distribution of the quantity X in the MCE with fixed E and V. Further more it is assumed that the distribution $P_\alpha(V)$ has the scaling form: $P_\alpha(V) = \overline{V}^{-1}\, \phi_\alpha(V/\overline{V})$, with the volume parameter $\overline{V}$ being the scale parameter of $P_\alpha(V)$. The MCE/sVF defined by Eq. (9) is a special case of α-ensembles (1). The volume integral in Eq. (9) can be conveniently rewritten as:

$$P_\alpha(X;E) = (\overline{V})^{-1} \int_0^\infty dV \phi_\alpha\left(V/\overline{V}\right) P_{\rm mce}(X;E,V)$$
$$= \int_0^\infty dy \psi_\alpha(y) P_{\rm mce}(X;E,y^4\overline{V}), \qquad (10)$$

where $\psi_\alpha(y) \equiv 4y^3\phi_\alpha(y^4)$. Choosing $\psi_\alpha(y) = \delta(y-1)$ one recovers the MCE with $V = \overline{V}$ and $\langle N\rangle_{\rm mce} = \overline{N}$. The scaling function $\psi_\alpha(y)$ will be required to satisfy two normalization conditions:

$$\int_0^\infty dy\, \psi_\alpha(y) = 1, \qquad \int_0^\infty dy\, y\, \psi_\alpha(y) = 1. \qquad (11)$$

The first condition guarantees the proper normalization of the volume probability density function, $\int_0^\infty dV P_\alpha(V) = 1$. The second condition is selected in order to keep the mean multiplicity in the MCE/sVF equal to the MCE mean multiplicity. The multiplicity distribution in the MCE/sVF is:

$$P_\alpha(N;\overline{N}) = \int_0^\infty dV P_\alpha(V) P_{\rm mce}(N;E,V)$$
$$= \int_0^\infty dy\, \psi_\alpha(y) P_{\rm mce}(N;E,y^4\overline{V}). \qquad (12)$$

At $\overline{N} \gg 1$, the particle number distribution (12) can be approximated as:

$$P_\alpha(N;\overline{N}) \cong \langle N\rangle_\alpha^{-1} \psi_\alpha(z), \qquad (13)$$

where $z \equiv N/\langle N\rangle_\alpha$, and the mean multiplicity $\langle N\rangle_\alpha$ is given by:

$$\langle N\rangle_\alpha = \sum_{N=1}^\infty N P_\alpha(N;\overline{N}) \cong \overline{N} \int_0^\infty dy\, y\, \psi_\alpha(y) = \overline{N}. \qquad (14)$$

The approximate equality of $\langle N\rangle_\alpha$ and $\overline{N}$ is satisfied for $\overline{N} \gg 1$ due to the second normalization condition (11). The KNO scaling of the multiplicity distribution $P_\alpha(N;\overline{N})$ follows from the assumption of the scaling of the volume fluctuations.

For convenience, a simple analytical form of the scaling function, ψ_α will be used:

$$\psi_\alpha(y) = \frac{k^k}{\Gamma(k)}\, y^{k-1} \exp(-k\, y), \tag{15}$$

where $\Gamma(k)$ is the Euler gamma function. The function (15) with $k = 4$ approximately describes the experimental data on KNO scaling in p+p interactions. Note, that the function (15) satisfies both normalization conditions (11) for any $k > 0$. The KNO scaling of the multiplicity distribution implies that the scaled variance of the distribution increases in proportion to the mean multiplicity. For $\overline{N} \gg 1$ one gets: $\omega_\alpha \cong \kappa\langle N\rangle_\alpha$, where $\kappa = \int_0^\infty dy(y-1)^2\psi_\alpha(y) > 0$. For the function $\psi_\alpha(y)$ defined by Eq. (15) one finds, $\kappa = k^{-1}$.

In Fig. 2 (*left*) the multiplicity distributions obtained within the MCE and the MCE/sVF for $\overline{N} = 50$ are compared. The scaled variance of the MCE/sVF distribution for $\overline{N} = 50$ is about 12.5, whereas the scaled variance of the MCE distribution is 1/4. This large difference in the width of the MCE/sVF and the MCE distributions is clearly seen in the figure. The volume fluctuations in the MCE/sVF significantly increase the width of the multiplicity distribution. They are also expected to modify the single particle momentum spectrum. This is because for a fixed system energy, the volume of the system determines the energy density, and consequently, the effective temperature of particles.

The single particle momentum spectrum within the MCE/sVF can be directly calculated from Eq. (9) and it reads [20]:

$$\begin{aligned} F_\alpha(p) &\equiv \frac{1}{\langle N\rangle_\alpha}\left\langle \frac{dN}{p^2dp}\right\rangle_\alpha \\ &= \frac{1}{\langle N\rangle_\alpha}\int_0^\infty dV\, P_\alpha(V)\frac{V}{2\pi^2}\, f(p;E,V) \\ &= \frac{1}{\langle N\rangle_\alpha}\frac{1}{2E^3}\sum_{N=2}^{\infty}\frac{N(3N-1)!}{(3N-4)!}\left(1-\frac{p}{E}\right)^{3N-4} P_\alpha(N;\overline{N}). \end{aligned} \tag{16}$$

The formal structure of the expression (16) is similar to the structure of the corresponding expression derived within the MCE (8). The only, but the crucial, difference is that the narrow MCE multiplicity distribution used for averaging the particle spectrum in Eq. (8) is replaced by the broad MCE/sVF multiplicity distribution in Eq. (16). The spectrum $F_\alpha(p)$ fulfills the normalization condition, $\int_0^E p^2dp\, F_\alpha(p) = 1$. From Eq. (16) one finds $F_\alpha(p=0) = a \cdot F_{\rm gce}(p=0)$, where $a \cong 3.28$ for the ψ_α function (15) with $k = 4$. Thus, in the MCE/sVF there is an enhancement of the momentum spectrum at $p \to 0$ compared to the GCE and MCE results.

The single particle momentum spectrum (16) calculated with the volume scaling function (15) is shown in Fig. 2 (*right*). A striking new feature of this spectrum is the presence of a long power law tail. In the momentum range from several GeV to about 20 GeV the spectrum can be approximated by:

$$F_\alpha(p) \cong C_p p^{-K_p}, \tag{17}$$

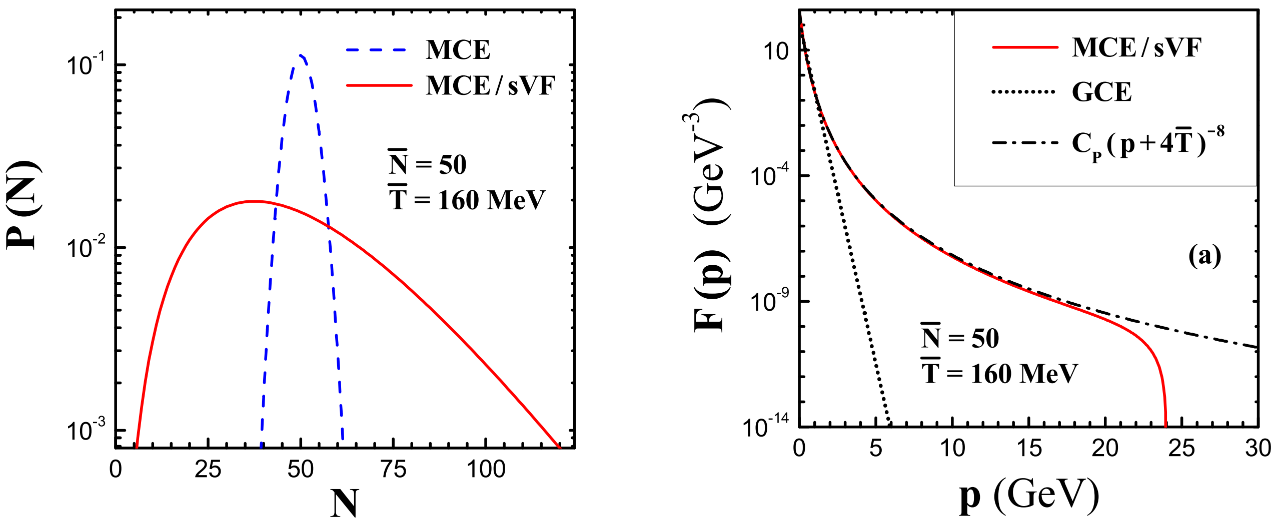

Fig. 2. (Color online) *Left:* A comparison of the multiplicity distributions of massless neutral particles calculated with the MCE/sVF (solid line) and the MCE (dashed line). The system energy is $E = 3\overline{N}\,\overline{T} = 24$ GeV. *Right:* The momentum spectrum of massless neutral particles calculated within the MCE/sVF (16), solid line, and the GCE (7), dotted line. The approximation (18) of the MCE/sVF spectrum is shown by the dashed-dotted line.

with C_p and $K_p = k + 4 = 8$ being the normalization and power parameters, respectively. For momenta smaller than 3 GeV the spectrum starts to deviate significantly from the power law parametrization and its local inverse slope parameter is close to the temperature of the corresponding GCE, $\overline{T} = 160\,\text{MeV}$. A rapid decrease of the spectrum starts at $p \geq 20\,\text{GeV}$, when the threshold value $p = 24\,\text{GeV}$ is approached. The above features of the MCE/sVF momentum spectrum resemble features of the transverse momentum spectrum of hadrons produced in high energy p+p interactions. The power law dependence (17) of the momentum spectrum at high momenta can be derived analytically, namely:

$$\begin{aligned} F_\alpha(p) &= \int_0^\infty F_{\text{mce}}(p)\,\psi_\alpha(y)\,dy \\ &\cong \frac{\overline{V}}{2\pi^2\overline{N}} \int_0^\infty dy\,\psi_\alpha(y) y^4 \exp\left(-\frac{p}{\overline{T}}\,y\right) \\ &= \frac{k^k\Gamma(k+4)}{2\Gamma(k)}\overline{T}^{k+1}(p+\overline{T}k)^{-k-4} \\ &\cong 11.27\text{GeV}^5(p+4\overline{T})^{-8}, \end{aligned} \tag{18}$$

where $\overline{T} = 160$ MeV and $k = 4$ is set in the last expression.

V. SEMI-INCLUSIVE QUANTITIES IN STATISTICAL MODELS

In Ref. [22] the selected properties of semi-inclusive events have been studied within statistical models: the GCE, CE, MCE, and MCE/sVF. In particular, the mean multiplicity of neutral particles and momentum spectra of charged particles are considered at a fixed charged particle multiplicity. Different statistical ensembles lead to qualitatively different results for these semi-inclusive quantities. For illustration the joint N_0 and N_- distributions of neutral and negatively charged particles are calculated within the GCE, CE, MCE, and MCE/sVF. They are shown in Fig. 3 for the system with the net charge $Q = N_+ - N_-$ equal to zero.

The multiplicities of neutral and negatively charged particles are uncorrelated in the GCE and CE (see Fig. 3 top panels). They are anti-correlated and correlated in the MCE and MCE/sVF, respectively. A positive correlation between N_0 and N_- in the MCE/sVF is caused by the scaling volume fluctuations. Note that finite size effects are seen in Fig. 3. For example, the Poisson distribution of the GCE is significantly asymmetric for large deviations from $\overline{N}$.

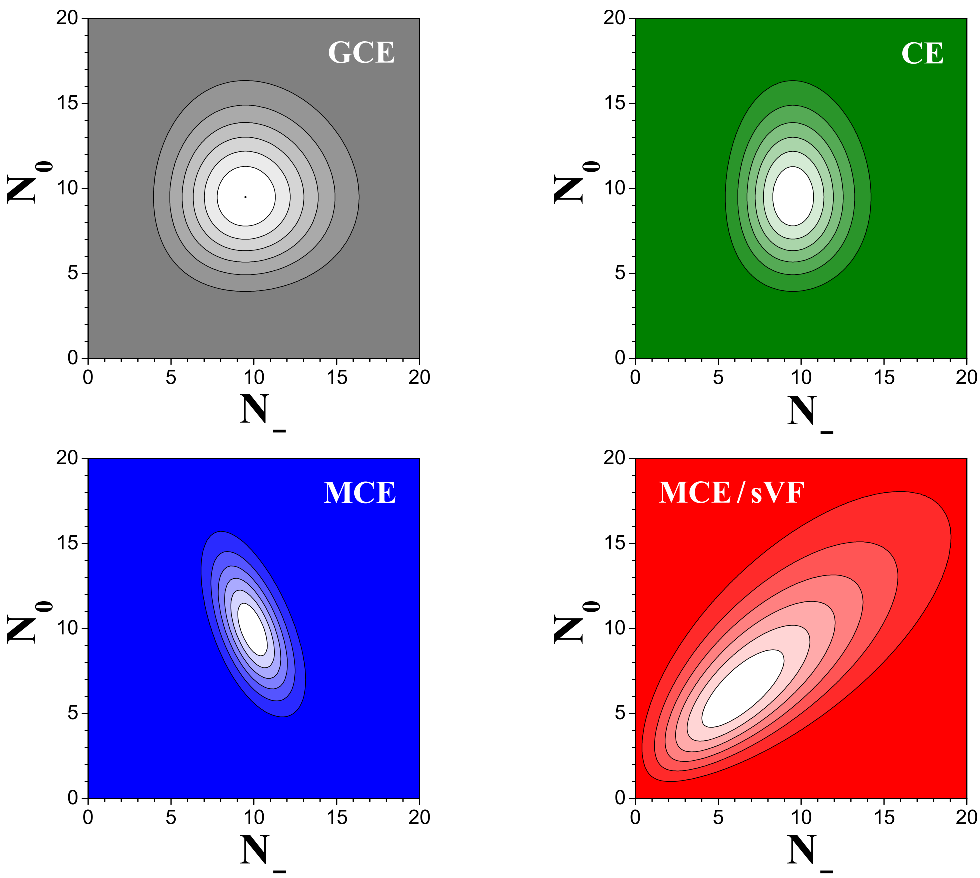

Fig. 3. (Color online) Examples of the joint N_0 and N_- distributions calculated within the GCE (*top left*), CE (*top right*), MCE (*bottom left*) and MCE/sVF (*bottom right*). The distributions are calculated assuming $Q = 0$ and average multiplicities of neutral and negatively charged particles equal to $\overline{N} = 10$ (see Ref. [22] for details).

VI. CONCLUSIONS

In this paper I presented the results of Refs. [16,17,20, 22]. It was suggested to extend the concepts of statistical ensembles. The class of ensembles defined by external distributions of extensive quantities was introduced. This construction was motivated by the statistical approach to the particle number fluctuations in high energy collisions. We believe also that the concept of statistical ensembles with fluctuating extensive quantities may be appropriate in other situations too.

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[1] J. Cleymans, H. Oeschler, K. Redlich, S. Wheaton, Phys. Rev. C **73**, 034905 (2006); F. Becattini, J. Manninen, M. Gaździcki, *ibid.* **73**, 044905 (2006); A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A **772**, 167 (2006).
[2] F. Becattini, Z. Phys. C **69**, 485 (1996); F. Becattini, U. Heinz, Z. Phys. C **76**, 269 (1997).
[3] J. Cleymans, K. Redlich, E. Suhonen, Z. Phys. C **51**, 137 (1991).
[4] M. I. Gorenstein, M. Gaździcki, W. Greiner, Phys. Lett. B **483**, 60 (2000).
[5] M. I. Gorenstein, A. P. Kostyuk, H. Stöcker, W. Greiner, Phys. Lett. B **509**, 277 (2001).
[6] F. Becattini, L. Ferroni, Eur. Phys. J. C **35**, 243 (2004); **38**, 225 (2004); V. V. Begun, L. Ferroni, M. I. Gorenstein, M. Gaździcki, F. Becattini, J. Phys. G **32**, 1003 (2006); F. Becattini, L. Ferroni, Eur. Phys. J. C **51**, 899 (2007); **52**, 597 (2007).
[7] V. V. Begun, M. Gaździcki, M. I. Gorenstein, O. S. Zozulya, Phys. Rev. C **70**, 034901 (2004); V. V. Begun, M. I. Gorenstein, O. S. Zozulya, Phys. Rev. C **72**, 014902 (2005); A. Keränen, F. Becattini, V. V. Begun, M. I. Gorenstein, O. S. Zozulya, J. Phys. G **31**, S1095 (2005); F. Becattini, A. Keränen, L. Ferroni, T. Gabbriellini, Phys. Rev. C **72**, 064904 (2005); J. Cleymans, K. Redlich, L. Turko, Phys. Rev. C **71**, 047902 (2005); J. Phys. G **31**, 1421 (2005); V. V. Begun, M. I. Gorenstein, Phys. Rev. C **73**, 054904 (2006).
[8] V. V. Begun, M. I. Gorenstein, A. P. Kostyuk, O. S. Zozulya, Phys. Rev. C **71**, 054904 (2005).
[9] V. V. Begun, M. I. Gorenstein, A. P. Kostyuk, O. S. Zozulya, J. Phys. G **32**, 935 (2006).
[10] V. V. Begun, M. I. Gorenstein, M. Hauer, V. P. Konchakovski, O. S. Zozulya, Phys. Rev. C **74**, 044903 (2006).
[11] V. V. Begun, M. Gaździcki, M. I. Gorenstein, M. Hauer, B. Lungwitz, V. P. Konchakovski, Phys. Rev. C **76**, 024902 (2007).
[12] M. Hauer, V. V. Begun, M. I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008).
[13] Yu. B. Rumer, M. Sh. Ryvkin, *Thermodynamics, Statistical Physics, and Kinetics*, (Nauka, Moscow,1972) (in Russian).
[14] K. B. Tolpygo, *Thermodynamics and Statistical Physics*, (Kiev University, Kiev, 1966) (in Russian).
[15] M. Hauer, Phys. Rev. C **77**, 034909 (2008).
[16] M. I. Gorenstein, J. Phys. G **25**, 125102 (2008).
[17] M. I. Gorenstein, M. Hauer, Phys. Rev. C **78**, 041902(R) (2008).
[18] Z. Koba, H. B. Nielsen, P. Olesen, Nucl. Phys. B **40**, 317 (1972).
[19] M. Gazdzicki, M. I. Gorenstein, Phys. Lett. B **517**, 250 (2001).
[20] V. V. Begun, M. Gaździcki, M. I. Gorenstein, Phys. Rev. C **78**, 024904 (2008).
[21] M. I. Gorenstein, Yad. Fiz. **31**, 1630 (1980); R. Hagedorn, Z. Phys. C **17**, 265 (1983); St. Mrowczynski, Z. Phys. C **27**, 131 (1985).
[22] V. V. Begun, M. Gaździcki, M. I. Gorenstein, arXiv: 0812.3078 [hep-ph].

НОВА КОНЦЕПЦІЯ СТАТИСТИЧНИХ АНСАМБЛІВ

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Запропоновано розширення стандартної концепції статистичних ансамблів. Уведено статистичні ансамблі, у яких екстенсивні величини флюктують відповідно до заданного ззовні розподілу. Показано застосування такого підходу в статистичних моделях множинного народження адронів у фізиці високих енерґій.