СЕМІНАР ІЗ СУЧАСНИХ ПРОБЛЕМ ФІЗИКИ
(Львів, 10–11 липня 2012 року)

WORKSHOP ON CURRENT PROBLEMS IN PHYSICS
(Lviv, 10–11 July 2012)

On 10–11 July 2012, the Physics Faculty of the Ivan Franko National University of Lviv hosted the Workshop on Current Problems in Physics. The representatives from the scientific institutions of Ukraine and Poland participated in the Workshop, which was the fifth meeting of this series. The talks covered quantum mechanics, condensed matter physics, statistical physics, astrophysics, and some other subjects. The abstracts of the presentations are given below.

FRACTIONAL PERTURBED VOLTERRA EQUATIONS OF THE CONVOLUTION TYPE

B. Bandrowski[1]*, A. Karczewska*[1] *and P. Rozmej*[2]
[1] Faculty of Mathematics, Computer Science and Econometrics
[2] Faculty of Physics and Astronomy, University of Zielona Góra, Poland

In the paper we study the following class of perturbed Volterra equations of the convolution type

$$u(x,t) = u(x,0) + \int_0^t [g_\alpha(t-s) + (g_\alpha * k)(t-s)]\Delta u(x,s)ds + \int_0^t b(t-s)u(x,s)ds, \tag{1}$$

where $x \in \mathbb{R}$, $t > 0$, $g_\alpha(t) = t^{\alpha-1}/\Gamma(\alpha)$, Γ is the gamma function, $\alpha \in [1,2]$, $b,\ k \in L^1_{loc}(\mathbb{R}_+;\mathbb{R})$ and Δ is the Laplace operator. The function g_α corresponds to the class of equations interpolating heat and wave equations. In other words, the time derivative in the differential equation corresponding to the Volterra equation with that kernel represents a fractional derivative of the order α. Functions $b,\ k$ introduce some additional perturbations. The problems described by equation (1) arise in the phenomena of anomalous transport, for instance in porous materials.

The paper provides approximate numerical solutions to the considered equations. The results obtained generalize our previous results [1] from 2010. The approximation consists in the application of a finite subspace of an infinite basis in time variable and discretization in space variables. This leads to a large-scale system of linear equations with a non-symmetric matrix that is solved with the use of the iterative GMRES method.

The presentation bases on paper [2].

[1] B. Bandrowski, A. Karczewska, P. Rozmej, Int. J. Appl. Math. Computer Sci. **20** 261 (2010).
[2] B. Bandrowski, A. Karczewska, P. Rozmej, *Numerical solutions to fractional perturbed Volterra equations*, submitted.

FLAT-BAND STRONGLY CORRELATED LATTICE SYSTEMS

O. Derzhko
Condensed Matter Physics of NAS of Ukraine, Lviv, Ukraine

Ten years ago J. Schulenburg et al. [J. Schulenburg, A. Honecker, J. Schnack, J. Richter, and H. - J. Schmidt, Phys. Rev. Lett. **88**, 167207 (2002)] reported interesting results concerning the ground-state magnetization curve for some quantum Heisenberg antiferromagnets. They found that for a wide class of highly frustrated lattices the magnetization curve at zero temperature for this spin model exhibits a jump just at the saturation field. Later on it was recognized that the examined quantum spin systems are intimately connected to another class of strongly correlated electron models, the so-called flat-band Hubbard ferromagnets, which were discussed even earlier [A. Mielke and H. Tasaki, Commun. Math. Phys. **168**, 341 (1993)]. For both spin and electron models the considered lattices support a completely dispersionless (flat) one-particle (one-magnon or one-electron) band and the states from the flat band are the lowest-energy

one-particle states. This circumstance opens an interesting perspective to construct and fully characterize *many-particle ground states* of the strongly correlated lattice systems. Moreover, it provides a possibility to map these strongly correlated systems onto the corresponding (highly non-trivial) *classical* systems and as a result to use completely different methods from a tool-box of classical statistical mechanics (no Green functions, no functional integrals, etc.) for a discussion of quantum many-body physics.

In the present talk I intend to discuss some recent results on flat-band strongly correlated lattice systems focusing mainly on the studies performed in collaboration with J. Richter (Magdeburg), A. Honecker (Göttingen), R. Moessner (Dresden), H.-J. Schmidt (Osnabrück), J. Schulenburg (Magdeburg), T. Krokhmalskii (Lviv), and M. Maksymenko (Lviv).

THE SCATTERING OF THE ELECTRON BY A MAGNETIC FIELD WITH CYLINDRICAL SYMMETRY: WHAT IS NEW AND UNEXPECTED?

Yu. A. Sitenko[1], *N. D. Vlasii*[2]
[1]Bogolyubov Institute for Theoretical Physics, NAS of Ukraine, Kyiv, Ukraine
[2]Physics Department, Taras Shevchenko National University of Kyiv, Ukraine

Quantum-mechanical scattering of nonrelativistic charged particles by a magnetic vortex of nonzero transverse size is considered. A feasible experiment can be performed involving only the particles with short, as compared with the transverse size of the vortex, wavelengths. We show that the flux of the vortex serves as a gate for the strictly forward propagation of such particles; this effect is the same for various configurations of the magnetic field distributed inside the vortex. We discuss the possibilities of the experimental detection of this effect.

RADIATIVE PROCESSES IN RELATIVISTIC MAGNETISED PLASMA

G. Melikidze
Institute of Astronomy, University of Zielona Góra, Poland

It is well known that plasma processes play a decisive role in the generation of emission from many astrophysical objects. An excellent example of a space laboratory for plasma physics is a magnetized neutron star and its environment. If the frequency of radiation is less than that of the plasma and/or cyclotron frequencies, the radiation mechanism should be coherent. Therefore, high energy (i.e. X- and γ-ray) emission can usually be explained by a single-particle radiation (excluding the black-body radiation from very hot spots), but the radio and optical radiation must be generated due to some plasma instabilities. We will overview linear and nonlinear processe the in such plasmas and addresses possibilities of their application for astrophysical purposes. It should be mentioned that generation of waves is a necessary but insufficient condition for radiation mechanisms. Though the plasma waves often possess the highest growth rate, they cannot escape the plasma. Quite often it is not enough to generate turbulence in the source, but it is also necessary to find a mechanism that ensures transformation of plasma waves into waves capable of escaping from the source and reaching the observer.

DO THE OBSERVATIONS PREFER PHANTOM DARK ENERGY?

B. Novosyadlyj, O. Sergijenko
Astronomical Observatory of Ivan Franko National University of Lviv, Ukraine

We analyze the dynamics of expansion and large scale structure formation of the Universe in the models with Λ-phantom scalar field as dark energy, which starts from Λ-term mimicry at the Big Bang inflation and slowly evolves to the Big Rip singularity. Such a model of dark energy is described by the three parameters — density and EoS parameter in the current epoch, Ω_{de} and w_0, and asymptotic value of the EoS parameter in a-infinity, w_{a-i} (a is the scale factor in Friedmann–Robertson–Walker metric). Their best-fit values were determined jointly with other important cosmological parameters by the MCMC method using available observational data on CMB anisotropy, SNe Ia luminous distances, BAO measurements, etc. They are: $\Omega_{de} = 0.72 \pm 0.04$, $w_0 = -1.043^{+0.043}_{-0.24}$ and $w_{a-i} = -1.12^{+0.12}_{-0.50}$ (denoted by $\mathbf{p_1}$) for the dataset including SDSS SNe Ia distance moduli, determined by SALT2 light curve fitting method, and $\Omega_{de} = 0.69 \pm 0.05$, $w_0 = -1.002^{+0.002}_{-0.14}$ and $w_{a-i} = -1.19^{+0.19}_{-0.42}$ (denoted by $\mathbf{p_2}$) for dataset with SDSS SNe Ia distance moduli, determined by MLCS2k2 light curve fitting method. For the model with best-fit parameters $\mathbf{p_1}$ the Big Rip singularity occurs ~ 170 Gyrs after Big Bang, while in the model with $\mathbf{p_2}$ 600 Gyrs after Big Bang. The similar simulations have been run for ΛCDM and the quintessential scalar field model of dark energy. It was shown that the dataset with SALT2 SNe Ia distance moduli

prefers phantom scalar field with parameters $\mathbf{p_1}$ as dark energy, while dataset with MLCS2k2 SNe Ia distance moduli prefers quintessential scalar field with parameters $\Omega_{de} = 0.70 \pm 0.05$, $w_0 = -0.83^{+0.22}_{-0.17}$ and $w_e = -0.88^{+0.88}_{-0.12}$ (w_e is EoS parameter in early epoch). But the differences of maximal likelihoods for them are statistically insignificant. Therefore, at the current level of accuracy of cosmological observational data we cannot clearly establish the type of dark energy — quintessence, phantom or Λ. Perhaps the new data from the current and planned observational programs will give a possibility to establish the type of dark energy.

QUANTUM BRACHISTOCHRONE PROBLEM FOR TWO SPIN-1/2

A. R. Kuzmak
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

We study the quantum brachistochrone evolution for a system of two spin-$\frac{1}{2}$ under the specific Hamiltonian. This Hamiltonian realizes quantum evolution in two subspaces which we express as "1" spanned by $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ and "2" ($|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$). These allows to consider our problem for each subspace separately and lead it to the quantum brachistochrone problem for a spin-$\frac{1}{2}$ in the magnetic field. Also, we obtain the time unitary operator and conditions require to generate an entanler gate and other gates. We demonstrate that this Hamiltonian allows to reach maximaly entangled states such as Bell states. This results might be important in quantum computing, quantum teleportation and quantum cryptography.

SURFACE WATER WAVES BEYOND THE KdV CASE

A. Karczewska [1], *P. Rozmej* [2], *Ł. Rutkowski* [2]
[1]Faculty of Mathematics, Computer Science and Econometrics
[2]Faculty of Physics and Astronomy
University of Zielona Góra

In the paper we study the propagation of surface waves on shallow water in an approximation which goes beyond that leading to the Korteveg–de Vries equation. Our aim is to solve the Euler equations for an irrotational flow of an incompressible fluid with a free surface [1]:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0, \quad \text{for} \quad -h < z < \eta(x,y,t) \tag{1}$$

$$\phi_z - (\eta_x\phi_x + \eta_y\phi_y + \eta_t) = 0, \quad \text{at} \quad z = \eta(x,y,t) \tag{2}$$

$$\phi_t + \tfrac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2) + g\eta = 0, \quad \text{at} \quad z = \eta(x,y,t) \tag{3}$$

$$\phi_n = 0, \quad \text{at} \quad z = -h. \tag{4}$$

Equation (1) is the Laplace equation for the velocity potential ϕ, equations (2) and (3) represent the so-called kinematic and dynamic boundary conditions at the fluid surface and (4) give the boundary condition at the bottom.

The derivation of the KdV equation bases on the assumptions that the bottom is flat, that is $h = \text{const}$, and $z + h \ll \lambda_0$, where λ_0 represents a mean value of surface wave length. Then one looks for the velocity potential in the form

$$\phi = \phi_0 + (z+h)\phi_1 + (z+h)^2\phi_2 + (z+h)^3\phi_3 + (z+h)^4\phi_4 + \cdots, \tag{5}$$

where functions $\phi_k(x,t), k = 0,1,2,\ldots$ do not depend on z. Further approximations consist in neglecting the terms of the orders $n > 3$ in $(z+h)^n$ in series (5) and abandoning some other small nonlinear terms which finally lead to the KdV equation for surface waves with soliton solutions.

We consider more general case in which $h = h(x,y)$, i. e. the bottom is not flat. In this case there is no analytic solution and we use numerical approach. We discuss several reasonable approximations in the spirit of the KdV approach and show that we can reproduce the KdV solutions when $h = \text{const}$. A more detailed presentation of the solutions of the above equations will be presented soon.

[1] M. Remoissenet, *Waves called solitons* (Springer, 1993).
[2] Ł. Rutkowski, A. Karczewska, P. Rozmej, in preparation.

QUASI-EXACTLY SOLVABLE POTENTIALS FOR A PARTICLE WITH THE POSITION-DEPENDENT MASS

O. Voznyak
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

The studies of the quantum-mechanical system with position-dependent effective mass constitute an important direction of modern physics. Particularly, such problems arise in the physics of inhomogeneously doped semiconductors, semiconductor heterostructures, quantum wells, superlattices, nuclear physics, quantum liquids, metal clusters, etc. Therefore, the methods of exact solutions of the quantum mechanical problems with position-dependent have been thoroughly studies with the use of different approaches.

Supersymmetric quantum mechanics provides a powerful method of the exact solution of Schrödinger equation. In [1] the supersymmetric method was extended to the case of position-dependent mass.

Unfortunately, there is a limited set of potentials which allows exact solvability of the Schrödinger equation. Therefore the concept of quasi-exact solvability was introduced where it is possible to compute a certain finite number of eigenvalues and eigenfunctions exactly, even though the exact expressions for the full set of eigenvalues do not exist.

In this paper we extend the supersymmetric method for the generation of the quasi-exactly solvable potentials [2] to the case of position-dependent mass. This method allows us to obtain different quasi-exactly solvable potentials with two known levels and the corresponding wave functions choosing different generating functions.

In the frame of this method the ground state was studied. It has been shown that position dependent mass can generate bound ground state even in the case of a constant potential.

The conditions for the existence of the ground and first excited state for different potentials and position dependent mass have been investigated.

[1] B. Bagchi, A. Banerjee, C. Quesne, V. M. Tkachuk, J. Phys. A **38**, 2929 (2005).
[2] V. M. Tkachuk, Phys. Lett. A **245**, 177 (1998).

ONE-PARTICLE DENSITY MATRIX OF THE BOSE-SYSTEM + IMPURITY ATOM AT FINITE TEMPERATURES

G. Panochko
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

In the contribution the study of the impurity states in quantum fluids based on the construction of the full density matrix for all temperatures is made.

The contribution taking into account the effect of impurities on the system states is singled out for the aggregate of interacting Bose-particles and impurity atoms of ^{3}He in the approximation of pair correlations in the expression for density matrix. Averaging over the states of the Bose-liquid the one-particle density matrix of the system is found. Its Fourier-image allows to find the momentum distribution of "liquid helium + impurity". The estimation of the effective mass of the impurity is made.

COMBINING TIGHT-BINDING AND MOLECULAR DYNAMICS METHODS TO MODEL THE BEHAVIOR OF METALS IN THE PLASTIC REGIME

M. Białoskórski[1,2], *M. Bobrowski*[1,2], *J. Dziedzic*[1], *J. Rybicki*[1,2]
[1]Faculty of Applied Physics and Mathematics, [2]TASK Computer Centre,
Gdańsk University of Technology, Poland

Ultra-precision machining of metals, the breaking of nanowires under tensile stress and thefracture of nanoscale materials are examples of technologically important processes which are both extremely difficult and costly to investigate experimentally. We describe a multiscale method for the simulation of such systems in which the energetically active region is modeled using a robust tight-binding scheme and the rest of the system is treated with the molecular dynamics method. We introduce a computer code implementing the method geared towards non-equilibrium, cross-scaled tight-binding and molecular dynamics simulations. Apart from the presentation of the method and implementation, we discuss the preliminary physical results obtained as well as their validity.

STRUCTURAL ASPECTS OF CARBON NANOTUBES-BASED COMPOSITES

S. Mudry[1], *I. Shtablavyi*[1], *Yu. Kulyk*[1], *R. Serkiz*[2]
[1]Physics of Metals Department,
[2]Scientific-Technical and Educational Center of Low-Temperature Studies,
Ivan Franko National University of Lviv, Ukraine

Nanocomposite materials with a metallic matrix and carbon nanotubes as a filler attract the attention of researchers due to their promising use in various areas of application and as inhomogeneous systems with unique physical-chemical properties which call for more detailed studies from the fundamental viewpoint.

The interrelation between the matrix and filler structure difference and the properties of a composite system are unclear.

It is also interesting how the boundary matrix-nanotubes can be modified in order to improve the properties of the nanocomposite.

In this work we represent the results of studying the nanocomposites which consist of carbon nanotubes covered with some metals (Cu, Ni) by means of the electrolytic method.

Such nanotubes were mixed with the metallic melt and then solidified upon cooling.

The structure changes at the formation of the composite have been studied by means of the X-ray diffraction method.

THE GENERALIZED UNCERTAINTY PRINCIPLE WITH MINIMAL LENGTH AND MOMENTUM AND BLACK HOLE'S THERMODYNAMICS

M. M. Stetsko
Department of Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

It is well known that Heisenberg's uncertainty principle allows one to obtain thermodynamical functions such as temperature and entropy for a black hole. We apply generalized uncertainty principle with minimal length and momentum to a Schwarzschild black hole. We calculate thermodynamical functions of a Schwarzschild black hole such as temperature, entropy and heat capacity. We also investigate the emission rate relation for a black hole and show that the generalized uncertainty principle leads to a shorter lifetime of a black hole.

PTICAL AND EPR SPECTROSCOPY OF THE Nd-DOPED BORATE GLASSES

B. V. Padlyak[1,2], *W. Ryba-Romanowski*[3], *R. Lisiecki*[3]
[1]Division of Spectroscopy of Functional Materials, Institute of Physics,
University of Zielona Góra, Poland
[2]Sector of Spectroscopy, Institute of Physical Optics, Lviv, Ukraine
[3]Institute of Low Temperatures and Structure Research, Polish Academy of Sciences, Wrocław, Poland

Optical absorption, luminescence (excitation and emission) spectra as well as luminescence kinetic of the Nd^{3+} centers in glasses with the $Li_2B_4O_7$:Nd, $LiCaBO_3$:Nd, and Ca B_4O_7:Nd compositions containing 0.5 and 1.0 mol. % Nd_2O_3 have been studied. By optical and electron paramagnetic resonance (EPR) spectroscopy it was shown that the Nd impurity is incorporated in the borate glass network as Nd^{3+} ($4f^3$, $^4I_{9/2}$) ions. All the observed transitions of the Nd^{3+} centers in optical absorption and luminescence spectra were identified. The oscillator strengths (f_{teor}) and phenomenological intensity parameters Ω_t ($t = 2, 4, 6$) for glasses containing 1.0 mol. % Nd_2O_3 are calculated using standard Judd-Ofelt theory. The radiative transitions rates (W_r), branching ratios (β), and radiative lifetime (τ_{rad}) for Nd^{3+} centers in the investigated glasses have been calculated and analyzed in comparison with the corresponding parameters for Nd^{3+} centers in other borate glasses with different compositions. Measured lifetimes for Nd^{3+} centers in the $^4F_{3/2}$ emitting level are compared with those calculated and quantum efficiencies (η) of the investigated glasses are estimated. Some perspectives of the application of the $Li_2B_4O_7$:Nd, $LiCaBO_3$:Nd, and CaB_4O_7:Nd glasses for Nd^{3+}-lasers ($F_{3/2} \rightarrow {}^4I_{11/2}$ channel), particularly for LED-pumped Nd^{3+}-lasers, are considered. Based on the structural data for borate glasses and corresponding crystals, the peculiarities of spectroscopic properties and local structure of the Nd^{3+} centers in the $Li_2B_4O_7$:Nd, $LiCaBO_3$:Nd, and CaB_4O_7:Nd glasses have been discussed.

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NON-FINITE DIFFERENCE APPROACH TO NONLINEAR DIFFERENTIAL EQUATIONS

M. R. Dudek, W. Wolak, B. Zapotoczny
Institute of Physics, University of Zielona Góra, Poland

A new efficient numerical algorithm for solving the nonlinear differential equations has been discussed. It is a generalization of the non-finite difference algorithm published for ODE [1],[2] and DDE [3]. Some particular examples have been discussed of the N-soliton solutions of the KdV equation with $N = 1, 2,$ and 3.

[1] M. R. Dudek, T. Nadzieja, Int. J. Mod. Phys. C **16**, 413 (2005).
[2] B. Brzostowski, M. R. Dudek, B. Grabiec, T. Nadzieja, Phys. Stat. Solidi (b) **244**, 851 (2007).
[3] M. R. Dudek, T. Nadzieja, in *Series on Advances in Mathematics for Applied Sciences* **79**, ed. by M. Lachowicz, J. Miekisz (World Scientific, 2009), p. 149.

DEFORMED HEISENBERG ALGEBRA WITH MINIMAL LENGTH AND EQUIVALENCE PRINCIPLE

V. Tkachuk
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

The studies in string theory and quantum gravity lead to the Generalized Uncertainty Principle (GUP) and suggest the existence of fundamental minimal length which, as it was established, can be obtained in the frame of a deformed Heisenberg algebra. The first look at the motion of bodies in the deformed space in the uniform gravitational field can give an impression that the bodies of different mass fall in different ways and that the equivalence principle is thus broken. Taking into account the result of paper [1] for the effective parameter of deformation for the center of mass of a macroscopic body as a composite system we show that in fact GUP is consistent with the equivalence principle.

[1] C. Quesne, V. M. Tkachuk, Phys. Rev. A **81**, 012106 (2010).

ON A POSSIBLE EMISSION OF COSMIC γ-RAYS BY LONG LIVING UNSTABLE PARTICLES

K. Urbanowski
Institute of Physics, University of Zielona Góra, Poland

Not all the mechanisms of the emission of a very high energy cosmic gamma-rays are clear. We find that charged unstable particles as well as neutral unstable particles with non-zero magnetic moment which live sufficiently long may emit electromagnetic radiation. This new mechanism is connected with the properties of unstable particles at the post-exponential time region. Analyzing the transition time region between the exponential and non-exponential form of the survival amplitude it is found that the instantaneous energy of the unstable particle can take very large values, much larger than the energy of this state for t from the exponential time region. Basing on the results obtained for the model considered it is shown that this purely quantum mechanical effect may be responsible for causing unstable particles to emit electromagnetic, X- or γ-rays at some time intervals from the transition time regions.

THE EXACT SOLUTION FOR THE QUANTUM SPIN-1/2 TWO-LEG LADDER WITH THE ISING INTERRUNG INTERACTION

T. Verkholyak[1], *J. Strečka*[2]
[1]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv, Ukraine,
[2]Department of Theoretical Physics and Astrophysics, Institute of Physics,
P. J. Šafárik University, Košice, Slovak Republic

The quantum spin-1/2 two-leg ladder with the anisotropic XYZ Heisenberg interaction along rungs and Ising interaction between the neighboring rungs is studied rigorously. Using the unitary transformation the model is reduced to the transverse Ising chain with composite spins so that the ground state of the ladder can be determined exactly. We calculate the ground state phase diagram for the cases when the interaction between the x or y spin components is dominating on the same rungs. The limit of $X - X$ intra-rung interaction corresponds to the quantum compass ladder where this interaction may destroy the z-ordering of spins along legs. We analyze how an additional diagonal Ising interaction affects the ground state of the model.

DFT STUDY OF HOMO- AND HETERO-NUCLEAR CHROMIUM-BASED MOLECULAR RINGS

B. Brzostowski[1], *T. Ślusarski*[2] *and G. Kamieniarz*[2]
[1]Institute of Physics, University of Zielona Góra, Poland,
[2]Faculty of Physics, Adam Mickiewicz University, Poznań, Poland

Based on the first principles density functional theory (DFT) calculations, as implemented in the SIESTA package, we present a comprehensive study of electronic and magnetic properties of octometallic homo- and hetero-metallic chromium-based molecular rings. We widely examine the electronic and magnetic properties of $Cr_7MF_8(O_2CH)_{16}$ where M = Cr, Cd and Ni. The total, local and orbital projected density of states are presented the magnetic moments are calculated using different approaches as well as the electron density and spin density maps are discussed. Also, depending on M, the highest occupied molecular orbital (HOMO) and the lowest unoccupied molecular orbital (LUMO) gaps for both spin channels are analyzed. All the non-equivalent spin configurations with $S = \pm 3/2$ are considered to extract exchange interaction parameter J from the spin model.

CRITICAL BEHAVIOUR OF SPIN MODELS ON COMPLEX NETWORKS

M. Krasnytska[1], *B. Berche*[2], *and Yu. Holovatch*[3]
[1]Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine,
[2]Institut Jean Lamour, Nancy Université, France,
[3]Institute for Condensed Matter Physics, NAS of Ukraine, Lviv, Ukraine

Potts and Ising models belong to the most popular spin models of statistical physics. In this report we discuss the case, when these models are considered not on the lattice sites, but on the sites of complex network-like structures with the varying node degree k. Possible applications of spin models on complex networks can be found in various segments of physics, starting from problems of sociophysics to physics of nanosystems, whose structure is often much better described not with the geometry of the lattice but by a network.

First, we review peculiarities of the critical behaviour of the Ising model on a complex network. Then, we use the mean-field approach to analyze the thermodynamical properties of the q-state Potts model on the uncorrelated scale-free network with the power-law decay of the node-degree distribution $P(k) \sim k^{-\lambda}$. We obtain thermodynamic functions and analyze the phase diagram of the model in different regions of the $q - \lambda$ plane. Depending on the particular values of q and λ one observes either the first-order or the second-order phase transition [1]. Analyzing the free energy for different q and λ, we observe that the behaviour of the thermodynamic functions in the vicinity of the critical point for $1 \leq q \leq 2$ and certain values of λ are governed by power-law singularities enhanced by the logarithmic corrections. For $q = 2$, the logarithmic corrections are observed at $\lambda = 5$ whereas for $1 \leq q < 2$ they appear at $\lambda = 4$. By our data obtained for $q = 2$ we recover familiar results for the Ising model. For $q = 1$ (and $1 < q < 2$), $\lambda > 4$ we recover the usual mean field results for the lattice percolation exponents. Furthermore, for $3 < \lambda < 4$ our results for the leading exponents reproduce the percolation exponents on a scale-free network [2].

We show that the logarithmic corrections exponents that appear at $\lambda = 4$ and $q = 1$, describe percolation on a scale-free network with the node-degree distribution exponent $\lambda = 4$. For the percolation problem, we also display the scaling functions for different observables in an explicit form.

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TOWARDS THE DEFINITION OF CRITICAL TEMPERATURES IN THE COMPLEX-VALUED FRACTIONAL STATISTICS

A. Rovenchak
Department for Theoretical Physics, Ivan Franko National University of Lviv, Ukraine

The possible approaches to the definition of critical temperature are analyzed for a system of D-dimensional harmonic oscillators ($D = 1, 2$) obeying the Polychronakos fractional statistics with a complex-valued parameter. The occupation number in this statistics is given by

$$n_i = \frac{1}{z^{-1}e^{\varepsilon_i/T} - e^{i\pi\nu}},$$

where z is fugacity, T is temperature, and ε_i is the energy of the ith level. The statistics parameter $\alpha = e^{i\pi\nu}$ stays on the unit circle ($\nu = 0 \div 1$).

The system of oscillators is shown to have temperature points corresponding to the non-analytical behavior of thermodynamic functions. For $\nu \neq 1$ similarity to the Bose-condensation phenomenon is observed to some extent.

RECENT DEVELOPMENTS ON AUXETIC MATERIALS

R. Caruana-Gauci[1], *R. Gatt*[1], *D. Attard*[1], *R. Cauchi*[2], *Ch. Zerafa*[2], *A. Casha*[1], *K. M. Azzopardi*[1], *A. Buttigieg*[1,3], *H. Vella*[3], *J. N. Grima*[1,2]
[1]Metamaterials Unit, Faculty of Science, University of Malta,
[2]Department of Chemistry, Faculty of Science, University of Malta,
[3]Halmann Vella Research & Development, Lija, Malta

Typically materials which we encounter in our everyday life have a tendency to get thinner when stretched (i. e. exhibit a positive Poisson's ratio). However, not all materials behave like this and there exist some materials and structures which get fatter when stretched (i.e. exhibit a negative Poisson's ratio). Such systems are referred to as auxetic, a word which derives from the Greek word *auxetos*, which means 'to grow'.

Auxeticity, though not a common occurrence, is a highly desirable property since it imparts on the materials several enhanced properties when compared to conventional counterparts. Amongst these properties are: increased indentation resistance, the ability to adopt dome shaped surfaces and enhanced vibration absorption properties. Due to these properties, auxetic materials have a wide variety of applications.

In this paper we present some recent developments made in the subject of auxetic materials. In particular we review the various models which are currently used to explain the Poisson's ratio (both positive and negative) in materials and structures and show that new materials can be produced to exhibit this atypical property.

ULTRARELATIVISTIC LIMIT OF THE LORENTZ–DIRAC EQUATION

Yu. Yaremko
Institute for Condensed Matter Physics, NAS of Ukraine, Lviv, Ukraine

The Fradkin's projection operators are applied to the problem of motion of a light charged particle in a very strong electromagnetic field. The equation of motion is derived which takes into account the radiation reaction (self-action). The equation on eigenvalues and eigenvectors of the electromagnetic field tensor arises as the ultrarelativistic limit of the well-known Lorentz–Dirac equation. The global structure of pulsar magnetosphere is considered in the context of this equation of motion.

ON THE SCHRÖDINGER EQUATION IN A COMPACTIFIED SPACE

A. Duviryak
Institute for Condensed Matter Physics, NAS of Ukraine, Lviv, Ukraine

The development of the modern string theory caused deep revision in the conception of physical space. At least two characteristic features of the space anticipated by the string theory are more profound than the theory itself: the existence of fundamental length and multi-dimensionality. Lack of evidence of the space multi-dimensionality is conventionally explained due to the compactification of extra space dimensions. A study of same peculiarities of such spaces by means of mathematically simple tools, without sophisticated techniques of the string theory, is the subject of the present report.

It is considered a potential of point-like mass or charge in the multidimensional partially compactified space. The behavior of the potential is studied at a distance much longer and much shorter than a compactification radius. An explicit form of the potential in the cases of 2- and 4-dimensional space with one compact dimension is found. An applied physical-engineering interpretation is proposed for the first example. The second example of the potential is used in the Schrödinger equation describing a state of the particle on a 3-D brane in the 4-D compactified space under the gravity of a point-like mass. The energy spectrum is found via various approximation methods: variational, perturbational and numerical integration. The relativistic aspect of the problem is discussed.

РОМАН ВАСИЛЬОВИЧ ЛУЦІВ (1937–2012)

ROMAN VASYLIOVYCH LUTSIV (1937–2012)

12 червня 2012 року пішов із життя відомий учений, лауреат Державної премії України в галузі науки і техніки, завідувач кафедри радіоелектронного матеріалознавства Львівського національного університету імені Івана Франка, професор Роман Васильович Луців.

Роман Васильович Луців народився 23 жовтня 1937 року в м. Львові. У 1959 році закінчив фізичний факультет Львівського університету. Трудову діяльність розпочав із посади лаборанта науково-дослідної лабораторії. У 1959 р. очолив науково-дослідну лабораторію росту та дослідження фізичних властивостей кристалів. На ту пору Р. В. Луців активно працював у галузі синтезу й вивчення електрофізичних властивостей напівпровідникових матеріалів типу A^2B^6, що стало основою напівпровідникових досліджень та передумовою утворення кафедри фізики напівпровідників. Продовженням робіт у цьому напрямі був синтез, вирощування й дослідження монокристалів систем на основі халькогенідів ртуті. Р. В. Луців зробив значний внесок у розробку методики вирощування монокристалів HgS, як чистих, так і з різними домішками, та в комплексне вивчення властивостей цього матеріалу. За його участі виконано великий цикл технологічних досліджень, унаслідок чого розроблено декілька методів одержання монокристалів і тонких шарів різних модифікацій сірчистої ртуті. Подальший розвиток напівпровідникової тематики був пов'язаний із синтезом і вирощуванням монокристалів систем HgSe–HgS, HgTe–HgS, HgTe–MnTe. Ці системи зайняли важливе місце в комплексних дослідженнях напівпровідників із вузькою забороненою зоною та великою рухливістю носіїв заряду. Так, удосконалено методику вирощування монокристалів подвійних сполук і твердих розчинів на їхній основі, проведено систематичне вивчення дефектів структури, електрофізичних, фотоелектричних та оптичних властивостей цих сполук. Результати досліджень лягли в основу кандидатської дисертації "Синтез та дослідження фізичних властивостей твердих розчинів халькогенідів ртуті (HgSe–HgS, HgTe–HgS)", яку Роман Васильович захистив у 1967 році.

У 60–70-х роках за активної участі вченого на кафедрі фізики напівпровідників досліджують вузькозонні напівпровідникові сполуки A^2B^6, зокрема тверді розчини кадмій–ртуть–телур (КРТ), плівкові системи на їхній основі, варізонні структури; упроваджують оригінальні оптимальні технологічні методи вирощування монокристалів твердих розчинів на основі халькогенідів ртуті CdTe–HgTe. Розроблена в ту пору технологія одержання досконалих об'ємних зразків $Cd_xHg_{1-x}Te$ дала змогу вивчити електрофізичні властивості цих кристалів, визначити й уточнити основні параметри їхньої зонної структури. На кафедрі запропоновано нові методи одержання епітаксійних шарів цих напівпровідників і технологію створення приладів на їхній основі.

З 1975 р. на фізичному факультеті Львівського університету під керівництвом Р. В. Луціва сформовано новий напрям досліджень із фізики силіцидів і германідів рідкісноземельних та перехідних металів з метою застосування їх у мікроелектроніці, з'ясування впливу компонент на характер міжатомних взаємодій та фізичні властивості. За цикл робіт, зокрема за дослідження з хімії, кристалохімії та фізики силіцидів і германідів рідкісноземельних та перехідних металів, розробку на їхній основі нових матеріалів електронної техніки Р. В. Луців у складі авторського колективу відзначений Державною премією УРСР у галузі науки і техніки за 1984 р. Наприкінці 80-х — на початку 90-х років ці роботи розвинулись у напрям фізики сильнокорельованих конденсованих систем. У результаті отримано важкоферміонні системи силіцидів церію з міддю, які мають "екзотичну" надпровідність.

Від 1987 року в коло наукових інтересів Р. В. Луціва входять технологічні, теоретичні та експериментальні дослідження електричних і магнітних властивостей високотемпературних надпровідникових матеріалів $YBa_2Cu_3O_{7-\delta}$, а з 1993 року — $HgBa_2Ca_{n-1}Cu_nO_{2n+2+\delta}$ $(n = 1-3)$ з температурами надпровідного переходу $T_c = 92 - 134$ K. Група науковців під його керівництвом провела великий обсяг результативної дослідницької роботи в напрямку вивчення особливостей формування надпровідної кераміки, дослідження фізичних властивостей ВТНП у нормальному та надпровідному станах. Зокрема встановлено взаємозв'язок між технологіями одержання надпровідних матеріалів і їхніми фізичними параметрами, оптимізовано процес синтезу Hg–1223 шляхом використання шихти на основі прекурсорів. Установлено особливості впливу лазерного випромінювання на кераміку Y-ВТНП з погляду зміни електрофізичних характеристик. З'ясовано можливості збільшення критичних струмів шляхом зміни морфології поверхні, ущільненням приповерхневих шарів, збільшен-

ням однорідного розподілу компонент, зменшенням домішкових вуглецевих фаз, зміною кисневого індексу, збільшенням ступеня текстурації в площині (a, b). Кількісно описано гістерезис магнітної сприйнятливості з урахуванням у моделі критичного стану Біна заложності від характеру міжзеренних зв'язків (надпровідник–ізолятор–надпровідник, надпровідник-нормальна фаза-надпровідник). Проаналізовано вплив розмірності гранул та розорієнтації на гістерезис $\chi_{AC}(H)$. З'ясовано закономірності трансформації електронної густини станів у межах феноменологічної моделі вузької зони. Проведено теоретичні розрахунки з використанням експериментальних даних щодо впливу леґування та дефектності кисневої підсистеми на особливості електронної густини станів тощо.

Науковий доробок професора Р. В. Луціва відображений у понад 400 публікаціях, 25 авторських свідоцтвах. Під його керівництвом захищено сім кандидатських дисертацій.

Професор Р. В. Луців проводив дуже значну педагогічну та організаторську роботу. Він автор великої кількості нових навчальних курсів для студентів фізичного факультету та факультету електроніки. У 1967–1968 рр. Роман Васильович працював начальником науково-дослідної частини, у 1970–1974 рр. — проректором з навчальної роботи Львівського університету. У 1995–1997 рр. виконував обов'язки заступника декана фізичного факультету. У 1997 р. обраний завідувачем кафедри радіоелектронного матеріалознавства й очолював її до 2012 р. Р. В. Луців протягом багато років був членом експертної ради з фізики Міносвіти України, членом редколегій "Вісника Львівського університету (серія фізична)", "Журналу фізичних досліджень". За багаторічну сумлінну працю Роман Васильович удостоєний звання Заслуженого професора Львівського національного університету імені Івана Франка.

Відчуття важкої та непоправної втрати сповнює друзів, колег та учнів Романа Васильовича Луціва. Його світлий образ назавжди залишиться в пам'яті та серцях усіх тих, хто його знав та мав змогу з ним спілкуватися.