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SHELL MODEL CALCULATIONS OF SOME NUCLEI NEAR ^{208}Pb REGION

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Untruncated large scale shell model calculations have been performed to study energy levels, reduced electric transition probabilities $B(E2;0_1^+ \rightarrow 2_1^+)$, and the binding energy for even-even of $^{210-212}$Pb, $^{210-212}$Po in the neutron deficit region $\pi(hfpi)\nu(igdsj)$ valance space above the ^{208}Pb core using four effective interactions *khpba*, *khpbu*, *khp* and *khpe*. The calculated energy spectra, reduced electric transition probabilities $B(E2;0_1^+ \rightarrow 2_1^+)$, and the binding energy are compared with the recently available experimental data. A very good agreement was obtained for all nuclei.

Key words: shell model, reduced electric transition probabilities, NushellX@MSU, binding energy.

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I. INTRODUCTION

The nuclei in the vicinity of the closed proton and neutron shells ^{208}Pb are of great interest; their neighbors are accessible to a variety of spectroscopic studies. This is not the case for other nuclei in the vicinity of closed shells like the ^{100}Sn and ^{132}Sn neighbors. These nuclei in fact, lie well away from the valley of stability and only recently our knowledge of their spectroscopic properties has significantly improved, thanks to the advent of large multidetector γ-ray arrays [1]. Nevertheless, a few valance particle systems around doubly magic nuclei are relativity easy to describe in terms of independent nucleons alone. The doubly magic closed nucleus of ^{208}Pb is known to exhibit the aspect of a strong core, which is parity seen in the large first 3^- and 2^+ excitation energies [2]. Zwarts *et al.*, studied the structure of $N = 126$ isotones with $208 \leq A \leq 215$ with the shell model including the configuration mixing with two different interactions [3]. Xu *et al.*, performed systematic calculations on low-lying states of 33 nuclei with $A = 202 - 212$, using the nucleon pair approximation of the shell model, The calculated binding energies, excitation energies, electric quadrupole and magnetic dipole moments of low-lying states, and $E2$ transition rates between low-lying states, their calculated results are reasonably consistent with the available experimental data [4]. Caurier *et al.*, had employed large-scale shell model calculations in the full $Z = 82 - 126$ proton model space π(hfipfp) by using the code NATHAN [5]. Their results were compared to experimental data including binding energies, level schemes, and electromagnetic transition rates. An overall excellent agreement has been obtained for states that can be described in their proposed model space [5]. Large scale shell model calculations have been performed by Teruya *et al.*, to calculate level schemes, and electromagnetic properties which were compared with the experimental data for even-even, odd-mass, and doubly odd nuclei of Pb, Bi, Po, At, Rn, and Fr isotopes in the neutron deficit region ($Z \geq 82, N \leq 126$) assuming ^{208}Pb doubly magic core [6]. Very recently Majeed *et al.* have performed large-scale shell model calculations without any restriction imposed on the valence nucleons outside the closed core ^{132}Sn for some selected exotic nuclei above the doubly-magic core ^{132}Sn to study the excitation spectra, binding energy and the reduced electric transition probabilities $B(E2;0_1^+ \rightarrow 2_1^+)$, their results were compared with the available experimental data [14], and a good global agreement was obtained for the selected isotopes [7].

As mentioned above, there have been several studies in this neutron deficient region. However, these studies are limited to few valence nucleons as particles or holes around the ^{208}Pb doubly magic core. Therefore, the main motivation for the present study is to carry out systematic study for some heavy isotopes above ^{208}Pb doubly magic core and to study some of their nuclear properties.

The present work is aimed to perform unrestricted large-scale shell model calculations to study the positive and negative energy levels, reduced transition probabilities $B(E2;0_1^+ \rightarrow 2_1^+)$ and binding energy for $^{210-212}$Pb and $^{210-212}$Po isotopes near the ^{208}Pb doubly-magic core. The calculations will be performed for selected isotopes using the recent shell model code NushellX@MSU for Windows by using *khpba*, *khpbu*, *khp* and *khpe* effective interactions and the theoretical results will be compared with the most recent available experimental data.

II. SHELL MODEL CALCULATIONS

To compare the theoretical results with experimental data large-scale shell model calculations were performed. The shell model code NushellX@MSU [8]was used, with *khpba*, *khpbu*, *khp* and *khpe* effective interactions. The full $jj67pn$ model space was utilised, including $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $1i_{11/2}$ for proton, $1i_{11/1}$, $2g_{9/2}$, $2g_{7/2}$, $3d_{5/2}$, $3d_{3/2}$, $4s_{1/2}$ and $1j_{15/2}$ for neutron. The cal-

culation of excitation energy levels, reduced transition probabilities and binding energy was compared with the recent available experimental data. In the nuclear shell model the central mean field potential is created by individual nucleons. Assuming two body interaction only, the nuclear Hamiltonian can be formally written as a sum of kinetic (T) and potential (V) and rearranged by introducing the one-body nucleon potential U_i.

$$H = T + V = \sum_{i=1}^{A} \frac{p^2}{2m_i} + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k), \quad (1)$$

$$H = \sum_{i=1}^{A} \left[\frac{p^2}{2m_i} + U_i(\mathbf{r}) \right] + \sum_{i>k=1}^{A} V_{ik}(\mathbf{r}_i - \mathbf{r}_k) - \sum_{i=1}^{A} U_i(\mathbf{r}) = H_0 + H_{\rm res}. \quad (2)$$

The solutions of the Schrödinger equation with H_0 are the nucleon single particle energies (SPE) in a central potential, as observed in single particle (hole) states outside a doubly-closed shell(CS) nucleus in its neighbours ($CS \pm 1$). The two body matrix element (TBME) of the residual interaction $H_{\rm res}$ represents the mutual interaction of the valence nucleons as observed in the $CS \pm 2$ neighbours of a magic nucleus [9].

The reduced transition probability B is defined by:

$$B(i \to f) = \frac{|\langle J_f \| O(\lambda) \| J_i \rangle|^2}{2J_i + 1}. \quad (3)$$

B depends upon the direction of the transition. For electromagnetic transitions J_i is the higher-energy initial state. But in Coulomb excitation the initial state is the lower state, and one often uses the notation $B(\uparrow)$ for this situation. If J_a is the lower state, J_b is the higher state, and $B(\uparrow)$ is given, then the value used for the electromagnetic transitions $J_b \to J_a$ is:

$$B(b \to a) = \frac{(2J_a + 1)}{(2J_b + 1)} B(\uparrow a \to b). \quad (4)$$

III. EFFECTIVE INTERACTIONS

The NushellX@MSU code comes with the library of effective interactions designed for many regions of the nuclear landscape. The calculations presented in Section IV utilize four different standard interactions for the $jj67pn$ shell, namely *khpba* [10], *khpbu* [10], *khp* [11], *khpe* [11]. The model space has the following orbits, for protons: $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{11/2}$, for neutrons: $1i_{11/2}$, $2g_{9/2}$, $2g_{7/2}$, $3d_{5/2}$, $3d_{3/2}$, $4s_{1/2}$ and $1j_{15/2}$.

A. *khpba* and *khpbu*

The *khpba* and *khpbu* effective interactions were developed by McGrory and Kuo [10] in 1975. They used "realistic" effective interactions derived from the Hamada–Johnston potential [12]. For the calculation of electromagnetic observables we used effective interactions which were consistent with the experimental information of electromagnetic observables in the "one-particle" systems. The observables which were considered were binding energies, excitation energies, strengths for one- and two-particle transfer reactions and E2 and M1 electromagnetic observables. The main difference between the *khpba* and *khpbu* effective interactions is that *khpba* is derived from the "bare" Kuo-Herling G-matrix [13], while *khpbu* utilizes the two-body matrix elements (TBME) of the "bare + one-particle one-hole ($1p1h$) bubble" Kuo-Herling (KH) [13].

B. *khp* and *khpe*

The *khp* and *khpe* effective interactions were developed by Warburton and Brown [11] in 1991 based on the Kuo-Herling realistic effective interactions for hole states and particle states relative to ^{208}Pb. They have modified the Kuo-Herling interactions by adding the core polarization correction varied to find the best fit to the energy spectra of $A = 204 - 206$ and 210–212 nuclei. The main difference between the *khp* and *khpe* effective interactions is that in the latter many TBME where adjusted empirically by adding the energy difference $\Delta(pp) = +49\,\text{keV}$ and $\Delta(nn) = +15\,\text{keV}$ to the diagonal pp and nn TBME, respectively.

IV. RESULTS AND DISCUSSION

In this section, the theoretical results are given for each nucleus. The excitation energies, $E2$ transition rate, and binding energy are calculated; in our calculations we assume that ^{208}Pb is a closed core and let the valance nucleons occupy the levels $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, $1i_{11/2}$ for protons, $1i_{11/2}$, $2g_{9/2}$, $2g_{7/2}$, $3d_{5/2}$, $3d_{3/2}$, $4s_{1/2}$ and $1j_{15/2}$ for neutrons using the code NushellX@MSU for Windows and compare the theoretical results with the most recent available experimental data.

A. Excitation Energies

Figure 1 shows the theoretical energy spectra for even-even ^{210}Pb isotope in comparison with the experimental data [14].With calculated values from *khpba*, *khpbu*, *khp* and *khpe* effective interactions, the results for four effective interactions agree with the experimental values. From Fig. 1 we can notice that *khp* and *khpe* are in excellent agreement with the experimental data unlike *khpba* and *khpbu*. There are many unconfirmed experimental energy levels that were confirmed by our calculations with different set of effective interactions. Also we can see that our calculations of *khp* and *khpe* effective interactions have the same results.

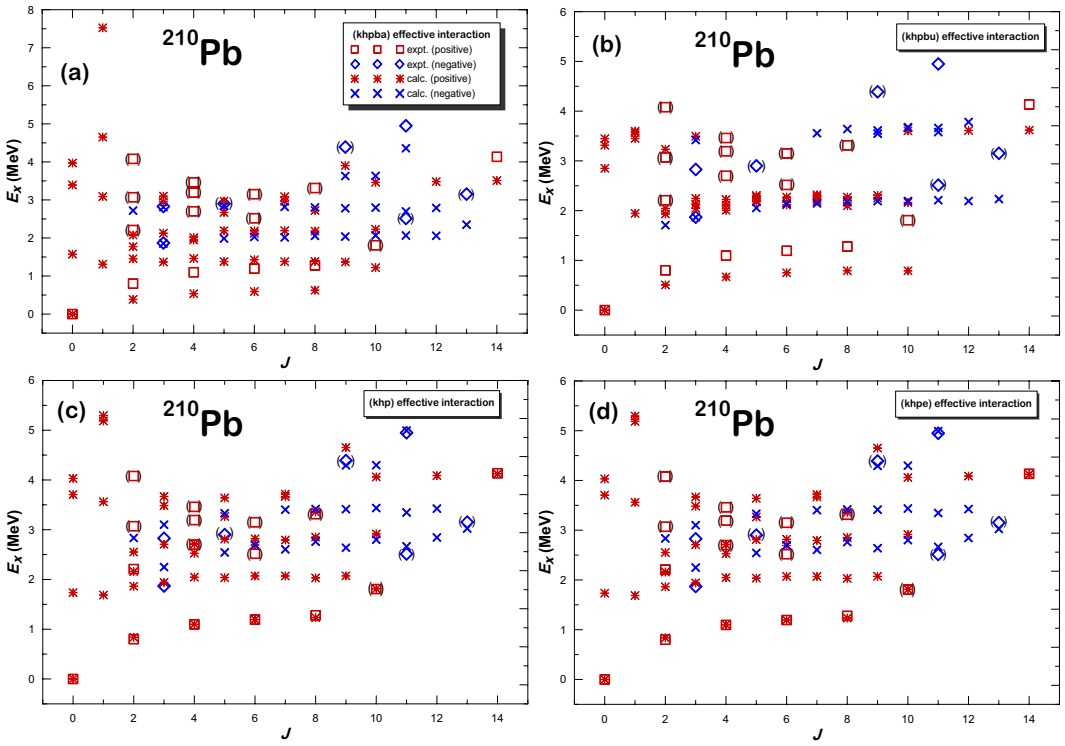

Fig. 1. The theoretical energy spectra for even-even ^{210}Pb isotope in comparison with the experimental data. The squares and diamonds represent experimental positive and negative parity states, respectively. The asterisks and crosses represent theoretical positive and negative parity states, respectively, for the *khpba*, *khpbu*, *khp* and *khpe* effective interactions. The experimental data are taken from [14].

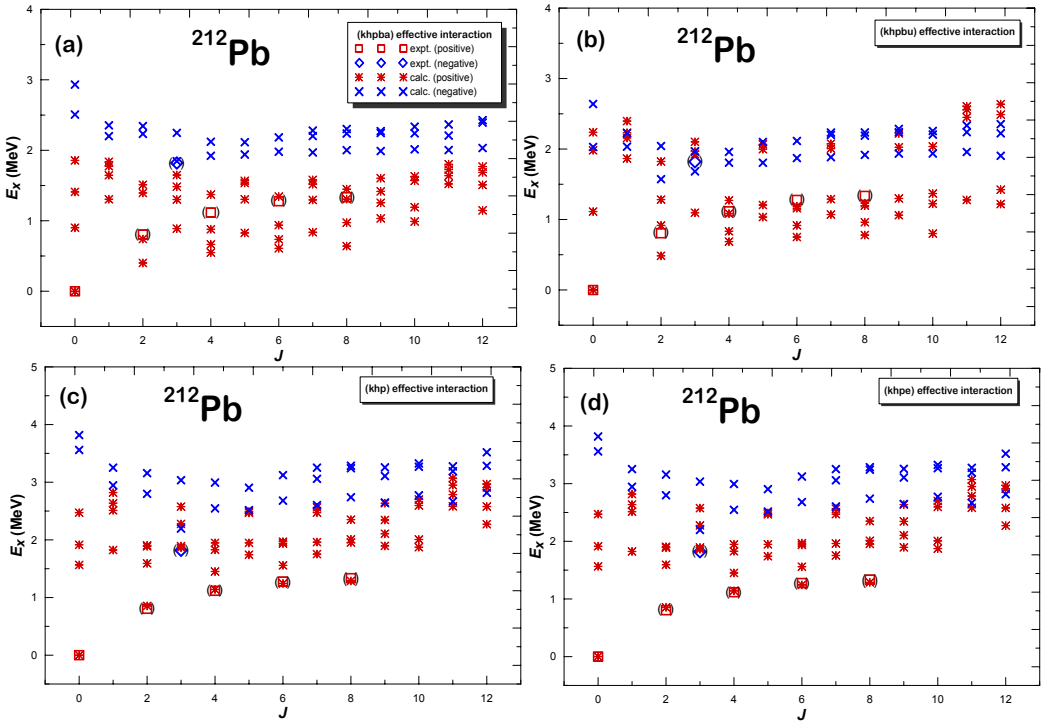

Fig. 2. The theoretical energy spectra for even-even ^{212}Pb isotope in comparison with the experimental data. The squares and diamonds represent experimental positive and negative parity states, respectively. The asterisks and crosses represent theoretical positive and negative parity states, respectively, for the *khpba*, *khpbu*, *khp* and *khpe* effective interactions. The experimental data are taken from [14].

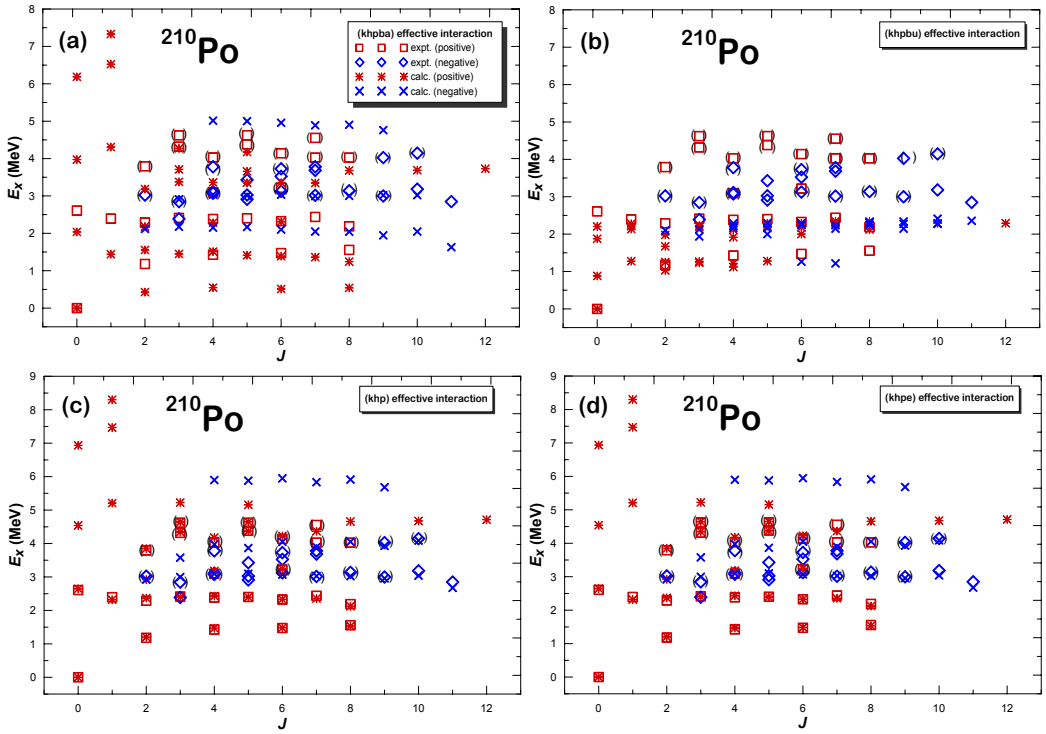

Fig. 3. The theoretical energy spectra for even-even ^{210}Po isotope in comparison with the experimental data. The squares and diamonds represent experimental positive and negative parity states, respectively. The asterisks and crosses represent theoretical positive and negative parity states, respectively, for the *khpba*, *khpbu*, *khp* and *khpe* effective interactions. The experimental data are taken from [14].

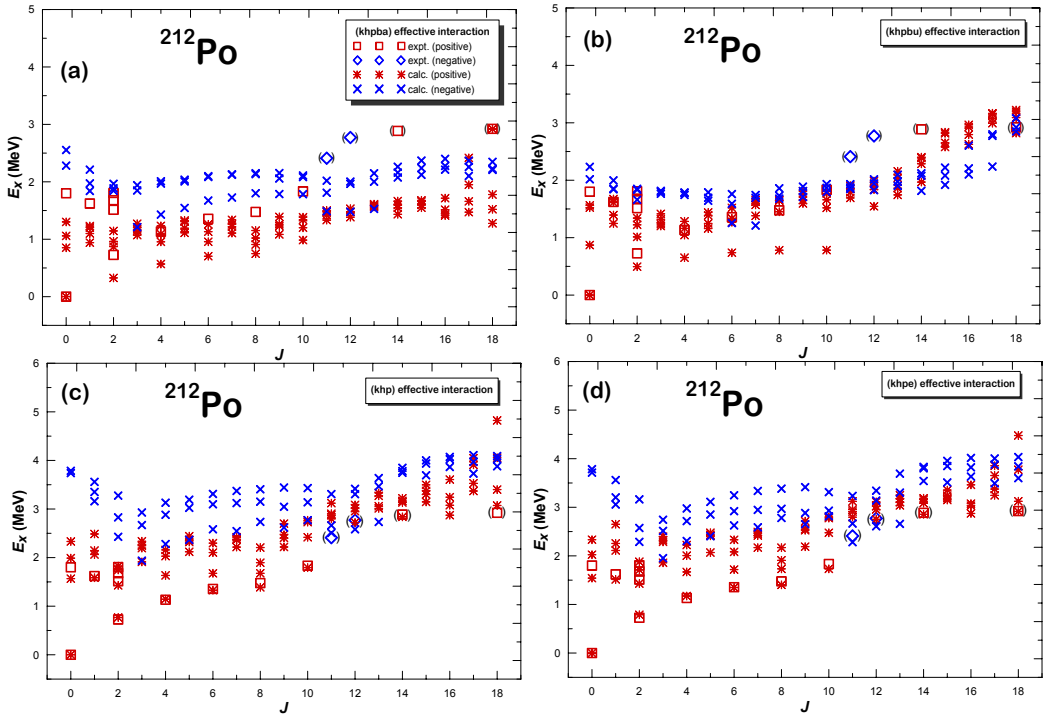

Fig. 4. The theoretical energy spectra for even-even ^{212}Po isotope in comparison with the experimental data. The squares and diamonds represent experimental positive and negative parity states, respectively. The asterisks and crosses represent theoretical positive and negative parity states, respectively, for the *khpba*, *khpbu*, *khp* and *khpe* effective interactions. The experimental data are taken from [14].

The comparison of the calculation from the present work with the experimental energy levels for the positive and negative parity states for ^{212}Pb is presented in the Fig. 2 by using *khpba*, *khpbu*, *khp* and *khpe* effective interactions, respectively. The best agreement between our theoretical calculations and experimental data was achieved by employing the *khp* and *khpe* effective interactions. Many unconfirmed experimental energy levels for this nucleus have been confirmed, Also we can see that our calculations of *khp* and *khpe* effective interactions have the same results.

Here $^{210-212}$Po isotopes are discussed. Fig. 3 and Fig. 4 respectively, show the theoretical energy spectra for even-even Po isotopes in comparison with the experimental data by using the same effective interactions. The four effective interactions give good results in comparison with the experimental values. There are many unconfirmed experimental energy levels that were confirmed by our calculations with different set of effective interactions. The best agreement between theory and experiment was achieved using *khp* and *khpe* effective interactions.

B. Reduced Transition Probabilities

The electromagnetic transition probability $B(E2; 0_1^+ \rightarrow 2_1^+)$ values calculated using $jj67pn$ model space for each effective interaction were adopted in the present work in units of e^2fm^4. These predicted theoretical values are compared with the experimental data for $^{210-212}$Pb and $^{210-212}$Po isotopes and the corresponding experimental data are displayed in Table 1. The core polarization effects were considered in the present work by means of effective proton and neutron charges. The average effective charge for the proton and neutron is taken to be constant for each isotope as tabulated in Table 1.

$J_i^\pi \rightarrow J_f^\pi$	Isotope	Exp.	*khpba*	*khpbu*	*khp*	*khpe*	e_π^{eff}	e_ν^{eff}
$0_1^+ \rightarrow 2_1^+$	^{210}Pb	510(15)	517.6	492.1	497.1	497.1	0.89	0.89
$0_1^+ \rightarrow 2_1^+$	^{212}Pb		266.7	282.7	317.5	317.5	0.5	0.5
$0_1^+ \rightarrow 2_1^+$	^{210}Po	200	456.4	751	460	460	0.001	0.1
$0_1^+ \rightarrow 2_1^+$	^{212}Po		1552	293.3	1233	1195	0.5	0.5

Table 1. The comparison of the calculated $B(E2; 0_1^+ \rightarrow 2_1^+)$ [e^2fm^4] of all nuclei under the study for each effective interaction with the experimental data taken from [15].

C. Binding energy

The ground state binding energies and spin parities are important observables, particularly for testing the effective interactions and for the application in the nucleosynthesis calculation. The comparison of the calculated binding energy for all nuclei under the study using *khpba*, *khpbu*, *khp* and *khpe* with the corresponding experimental binding energy [16] are tabulated in Table 2. All the effective interactions used in the present work are able to predict the binding energy precisely for all isotopes under the study.

Isotope	Exp.	*khpba*	*khpbu*	*khp*	*khpe*
^{210}Pb	1645.55	1644.97	1638.01	1645.54	1645.52
^{212}Pb	1654.51	1653.65	1639.01	1654.55	1654.46
^{210}Po	1645.21	1644.37	1636.61	1645.24	1645.19
^{212}Po	1655.77	1654.32	1637.91	1655.50	1655.64

Table 2. The comparison of the calculated binding energy for each effective interactions in MeV with the experimental data taken from Ref. [16].

V. CONCLUSIONS

In the present study, we emphasize that there is an overall reasonably good agreement of the calculated binding energies, excitation and $B(E2)$ values with the experimental data for $^{210-212}$Pb and $^{210-212}$Po, using the shell model code NushellX@MSU by adopting the model space $jj76pn$ with *khpba*, *khpbu*, *khp* and *khpe* residual effective interactions. It is found that *khp* and *khpe* effective interactions is best fitted the experimental data for excitation energy and they have identical results for all isotopes. We have found a good agreement with the experimental data for the excitation spectra and reduced electric transition probabilities $B(E2)$ for all the nuclei under study. The experimental binding energies are very well reproduced by the current large-scale shell model calculations.

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РОЗРАХУНКИ ОБОЛОНКОВОЇ МОДЕЛІ ДЕЯКИХ ЯДЕР ПОБЛИЗУ ОБЛАСТІ ^{208}Pb

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У межах оболонкової моделі розраховано енергетичні рівні, ймовірності переходів $B(E2; 0_1^+ \to 2_1^+)$ та енергії зв'язку для $^{210-212}$Pb, $^{210-212}$Po в області нейтронного дефіциту $\pi(hfpi)\nu(igdsj)$ валентного простору над ядром ^{208}Pb із використанням чотирьох ефективних взаємодій *khpba*, *khpbu*, *khp* та *khpe*. Розраховані величини порівняно з нещодавно отриманими експериментальними даними. Досягнуто дуже доброго узгодження для всіх ядер.