L.I. Anatychuk

THE ACCURACY OF THERMOEMF MEASUREMENT BY THE PROBE METHOD

L.I. Anatychuk, R.M. Mochernyuk
(Institute of Thermoelectricity, 1, Nauky Str.,
Chernivtsi, 58029, Ukraine)

R.M. Mochernyuk

- *Computer model of measuring thermoelectric coefficient using the hot probe method has been created that permits to investigate measurement errors caused by eddy currents, sample thermal conductivity and the tip end diameter.*

Introduction

General characterization of the problem. The accuracy of measuring parameters of physical quantities is an important problem of modern science. For a rational use of thermoelectric materials (TEM) and creation of precision thermoelectric instruments the quality of TEM should be controlled. For this purpose the basic material parameters, and, in particular, thermoelectric coefficient, are measured.

Analysis of the literature. To determine the thermoelectric coefficient, steady-state methods of measuring regular-shaped samples are mostly used, specifically the hot probe method [1]. The temperature difference, as a rule, is 10 to 15 K. The thermoelectric coefficient is found from the relation:

$$\alpha = \frac{E}{\Delta T}, \tag{1}$$

E is electromotive force (EMF) that is generated in the sample under the influence of temperature difference ΔT.

In conformity with the hot probe method, a heated probe is arranged on the sample surface for a local heat-up of the sample. This leads to generation of thermoEMF which is measured relative to another probe located beyond the heated area [1].

Ref. [2] describes a setup which enables thermoEMF to be measured in different sample areas. Using this method, one can get information on the presence of impurities in the sample. The method is based on the fact that a probe, heated to certain temperature, with a thermocouple embedded into it, moves along the surface of the sample. Owing to sample area heating, a thermoEMF is generated in it which is recorded relative to another, immobile probe.

In Ref. [3] investigation was performed to measure α of films with the use of thermal probe. The temperature distribution was calculated, as well as dependence of thermoEMF on the probe diameter was studied.

In the general case the accuracy of α measurement is 2 to 5% [1], and the repeatability of results is about 10% [2].

However, up to now no investigation has been performed on the influence of sample thermal conductivity, eddy currents, contact size and probe geometry on the accuracy of α measurement. Therefore, the purpose of this work is to study the effect of these factors on the accuracy of thermoelectric coefficient measurement by the probe method.

Physical model

To study the errors in measuring α by the hot probe method, it is necessary to find the temperature and potential distribution in the probe and sample. For this purpose a physical model should be built.

A physical model consists of arbitrary-shaped thermoelectric material 1 and cone-shaped probe 2. A sample of thermoelectric material is located on a thermostat which maintains the sample lower surface at temperature T_1. The probe upper surface is maintained at temperature T_2. The probe and sample lateral surfaces are thermally and electrically insulated (Fig. 1).

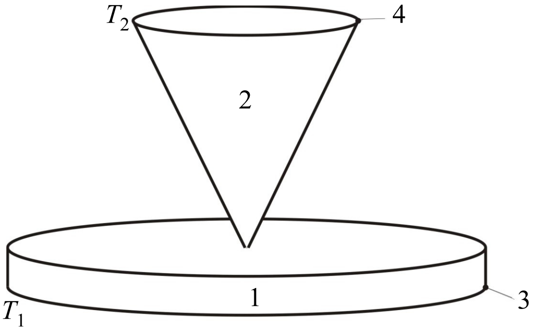

Fig. 1. A physical model of thermoelectric coefficient measurement.

Temperature drop created in the sample due to the presence of temperature difference between the upper and lower sample surfaces, is measured between two points 3 (potential $U = 0$) and 4 (Fig. 1).

Mathematical and computer model

Simulation of thermal fluxes in the probe will be done in the Femlab program. For this purpose, a mathematical model was preliminarily constructed on the basis of which the distributions of temperature, potential and currents in the probe and sample were described.

Equations of physical field distribution in the thermoelement were obtained from the laws of conservation of energy

$$div\vec{W} = 0, \tag{2}$$

and electrical charge

$$div\vec{j} = 0, \tag{3}$$

where

$$\vec{W} = \vec{q} + U\vec{j} \tag{4}$$

$$\vec{q} = \kappa\vec{\nabla}T + \alpha T\vec{j} \tag{5}$$

$$\vec{j} = -\sigma\vec{\nabla}U - \sigma\alpha\vec{\nabla}T \tag{6}$$

$\vec{W}$ is energy flux density, $\vec{j}$ is electric current density, U is electric potential, T is temperature, α, σ, κ are thermoelectric coefficient, electric conductivity and thermal conductivity. As long as temperature difference in the sample is not large (10 K), the temperature dependence of kinetic coefficients was ignored.

Taking into account expressions (5), (6) in (4) gives:

$$\vec{W} = -(\kappa + \alpha^2\sigma T + \alpha U\sigma)\vec{\nabla}T - (\alpha\sigma T + U\sigma)\vec{\nabla}U. \tag{7}$$

Then the laws of conservation (2), (3) will take on the form:

$$-\vec{\nabla}\left[(\kappa + \alpha^2\sigma T + \alpha U\sigma)\vec{\nabla}T\right] - \vec{\nabla}\left[(\alpha\sigma T + U\sigma)\vec{\nabla}U\right] = 0, \tag{8}$$

$$-\vec{\nabla}(\sigma\alpha\vec{\nabla}T) - \vec{\nabla}(\sigma\vec{\nabla}U) = 0. \tag{9}$$

Equations (8) and (9) are second order differential equations in partial derivatives for the desired functions *U* and *T*.

The next simulation step was reduction of differential equations (7) and (8) to one of standard forms (10) of the Femlab program.

$$\nabla(-C\nabla M) = 0, \tag{10}$$

where
$$M = \begin{bmatrix} U \\ T \end{bmatrix},\ C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{11}$$

$$\begin{aligned} &\nabla(-C_{11}\nabla U) + \nabla(-C_{12}\nabla T) = 0 \\ &\nabla(-C_{21}\nabla U) + \nabla(-C_{22}\nabla T) = 0, \end{aligned} \tag{12}$$

where
$$\begin{aligned} &C_{11} = \alpha\sigma T + U\sigma \\ &C_{12} = \kappa + \alpha^2\sigma T + \alpha\sigma U \\ &C_{21} = \sigma\alpha \\ &C_{22} = \sigma \end{aligned} \tag{13}$$

(13) are coefficients for the Femlab computer program.

Computer simulation results

Computer simulation was done using the Femlab program. The distributions of temperature (Fig. 2) and potential fields (Fig. 3) created in sample under study and thermal probe were obtained.

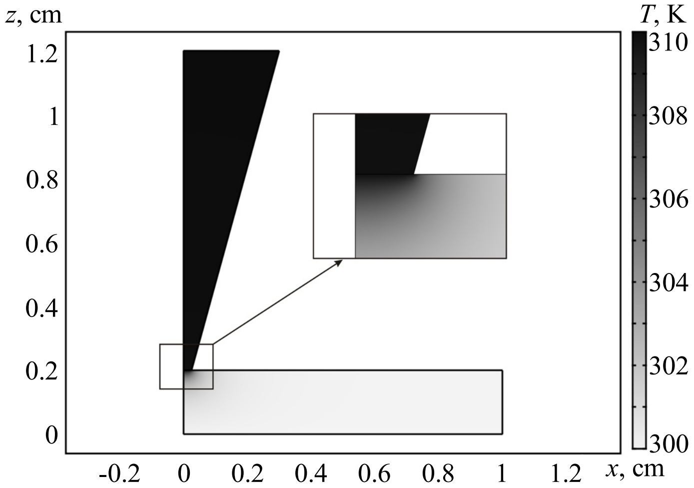

Fig. 2. Temperature distribution in the probe and sample.

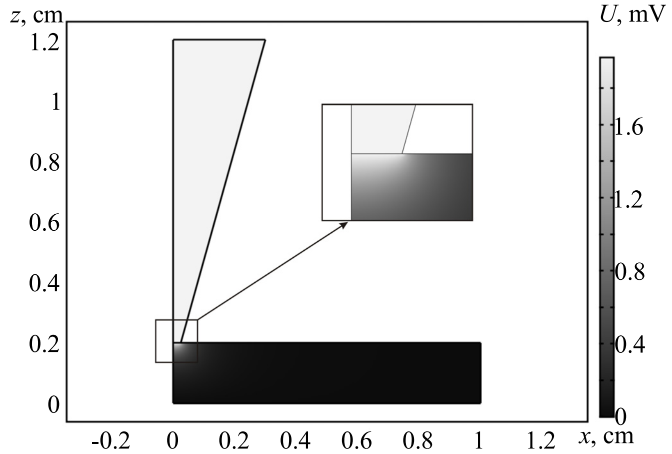

Fig. 3. Potential distribution in the probe and sample.

Effect of sample thermal conductivity

The next simulation step was to study the effect of sample thermal conductivity on the accuracy of measuring thermoelectric coefficient α. A broad range of thermal conductivity from 0.001 W·cm^{-1}K^{-1} to 4 W·cm^{-1}K^{-1} for different probe diameters was considered. The distributions of temperature field in the probe are given in Figs. 4 – 6, the effect of thermal conductivity – in Table 1.

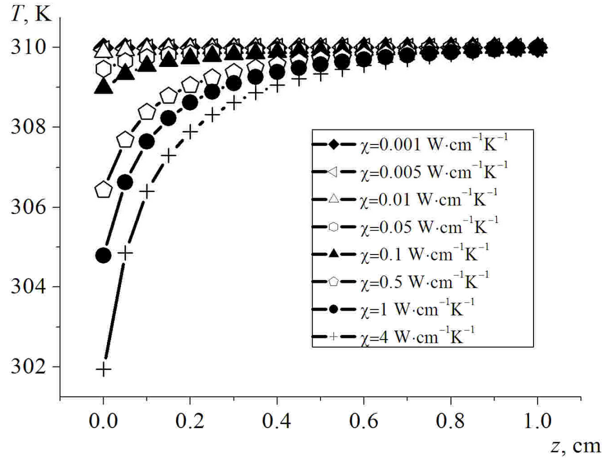

Fig. 4. Temperature distribution along the probe axis for different values of sample thermal conductivity. Probe tip diameter 0.5 mm.

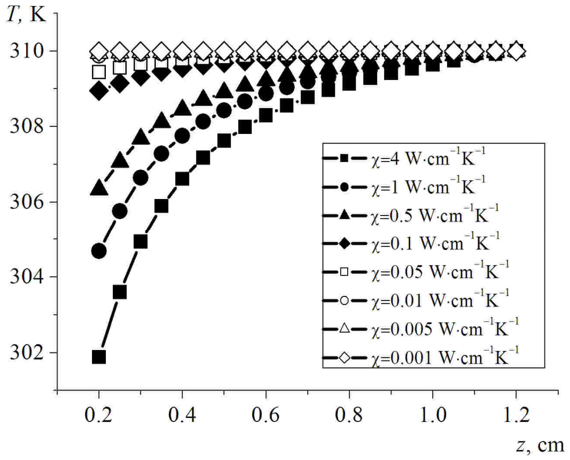

Fig. 5. Temperature distribution along the probe axis at different values of sample thermal conductivity. Probe tip diameter 1 mm.

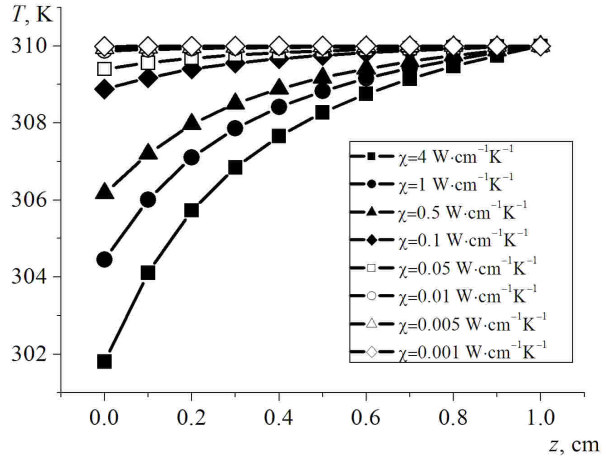

Fig. 6. Temperature distribution along probe axis at different values of sample thermal conductivity. Probe tip diameter 1.5 mm.

Table 1

Dependence of potential and temperature difference at "probe-sample" boundary on thermal conductivity of sample under study
(α = 200 μV/K, probe diameter 0.5 mm)

№	χ, W·cm^{-1}K^{-1}	Temperature at "probe-sample" boundary", K	E, μV	Error, %
1	4	301.946	374	81.3
2	1	304.788	911	54.5
3	0.5	306.436	1251	37.5
4	0.1	308.386	1786	12.7
5	0.05	309.465	1887	5.7
6	0.01	309.888	1976	1.2
7	0.005	309.943	1988	0.6
8	0.001	309.988	1998	0.1

Analysis of the results shows that measuring α by the probe method in the samples with a high thermal conductivity is involved with considerable heat dissipation. Due to this, the errors of measuring thermoelectric coefficient can reach 80%. This fact should be taken into account when measuring α in the samples with a high thermal conductivity.

Additional studies of the effect of tip end diameter on the error value have shown that a change in diameter within 0.1 – 1 mm does not result in the reduction of the error in measuring thermoelectric coefficient by thermal probe.

Effect of eddy currents

As a result of simulation it was established that eddy thermoelectric currents are generated in the area of "probe-sample" contact (Fig. 7).

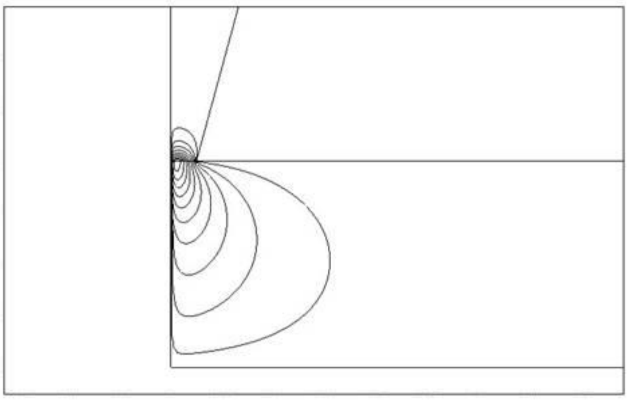

Fig. 7. Eddy thermoelectric currents generated in the area of probe-sample contact.

With this in mind, the effect of such currents on the accuracy of α measurement was studied. For this purpose we compared the thermoEMF generated in the sample with account of eddy currents to the thermoEMF calculated by formula (1) at a known temperature of probe tip end. Studies were performed for different material parameters. The results obtained are listed in Table 2.

Table 2

Effect of eddy currents on the accuracy of α *measurement by the hot probe method*

№	α, μV/K	σ, Ω^{-1}cm^{-1}	χ, W·cm^{-1}K^{-1}	$E_{eddy\ cur}$, μV	E, μV	ΔE, μV	Error, %
1	200	1000	0.014	1997.4	1997.56	0.16	0.01
2	100	10000	0.2	806.5	816.70	10.20	1.25
3	50	100000	0.4	339.4	345.95	6.55	1.89

As can be seen from the Table, the error caused by the effect of eddy currents grows when measuring α of samples with a high electrical conductivity.

Effect of probe material and base angle on temperature field distribution

The effect of probe material and base angle on heat dissipation when measuring thermoelectric coefficient by the hot probe method was calculated. Investigations were carried out for two probe materials: copper and tungsten. The measurement results are given in Fig. 8 and Fig. 9.

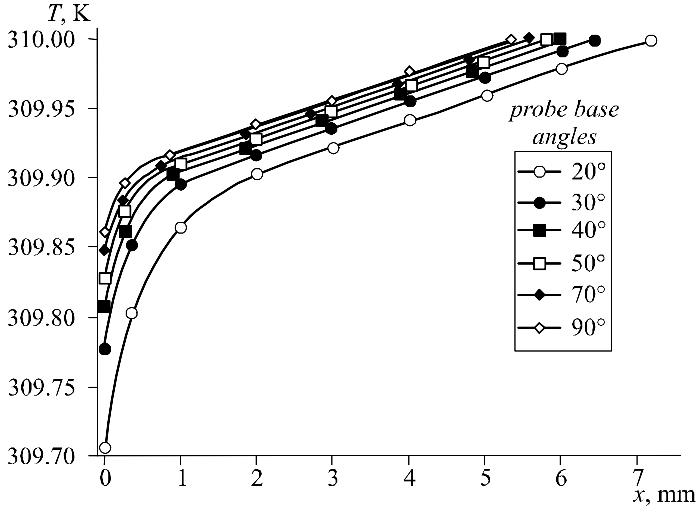

Fig. 8. Temperature distribution along copper probe with the tip end diameter 0.2 mm for different probe base angles.

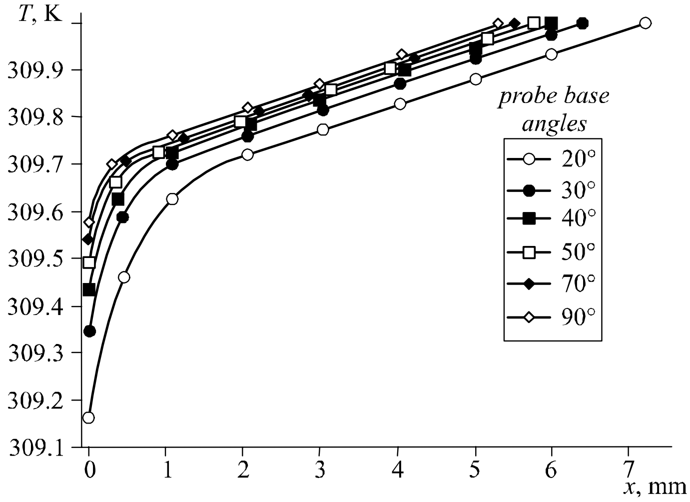

Fig. 9. Temperature distribution along tungsten probe with the tip end diameter 0.2 mm for different probe base angles.

The above plots show temperature distribution in a probe as a function of its base angle for tungsten and copper with the tip end diameter 0.2 mm. It is shown that the losses in tungsten probe are factor of three greater compared to copper probe. Hence, it is more reasonable to use copper as a probe material.

It has been established that with the angle of 40° and more the curves are little different, which enables determination of the optimal angle at the top of the probe.

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Conclusions

1. Computer model for measuring thermoelectric coefficient was created which allowed finding the distributions of temperature, potential and currents in the probe and thermoelectric material sample.
2. It was established that a change in diameter of measuring installation probe within 0.1 – 1 mm does not affect the accuracy of α measurement.
3. It is shown that when measuring α in the samples with a high thermal conductivity there is considerable distortion of temperature field in the probe, owing to which the errors in measuring thermoelectric coefficient can reach 80%.
4. It was established that when measuring α in the samples with a high electrical conductivity the error caused by the effect of eddy currents increases to 2%.
5. The effect of probe material on the distribution of temperature field in the sample has been studied. It has been established that the probe must be made of material with a high thermal conductivity (such as copper).

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