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**ANISOTROPIC SCATTERING THEORY  
AND RELEVANT PROBLEMS OF THE  
KINETICS OF ELECTRON PROCESSES  
IN MANY-VALLEY SEMICONDUCTORS**

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*This paper deals with the physics of anisotropic scattering theory. Researchers' attention is drawn to the advantageous features of this theory which made it suitable for practical application in the region of intermediate magnetic fields ( $\mu H/c \approx 1$ ) where the use of other theories proved to be inefficient.*

**Key words:** theory of anisotropic scattering, many-valley semiconductors, kinetic phenomena, electric and magnetic fields.

## **Introduction**

Without exaggeration one may state that a real prerequisite to progress in some divisions of the kinetics of electron processes in semiconductors in our country (and abroad) over the last three to four decades has been development of anisotropic scattering theory (AST) efficiently pursued by its author, professor A.G. Samoilovich and his school of thought in Chernivtsi since 1960 and actually to the last day of his creatively rich and far from being easy life.\*

Some AST divisions appeared in special editions [1-4] dealing with dedicated problems. Therefore, in this review it would not be inappropriate to set forth, possibly briefly and consecutively, the main postulates of AST, paying special attention to specific features of this theory assuring the opportunity of its application in such regions of magnetic field intensity (satisfying, in particular, criterion  $\mu H/c \approx 1$ ) in which a description of electron gas kinetics for other theories is virtually impossible.

The most detailed and complete (so to speak, first-hand) AST outline can be found in the compendium of lectures by A.G. Samoilovich [5] prepared for publication by Anatoliy Grygorovich's pupils – L.N. Vikhor, O.A. Okhrem and A.O. Snarskii.

Note that the theoretical special course of AST (published as a compendium of lectures) is certainly self-sufficient and complete. However, being restricted by the scope of university program, it surely did not embody the results of experimental research pursued at the Lashkaryov Institute of Semiconductor Physics, NAS Ukraine with the active participation of theoreticians from Chernivtsi State University. It is exactly this gap in the interpretation of AST interaction with the experiment that the authors of this review would like to fill to some extent, referring, of course, only to scientific papers and monographs published with A.G. Samoilovich and the research officers of the department he was in charge of.

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\* At the age of 5 A.G. Samoilovich lost the opportunity not only to move without exterior help, but even to eat.

## **1. Physics of anisotropic scattering theory and its contribution to research on electron gas kinetics in many-valley semiconductors (*n*-type *Ge* and *Si*)**

Research on the electric and galvanomagnetic properties of solids yields important information on the energy spectrum structure of carriers, the character of their scattering and other values describing transport effects. Knowledge of precise quantitative theory of these effects allows determination of a variety of parameters and enables a wide practical use of the electric and galvanomagnetic properties of solids in science and technology.

When considering electron motion in crystal lattice, it is necessary to take into account correctly the effective mass anisotropy and the anisotropy of electron scattering both on crystal lattice vibrations and on the ionized impurities.

The first successful step in this direction was taken in the work by Herring and Vogt [6]. In this work, the non-equilibrium addition to distribution function  $n_{\vec{k}}$  was taken in the form of a linear function of quasi-pulse  $\vec{k}$

$$n_{\vec{k}}' = \vec{A}(\varepsilon) \vec{k}, \quad (1)$$

which is equivalent to taking into account only the first harmonic in the expansion of  $n_{\vec{k}}$  with respect to spherical functions, that is, use was made of a linear approximation. Being restricted to a linear approximation, it is impossible to evaluate the errors that arise. In the case of strongly anisotropic scattering (scattering on ionized impurities) such an approach is unacceptable altogether.

As will be shown below, within AST the solution of the kinetic equation has a reliable mathematical background. This method is equivalent to variation one. We will restrict ourselves to consideration of elastic scattering of electrons that have isoenergetic surfaces in the form of ellipsoids of revolution. The solution of the kinetic equation is sought as a series of expansion with respect to spherical functions and reduced to an infinite system of linear algebraic equations with respect to coefficients of this expansion.

To calculate the fluxes, it is necessary to determine the first harmonic coefficients in the system. When scattering probability in the axes of mass ellipsoid does not depend on the azimuth of vector  $\vec{q}$  equal to a change in quasi-pulse at scattering, and the magnetic field is equal to zero, the first harmonic coefficients can be represented as a series where the first term gives a linear Herring-Vogt approximation. In the case of elongated isoenergetic ellipsoids of revolution ( $m_{\parallel} > m_{\perp}$ ) at scattering on ionized impurities these series rapidly converge for any (considerable)  $m_{\parallel} / m_{\perp}$  ratios, so it is enough to preserve two terms of the series. In the case of *Si* and *Ge* (as shown by estimates) at scattering on acoustic phonons it is enough to be restricted to the first term of expansion, because the second term is two orders of magnitude smaller than the first one. According to estimates, the last statement remains valid with a mixed scattering as well.

## **2. Solution of the kinetic equation with anisotropic electron scattering**

### **2.1. Reduction of the kinetic equation to a system of algebraic equations (according to data reported in works [1 – 10])**

Under anisotropic scattering conditions, the probability of  $W_{\vec{k}\vec{k}'}$  transition from  $\vec{k}$  state to  $\vec{k}'$  state depends on  $\vec{k}$  and  $\vec{k}'$  directions. Such scattering can be caused either by the anisotropy of electron energy spectrum or by the anisotropic character of scatterer. We will consider only elastic

scattering. Let us assume that the energy spectrum of electrons in the external electric field in the absence of a magnetic field ( $H = 0$ ) is given by:

$$\varepsilon = \sum_{i=1}^3 \frac{\hbar^2 k_i^2}{2 m_i}. \quad (2)$$

Then with the weak electric field and temperature gradient ( $\text{grad } T$ ), in conformity with [2, 3, 7 – 10], the kinetic equation can be written as

$$\hat{D} n_k^{(0)} + \hat{R} n_k' = 0, \quad (3)$$

where

$$\hat{R} n_k' = \sum_{k'} W_{kk'} (n_{k'}' - n_k'), \quad (4)$$

where  $n_k^{(0)}$  is equilibrium distribution function,  $n_k'$  is nonequilibrium addition to distribution function,  $\hat{R}$  is collision operator,  $\hat{D} n_k^{(0)}$  is a free term of the kinetic equation

$$\hat{D} n_k^{(0)} = \frac{1}{\hbar} \frac{\partial n_k^{(0)}}{\partial \varepsilon} \sum_i \left( \frac{\partial \mu}{\partial x_i} + \frac{\varepsilon - \mu}{T} \frac{\partial T}{\partial x_i} - e E_i \right) \frac{\partial \varepsilon}{\partial k_i}, \quad (5)$$

$\mu$  is chemical potential,  $E$  is external electric field.

The kinetic equations are solved for each isoenergetic ellipsoid taken individually (Umklapp processes between minima are disregarded).

Let us introduce “deformed” coordinates in quasi-pulse space

$$\xi_i = \frac{\hbar}{\sqrt{2 m_i \varepsilon}} k_i. \quad (6)$$

Then expression (2) will be of the form

$$\sum_i \xi_i^2 = 1. \quad (7)$$

Let us introduce a spherical coordinate system with a polar axis oriented along 0Z axis of constant energy ellipsoid

$$\xi_1 = \sin \vartheta_0 \cos \phi_0, \quad \xi_2 = \sin \vartheta_0 \sin \phi_0, \quad \xi_3 = \cos \vartheta_0. \quad (8)$$

In this case, denoting through

$$K_i = -\frac{\partial \mu}{\partial x_i} - \frac{\varepsilon - \mu}{T} \frac{\partial T}{\partial x_i} + e E_i, \quad (9)$$

one can write a free term of the kinetic equation as

$$\hat{D} n_k^{(0)} = \sum_m D_m Y_{1m}(\vartheta_0, \phi_0), \quad (10)$$

where  $Y_{lm}(\vartheta_0, \phi_0)$  is a spherical function normalized to unity.

$$\left. \begin{aligned} D_1 &= \sqrt{\frac{4\pi\varepsilon}{3}} \frac{\partial n_k^{(0)}}{\partial \varepsilon} \left( \frac{K_1}{\sqrt{m_1}} - \frac{i}{\sqrt{m_2}} K_2 \right) \\ D_0 &= \sqrt{\frac{8\pi E}{3 m_3}} \frac{\partial n_k^{(0)}}{\partial \varepsilon} K_3, \quad D_{-1} = -D_1^* \end{aligned} \right\}. \quad (11)$$

With regard to equation (10), it is natural to seek the solution of the kinetic equation as expansion with respect to spherical functions

$$n'_k = \sum_{km} X_{km}(\varepsilon) Y_{km}(\vartheta_0, \phi_0). \quad (12)$$

As long as under elastic scattering conditions a collision operator affects only the angular part of function  $n'_k$ , we get

$$\hat{R} n'_k = - \sum_{jkm p} X_{km}(\varepsilon) B_{jk}(pm) Y_{jp}(\vartheta_0, \phi_0), \quad (13)$$

where

$$- \sum_{jp} B_{jp}(pm) Y_{jp}(\vartheta_0, \phi_0) = \hat{R} Y_{km}(\vartheta_0, \phi_0). \quad (14)$$

Substituting (10) and (13) into (3), we obtain a system of equations in unknowns  $X_{km}(\varepsilon)$

$$\sum_{km} B_{jk}(pm) X_{km} = D_p \delta_{j1}. \quad (15)$$

Thus, the problem of finding  $n'_k$  from equation (3) came down to determination of  $X_{km}$  from the infinite system of linear algebraic equations (15). Current components are proportional to first-order spherical functions. Hence, for current calculation it is enough to find  $X_{1m}$  from the reduced system. Part of distribution function  $n'_k$  whereby current is determined is of the form

$$n'_k = \sum_m X_{1m}(\varepsilon) Y_{1m}(\vartheta_0, \phi_0). \quad (16)$$

Based on the principle of microscopic reversibility it can be shown that expansion (12) includes only unpaired harmonics.

Note that a system of equations (15) can be also obtained using the principle of entropy growth rate maximum.

As a result of collision operator action on the angular part of function  $n'_k$  and performing the necessary calculations [8, 9], the expression for coefficients  $B_{jk}(pm)$  was obtained as below:

$$B_{jk}(pm) = \frac{4\sqrt{2m_1 m_2 m_3} \varepsilon}{(2\pi\hbar)^3} i^{m-p} \sum_{s_{paired}} \sqrt{\frac{(2j+1)(2k+1)(j-s)!(k-s)!}{(j+s)!(k+s)!}} \times \int d\Omega \int_0^{\pi/2} d\theta \sin\theta \cos\theta W(\theta\vartheta\phi) P_j^s(\cos\theta) P_k^s(\cos\theta) P_{sp}^j(\cos\vartheta) P_{sm}^{*k}(\cos\vartheta) e^{i(m-p)\phi}. \quad (17)$$

Summation in (17) over  $S_{pair}$  is done from  $-k+1$  to  $k-1$ , when  $k < j$ ; and from  $-j+1$  to  $j-1$ , when  $j < k$ ;  $P_j^s$  is an associated Legendre function;  $P_{sp}^j$  is an orbital part of generalized spherical function (Wigner function) determined in [11].  $d\Omega = \sin\vartheta d\vartheta d\phi$ .  $\vartheta_0$  and  $\phi_0$  determine a direction  $\vec{\xi}$  of electron quasi-pulse in "deformed" coordinate system (6) up to scattering,  $\vartheta$  and  $\phi$  determine a direction of vector  $\vec{q} = \vec{\xi} - \vec{\xi}'$  equal to a change in quasi-pulse on scattering,  $\chi = \pi - 2\theta$  is scattering angle, that is, the angle between  $\vec{\xi}$  and  $\vec{\xi}'$  (Fig. 1).

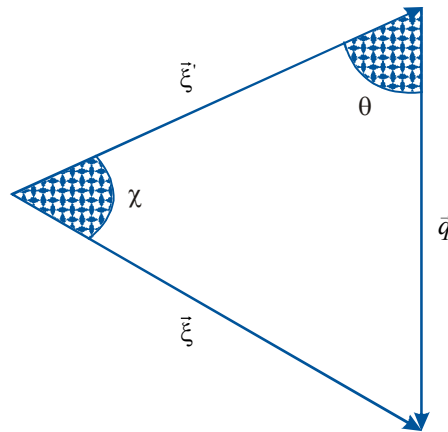


Fig. 1. Schematic of scattering angle  $\chi$ .

When probability of scattering in the axis of mass ellipsoid does not depend on vector azimuth  $\bar{q}$  (on angle  $\varphi$ ), then matrix  $\|B_{jk}(pm)\|$  is diagonalized in  $p$  and  $m$ :

$$B_{jk}(pm) = B_{jk}(p) \delta_{mp}. \quad (18)$$

In so doing, system (15) is disintegrated into independent systems with different  $p$ :

$$\sum_k B_{jk}(p) X_{kp} = D_p \delta_{j1}. \quad (19)$$

Accordingly,  $W(\theta, \vartheta, \varphi)$  in (17) will not depend on  $\varphi$  and we will get:

$$B_{jk}(pm) = B_{jk}(m) \delta_{mp} = \frac{\sqrt{2m_1 m_2 m_3} \varepsilon}{\pi^2 \hbar^3} \sum_{s \text{ paired}} \sqrt{\frac{(2j+1)(2k+1)(j-s)!(k-s)!}{(j+s)!(k+s)!}} \times \int_0^\pi d\vartheta \sin \vartheta \int_0^{\pi/2} d\theta \sin \theta \cos \theta W(\theta, \vartheta) P_j^s(\cos \theta) P_k^s(\cos \theta) P_{sm}^j(\cos \vartheta) P_{sm}^{*k}(\cos \vartheta) \times \delta_{mp}. \quad (20)$$

As is evident from (17),

$$B_{jk}(pm) = B_{kj}^*(mp). \quad (21)$$

Using an explicit expression for functions  $P_{sm}^j(\cos \vartheta)$  [11], it is readily apparent from (20) that coefficients  $B_{jk}(m)$  (as real ones) meet the condition

$$B_{jk}(m) = B_{kj}(m), \quad (22)$$

and, with regard to (20), one can also write

$$B_{jk}(m) = B_{jk}(-m). \quad (23)$$

## 2.2. Iteration method for determination of $X_{1m}$

A method for calculation of  $X_{1m}$  that will be set forth below can be efficiently used to find  $X_{1m}$  from system (19).

Let  $B(XX)$  be a quadratic form of variables  $X_{lm}$

$$B(XX) = \sum_{jk} B_{jk} X_j X_k \quad (24)$$

(index  $m$  is omitted as yet).

Let us introduce the notation

$$A_i(X) = \sum_k B_{ik} X_k, \quad (25)$$

$$Z_1 = A_1(X), \quad Z_k = \begin{vmatrix} B_{11} & B_{13} & \dots & B_{1,2k-3} & A_1(X) \\ B_{31} & B_{33} & \dots & B_{3,2k-3} & A_3(X) \\ \dots & \dots & \dots & \dots & \dots \\ B_{2k-1,1} & B_{2k-1,3} & \dots & B_{2k-1,2k-3} & A_{2k-1}(X) \end{vmatrix}. \quad (26)$$

In conformity with the Jacobi formula

$$B(X X) = \sum_l \frac{Z_l^2}{\Delta_l \Delta_{l-1}}, \quad (27)$$

where  $\Delta_l$  is determinant of the principal " $l$ "-th minor matrix  $\|B_{jk}\|$ . However, in our case variables  $X_{1m}$  meet system (18), therefore

$$A_1(X) = D, \quad A_{i>1} = 0, \quad B(X X) = D X_1. \quad (28)$$

Thus, for  $c$  we obtain from (27) the following series

$$X_{1m} = D_m \sum_l \frac{\bar{Z}_{l-1}^2}{\Delta_l(m) \Delta_{l-1}(m)}, \quad (29)$$

where

$$\bar{Z}_{l-1}^{(m)} = \begin{vmatrix} B_{31}(m) & B_{33}(m) & \dots & B_{3,2l-3}(m) \\ \dots & \dots & \dots & \dots \\ B_{2l-1,1}(m) & B_{2l-1,3}(m) & \dots & B_{2l-1,2l-3}(m) \end{vmatrix}; \quad \bar{Z}_0 = 1. \quad (30)$$

The sum of the first  $n$  terms of series for  $X_{1m}$  is a solution of system (19), if we set  $X_{1m} = 0$  at  $l > n$ . In this sense, said method for calculation of  $X_{1m}$  can be referred to as iteration. Note that series (29) when considering scattering on phonons and impurity ions is rapidly converging. For instance, at scattering on ionized impurities in the case of strongest anisotropy (when scattering is mainly forward and  $m_1/m_3 \ll 1$ ) the ratio of the first four terms in this series is as follows: 1: 0.72: 0.015: 0. Here, the nondiagonal matrix elements  $\|B_{jk}(m)\|$  are not small.

### 2.3. Scattering on ionized impurities

Let us consider a dielectric constant  $\zeta$  isotropic and assume impurity potential as

$$V = \frac{e_0^2}{\zeta r} e^{-r/a}, \quad (31)$$

where  $e_0$  is electron charge,  $a$  is screening radius which in the degenerate case is calculated by the formula

$$a^{-2} = \frac{4\pi e_0^2 n'}{\zeta k T}, \quad (32)$$

where  $n' = n + (n + N_A) \left( 1 - \frac{n + N_A}{N_D} \right)$ . Here  $n$  is electron concentration in conduction band,  $N_D$  and  $N_A$

are donor and acceptor concentrations. The  $n'$  value, according to Brooks [12], takes into account the effect of compensating impurity on screening.

Under these conditions, in the Born approximation, for matrix transition element we get

$$V_{\vec{k}\vec{k}'} = \frac{e_0^2}{\varsigma} \int e^{i(\vec{k}-\vec{k}', \vec{r})} \frac{e^{-r/a}}{r} d\vec{r} = \frac{4\pi e_0^2}{\varsigma} \frac{1}{(\vec{k}-\vec{k}')^2 + a^{-2}}. \quad (33)$$

If  $(\vec{k}-\vec{k}')^2$  is expressed through angles  $\theta$ ,  $\vartheta$  and  $\varphi$  (see explanations to (17)), in the case of ellipsoid of revolution we obtain the following expression for transition probability

$$W(\theta \vartheta \varphi) = \frac{2\pi}{\hbar} N |V_{\vec{k}\vec{k}'}|^2 = \frac{\pi^3 e_0^4 N \hbar^3}{2 \varepsilon^2 m_3^2 \chi^2 \left[ \left( \cos^2 \vartheta + \frac{m_1}{m_3} \sin^2 \vartheta \right) \cos^2 \theta + \gamma^2 \right]^2}, \quad (34)$$

where

$$\gamma^2 = \frac{\hbar^2}{8 a^2 m_3 \varepsilon}, \quad (35)$$

$N$  is the number of ionized impurities per  $1 \text{ cm}^3$ ,  $m_1 = m_2 < m_3$ .

Parameter  $\gamma^2$  in the region of  $5 \leq T \leq 300 \text{ K}$  and  $10^{14} \leq N_D \leq 10^{17} \text{ cm}^{-3}$  is in the range of  $10^{-7} \leq \gamma^2 \leq 10^{-2}$ . With that sort of low  $\gamma^2$  an angular summand in denominator (34) is essential, since the probability of scattering for certain angle  $\chi$  (scattering angle  $\chi = \pi - 2\theta$ ) with real values  $m_1/m_3$  is strongly dependent on angle  $\vartheta$ , that is, on the direction of electron motion.

As long as in our case  $W(\theta \vartheta \varphi)$  does not depend on  $\varphi$ , coefficients  $B_{jk}(pm) = B_{jk}(m) \delta_{pm}$  (see (18)) –  $X_{1m}$  can be determined by means of series (29). In the boundary case of low but finite  $\gamma^2$  and  $m_1/m_3$  we get

$$X_{10} = \frac{D_0}{\tilde{B}_{11}(0)} (1 + 0.72 + 0.015 + 0.00018 + \dots). \quad (36)$$

The values of  $\tilde{B}_{jk}(m)$  for  $m = 0.1$  are given in Tables 1 and 2.

Table 1

The values of coefficients  $\frac{\tilde{B}_{jk}(0)}{\tilde{B}_{11}(0)}$  at certain  $j$  and  $k$

$j$	$k$			
	1	3	5	7
1	1	3.44	6.74	10.7
3	3.44	28.2	59.2	96.0
5	6.74	59.2	169	287
7	10.7	96.0	287	576

Table 2

The values of coefficients  $\frac{\tilde{B}_{jk}(1)}{\tilde{B}_{11}(1)}$  at certain  $j$  and  $k$

$j$	$k$		
	1	3	5
1	1	1.40	1.74
3	1.40	4.70	6.24
5	1.74	6.24	11.4

For larger  $\gamma^2$  and  $m_1/m_3$  values, when the anisotropy is lower, series (29), naturally, converges even better. If we use an evaluation formula where we substitute  $m_1/m_3 = 0.052$  for *Ge* and  $m_1/m_3 = 0.196$  for *Si*, the second term in (29) will be equal to 0.38 and 0.15 respectively, and the third term  $\cong 10^{-3}$  in both cases.

Similarly, for  $X_{11}$  in the boundary case of low but finite  $\gamma^2$  and  $m_1/m_3$ , we obtain

$$X_{11} = \frac{D_1}{\tilde{B}_{11}(1)} (1 + 0.72 + 0.015 + \dots). \quad (37)$$

For *Ge* and *Si* the second term in (37) will be equal to 0.50 and 0.18, respectively. So, both for  $X_{10}$  and  $X_{11}$  we can restrict ourselves to two terms of the series.

Restricting ourselves in (29) to the second term, we will have

$$X_{1m} = D_m \chi_m, \quad (38)$$

where

$$\chi_m = \frac{1}{B_{11}(m)} \left[ 1 + \frac{B_{13}^2(m)}{B_{11}(m) B_{33}(m) - B_{13}^2(m)} \right], \quad (39)$$

in which case  $\chi_1 = \chi_{-1}$ . Taking into account (21) and (38), we will get the following expressions for components of relaxation time tensor

$$\left. \begin{aligned} \tau_{\parallel} = \tau_{33} = \chi_0 &= \frac{1}{B_{11}(0)} (1 + g_0) \\ \tau_{\perp} = \tau_{11} = \chi_1 &= \frac{1}{B_{11}(1)} (1 + g_1) \end{aligned} \right\}, \quad (40)$$

where

$$g_m = \frac{B_{13}^2(m)}{B_{11}(m) B_{33}(m) - B_{13}^2(m)}. \quad (41)$$

In the isotropic case ( $m_1/m_3 = 1$ ) all  $B_{jk}(m) = 0$  at  $j \neq k$  and formulae (40) give us relaxation time obtained by Brooks and Herring [12] and Dingle [13]:

$$\tau^{-1} = \frac{\pi e_0^4 N}{\zeta^2 \sqrt{2m^* \varepsilon^3}} \left( \ln \frac{1 + \gamma^2}{\gamma^2} - \frac{1}{1 + \gamma^2} \right). \quad (42)$$

Note that all coefficients  $B_{jk}(m)$  can be precisely calculated and are rather complicated and cumbersome expressions. However, as it turned out, in the calculation of  $B_{jk}(m)$  for *Ge* and *Si* one can make an expansion with respect to parameter  $\alpha^2 = \gamma^2 m_3 / m_1$  and omit all  $\alpha^2$  and higher-order terms, since, in practically most important region,  $\gamma^2 \leq 10^{-2}$ . The obtained coefficients  $B_{jk}(m)$  can be used for calculation of correction  $g_m$  by formula (39) in the cases when scattering on impurity ions prevails over other scattering mechanisms (heavily compensated samples, low temperatures, etc.).



## 2.4. Scattering on acoustic phonons

Based on deformation potential theory [6] and the results of Ref. [14], the probability of scattering for *Ge* and *Si* is of the form

$$W(\vartheta) = \frac{2\pi k T C_1^2}{C_{11}' \hbar} \left\{ 1 + \frac{2C_2 m_3 \cos^2 \vartheta}{C_1 (m_1 \sin^2 \vartheta + m_3 \cos^2 \vartheta)} + \frac{C_2^2 m_3^2 \cos^4 \vartheta}{C_1^2 (m_1 \sin^2 \vartheta + m_3 \cos^2 \vartheta)^2} + \frac{C_2^2 C_{11}' m_1 m_3 \sin^2 \vartheta \cos^2 \vartheta}{C_1^2 C_{44}' (m_1 \sin^2 \vartheta + m_3 \cos^2 \vartheta)^2} \right\}, \quad (43)$$

where  $C_1$  and  $C_2$  are deformation potential constants,  $C_{11}'$  and  $C_{44}'$  are averaged elastic constants determined in [14].

As can be seen from (43),  $W$  does not depend on angle  $\theta$ , hence, on scattering angle either. Coefficients  $B_{jk}(m)$  are calculated by the formula

$$B_{jk}(m) = \frac{4m_1 \sqrt{2m_3 \varepsilon}}{(2\pi \hbar)^3} \sum_{s \text{ paired}} L_{jk}^s R_{jk}^s(m), \quad (44)$$

where

$$\left. \begin{aligned} L_{jk}^s &= 2 \sqrt{\frac{(j-s)!(k-s)!}{(j+s)!(k+s)!}} \int_0^{\pi/2} d\theta \sin \theta \cos \theta P_j^{*s}(\cos \theta) P_k^s(\cos \theta) \\ R_{jk}^s(m) &= \int d\Omega \bar{P}_{sm}^j(\cos \vartheta) \bar{P}_{sm}^{*k}(\cos \vartheta) \end{aligned} \right\}, \quad (45)$$

$\bar{P}_{sm}^j(\cos \vartheta)$  are functions normalized to unity.

A matrix of coefficients  $B_{jk}(m)$  in this case is very close to the diagonal one, so in series (29) for  $X_{1m}$  the second term can be already omitted, since it is two orders of magnitude smaller than the first one for *Ge* and *Si*.

The respective  $B_{11}(m)$  values, according to (44) and (45), will be given by

$$B_{11}(m) = \frac{1}{\tau_0} S_m, \quad (46)$$

where "isotropic" relaxation time  $\tau_0$ :

$$\tau_0 = \frac{\pi C_{11}' \hbar^4}{k T C_1^2 \sqrt{2m_1^2 m_3 \varepsilon}}, \quad (47)$$

$$\begin{aligned} S_0 &= 1 + \frac{2C_2 m_3}{C_1 m_1 \beta^2} \left( 1 - \frac{3}{\beta^2} + \frac{3}{\beta^3} b \right) + \frac{m_3 C_2^2}{m_1 C_1^2 \beta^4} \times \\ &\times \left\{ \frac{m_3}{m_1} \left( 1 - \frac{6}{\beta^2} - \frac{3m_1}{2\beta^2 m_3} + \frac{15}{2\beta^3} b \right) + \frac{C_{11}'}{C_{44}'} \left[ 2 + \frac{15}{2\beta^2} - \frac{3b}{2\beta^3} (5 + 3\beta^2) \right] \right\}, \end{aligned} \quad (48)$$

$$\begin{aligned} S_1 &= 1 + \frac{C_2 m_3}{C_1 m_1 \beta^2} \left( 2 - \frac{3}{\beta^2} + \frac{3b m_3}{m_1 \beta^3} \right) + \frac{m_3 C_2^2}{m_1 C_1^2 \beta^4} \times \\ &\times \left\{ \frac{m_3}{m_1} \left( 1 + \frac{15}{4\beta^2} - \frac{3b}{4\beta^3} (5 + 3\beta^2) \right) - \frac{C_{11}'}{4C_{44}'} \left[ 13 + \frac{15}{\beta^2} - \frac{3}{\beta^3} (5 + 6\beta^2 + \beta^4) b \right] \right\}. \end{aligned} \quad (49)$$

Here,  $\beta^2 = \frac{m_3 - m_1}{m_1}$ ,  $b = \text{arctg } \beta$ .

Relaxation time tensor is given by

$$\tau_{11} = \tau_{22} = \tau_{\perp} = \frac{1}{B_{11}(1)} = \frac{\tau_0}{S_1}; \quad \tau_{33} = \tau_{\parallel} = \frac{1}{B_{11}(0)} = \frac{\tau_0}{S_0}. \quad (50)$$

## 2.5. Mixed scattering

In the case of mixed scattering, coefficients  $B_{jk}(m)$  (owing to summation of transition probabilities) will be as follows

$$B_{jk}(m) = B_{jk}^f(m) + B_{jk}^{ion}(m), \quad (51)$$

where  $B_{jk}^f(m)$  and  $B_{jk}^{ion}(m)$  are coefficients corresponding to scattering on phonons or impurity ions only. The second term of series (29) for  $X_{1m}$  (designated as  $g_m$ ) will be determined by the relationship

$$g_m = \frac{(B_{13}^f(m) + B_{13}^{ion}(m))^2}{(B_{11}^f(m) + B_{11}^{ion}(m))(B_{33}^f(m) + B_{33}^{ion}(m)) - (B_{13}^f(m) + B_{13}^{ion}(m))^2}. \quad (52)$$

As long as a matrix of coefficients  $\|B_{jk}^{\phi}(m)\|$  (as mentioned above) is almost diagonal, from (52) it is seen that correction  $g_m$  in this case is much smaller than in the case of purely ion scattering. Analysis shows that  $g_m$  with a mixed scattering is of the order of several percent for *Ge* and *Si* in the entire practically important range of temperatures and impurity concentrations. Therefore, for the case of mixed scattering (and this kind of scattering in actual practice is very frequent) it is enough in series (29) for  $X_{1m}$  to be restricted to the first term (linear approximation).

## 3. The use of AST for study of thermoelectric and thermomagnetic effects in anisotropic semiconductors

As long as the theory of thermoelectric and thermomagnetic effects within the assumptions used here was published in full scope as a separate edition [3], we will restrict ourselves to several general comments related to practical use of this theory (by comparison of its conclusions to the experiment).

In the framework of assumptions accepted in the band theory and AST, in said edition a general theory of galvano- and thermomagnetic effects was built for arbitrary-value (but non-quantizing) magnetic fields. In so doing, the kinetics of electron processes in many-valley semiconductors was considered both in the presence and absence of electron phonon drag effects.

It turned out that qualitative and quantitative analyses of the entire combination of considered effects can be carried to completion with restriction to two parameters that are well measured experimentally: mobility anisotropy parameter  $K = \mu_{\perp} / \mu_{\parallel}$  and drag thermoEMF anisotropy parameter  $M = \alpha_{\parallel}^{\phi} / \alpha_{\perp}^{\phi}$ , which characterize an isoenergetic ellipsoid taken individually.

The use of uniaxial elastic deformation (with application to crystal and a change in deforming mechanical stress  $X$  in rather wide limits) assures the analysis of effects under study with a different number of efficient isoenergetic ellipsoids: from one (*n-Ge*, deformed in  $\langle 111 \rangle$  direction) to six (*n-Si* – in the non-deformed crystal).

However, there are many semiconductors whose isoenergetic surfaces are shaped not as ellipsoids of revolution, but as three-axial ellipsoids (and even more complicated). In monograph [3], the necessary

generalizations of the theory appropriate for the description of kinetic effects in these (far from being simple) cases were made and can employed for comparison of experimental data to the theory.

## Conclusions

1. The basic postulates of anisotropic scattering theory (AST) are formulated and the most important formulae of this theory are given in a form convenient for practical use by comparison to the experimental data.
2. Possibilities of AST practical use in the investigation of thermoelectric and thermomagnetic effects in many-valley semiconductors of the type *n-Si* and *n-Ge* are explored.
3. The specific feature of AST is pointed out, namely its suitability for the description of the kinetics of electron processes not only in the region of near-boundary small or boundary large (non-quantizing) magnetic fields *H*, but also in the region of intermediate magnetic fields (that is, at  $\mu H/c \approx 1$ ), where the use of other theories is inefficient.

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