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STUDY OF THE UNSTEADY-STATE DISTRIBUTION OF TEMPERATURE AND THERMOPOWER OF ANISOTROPIC OPTICAL THERMOELEMENT UNDER PARALLEL DIRECTIONS OF RADIANT FLUX AND TEMPERATURE GRADIENT

The solution of the inhomogeneous unsteady-state thermal conductivity equation under parallel directions of temperature gradient and radiant flux of $ZnAs_2$ anisotropic plate with regard to the Bouguer-Lambert law is presented, and the dependence of temperature distribution on coordinates and time is analyzed. The expressions for the transverse thermopower are obtained for the cases of optical transmission and surface absorption.

Key words: anisotropic medium, anisotropic optical thermoelement, thermostat, radiant flux, temperature gradient, transverse thermopower.

Introduction

The prospects of using anisotropic media with different optical transmission values for recording and conversion of high-intensity radiant fluxes give an impetus to a thorough study of their properties. One of the methods of electromotive force generation is a transverse thermoelectromotive force (thermopower). Despite the fact that this effect had been long ago studied by Thomson [1], it was not until nearly a century later that Samoilovich with colleagues succeeded in implementing this idea in the form of anisotropic thermoelement [2]. The absence of conventional junction, availability of only one section, mutual normality of heat flux and thermoelectric field determined their promising outlook and resulted in the appearance of new generations of various instruments and devices having no domestic and foreign equivalents [3-6]. At the present time there are sources whose energy is rather difficult to be recorded and converted by the existing methods. For this purpose, here we use media with different optical transmission values. Such media make it possible to convert the absorbed part of radiant energy by means of thermal pyrocalorimetric effects. Analysis shows that for radiant fluxes of UV, visible and IR regions implementation of this method offers the greatest promise for the case of using the transverse thermopower effect [1-5]. This resulted in the development of anisotropic optical thermoelements (AOT) [7]. The choice of specific AOT and the necessary operating modes is dictated by service conditions and is a function of parameters of materials employed, as well as of mutual directions of radiant and heat fluxes propagation with respect to selected crystallographic orientations of material.

Unsteady-state distribution of AOT temperature and thermopower

Let us consider AOT in the form of a rectangular plate of dimensions a, b, c (Fig. 1), made of material whose thermal conductivity coefficient χ and the Seebeck coefficient α in the laboratory coordinate system (XYZ) rotated by an angle φ in XOY plane with respect to crystallographic plane (X′Y′Z′) are given by

$$\chi = \begin{vmatrix} \chi_{\parallel}\sin^2\phi + \chi_{\perp}\cos^2\phi & \left(\chi_{\parallel} - \chi_{\perp}\right)\sin\phi\cos\phi & 0 \\ \left(\chi_{\parallel} - \chi_{\perp}\right)\sin\phi\cos\phi & \chi_{\parallel}\cos^2\phi + \chi_{\perp}\sin^2\phi & 0 \\ 0 & 0 & \chi_{\perp} \end{vmatrix},$$
$$\alpha = \begin{vmatrix} \alpha_{\parallel}\sin^2\phi + \alpha_{\perp}\cos^2\phi & \left(\alpha_{\parallel} - \alpha_{\perp}\right)\sin\phi\cos\phi & 0 \\ \left(\alpha_{\parallel} - \alpha_{\perp}\right)\sin\phi\cos\phi & \alpha_{\parallel}\cos^2\phi + \alpha_{\perp}\sin^2\phi & 0 \\ 0 & 0 & \alpha_{\perp} \end{vmatrix}, \quad (1)$$

where, $\chi_{\parallel}$, $\chi_{\perp}$ and, $\alpha_{\parallel}$, $\alpha_{\perp}$ are tensor components χ and α .

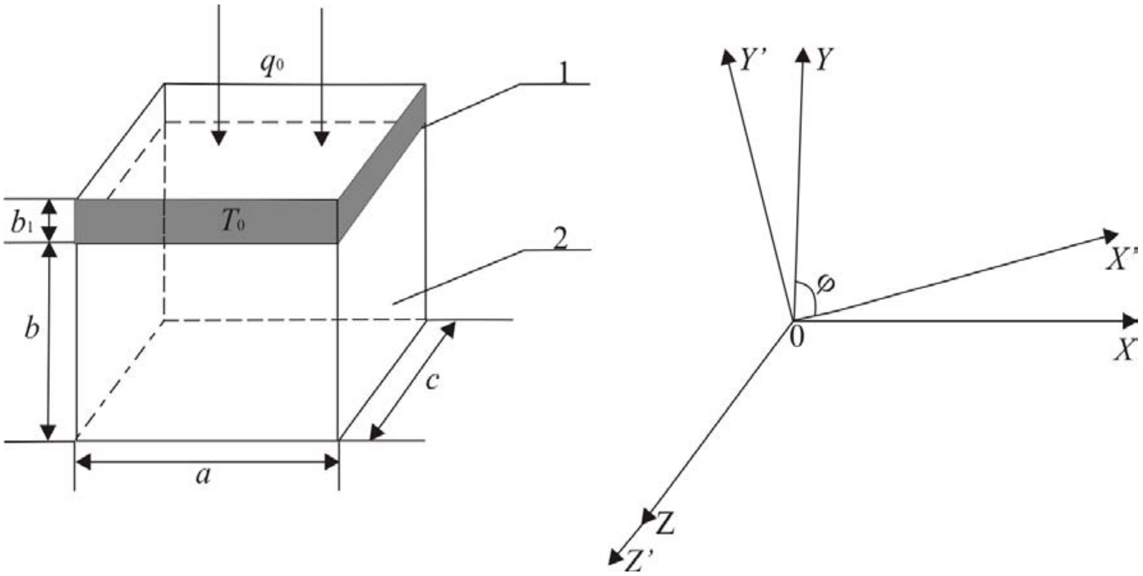

Fig.1. Schematic of AOT: thermostat 1, anisotropic plate 2. To the right – laboratory coordinate system XYZ and orientation of crystallographic axes X'Y'Z' of plate 2.

A uniform monochromatic radiant flux of density q_0 is incident on the upper face of thermostat 1 of thickness b_1 made of optically transparent material in the required wavelength spectrum with absorption coefficient γ_1. The lower face of the thermostat is in thermal optical contact with the upper face 2 of AOT at temperature $T = T_0$. The lateral and lower faces of plate 2 are adiabatically isolated. In so doing, the boundary effects are disregarded ($a = c >> b$) [8]. On passing through such a plate, the uniform monochromatic radiant flux of density q_0 and wavelength λ_0 causes the appearance of temperature gradient and transverse thermoEMF unambiguously related to it.

In the presence of internal heat sources, the unsteady-state temperature distribution in AOT can be found from the basic Fourier conduction law [9]

$$\frac{\partial T}{\partial t} = \frac{1}{d\,C_0}\sum_{i,k=1}^{3}\chi_{ik}\,\frac{\partial^2 T}{\partial x_i \partial x_k} + \frac{q_v}{d\,C_0}, \quad (2)$$

where d is AOT material density, C_0 is specific heat, χ_{ik} are corresponding components of thermal conductivity tensor found from another relation of system (1), q_v is the amount of heat released by internal sources in unit volume per unit of time and defined by the Bouguer-Lambert law.

For the unsteady-state temperature distribution in the approximation

$$\frac{\partial T}{\partial \mathrm{x}} = \frac{\partial T}{\partial \mathrm{z}} = 0, \quad \chi_{12} < \chi_{22}$$

thermal conductivity equation (2) is of the form

$$\frac{\partial T}{\partial t} = R^2 \frac{\partial^2 T}{\partial y^2} + P\, e^{-\gamma(b-y)}, \tag{3}$$

where $R^2 = \frac{\chi_{22}}{C_0 d}$, $P = \frac{q_0 \gamma}{C_0 d} \exp\left[-\gamma_1 b_1\right]$, γ is absorption coefficient of AOT material.

Solution of inhomogeneous equation (3) under the boundary and initial conditions

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0; \;\; T\big|_{y=b} = T_0;\; T\big|_{t=0} = T_0 \tag{4}$$

will be sought for as

$$T(y,t) = T_0 + V(y,t). \tag{5}$$

Substituting (5) into (3), we will obtain the thermal conductivity problem

$$\frac{\partial V}{\partial t} = R^2 \frac{\partial^2 V}{\partial y^2} + P\, e^{-\gamma(b-y)} \tag{6}$$

for function $V(y,t)$ under the homogeneous boundary and initial conditions

$$\left.\frac{\partial V}{\partial y}\right|_{y=0} = 0;\; V\big|_{y=b} = 0;\; V\big|_{t=0} = 0\,. \tag{7}$$

General solution of inhomogeneous equation (6) will be found as a sum of general solution of the homogeneous and partial solution of the inhomogeneous solutions (6), i.e.

$$V(y,t) = V_{\mathrm{hom}}(y,t) + V_{\mathrm{in\,hom}}(y,t). \tag{8}$$

General solution of homogeneous equation (6) under zero boundary conditions (7) results in Sturm-Liouville problem that has the eigenfunction $\cos\left[\frac{(2n+1)\pi}{2b} y\right]$ corresponding to the eigenvalues $\lambda_n = \frac{(2n+1)\pi}{2b}$, where $n = 0,1,2,\ldots$. Partial solution of inhomogeneous equation (6) will be sought for as expansion into a Fourier series in terms of eigenfunctions $\cos\left[\frac{(2n+1)\pi}{2b} y\right]$ of homogeneous:

$$V(y,t)=\sum_{n=0}^{\infty}\Phi_n(t)\cos\left[\frac{(2n+1)\pi}{2b}y\right]. \tag{9}$$

Substituting (9) and the expansion of thermal conductivity equation (6) inhomogeneity into a Fourier series in terms of eigenfunctions of homogeneous problem, we obtain an equation for determination of $\Phi_n(t)$

$$\sum_{n=0}^{\infty}\left\{\Phi_n{}'(t)+\left[\frac{(2n+1)\pi}{2b}R\right]^2\Phi_n(t)-\varphi_n\right\}\cos\left[\frac{(2n+1)\pi}{2b}y\right]=0, \tag{10}$$

where $\varphi_n=\frac{2}{b}\int_0^b Pe^{-\gamma(b-y)}\cos\left[\frac{(2n+1)\pi}{2b}\mathrm{y}\right]dy$. The resulting equality can be considered as zero function expansion into a Fourier series in terms of eigenfunctions $\cos\left[\frac{(2n+1)\pi}{2b}y\right]$. Substituting $\Phi_n(t)$ into equation (9), we obtain a solution of thermal conductivity problem (6)-(7) as below

$$V(y,t)=\int_0^t d\tau\int_0^b\left[\frac{2}{b}\sum_{n=0}^{\infty}\exp\left[-\left[\frac{(2n+1)}{2b}R\right]^2(t-\tau)\right]\cos\left[\frac{(2n+1)\pi}{2b}y\right]\cos\left[\frac{(2n+1)\pi}{2b}y\right]\xi\right]Pe^{-\gamma(b-\xi)}d\xi. \tag{11}$$

Let us introduce the influence function of instantaneous point heat source

$$G(y,\xi,t-\tau)=\frac{2}{b}\sum_{n=0}^{\infty}\exp\left[-\left[\frac{(2n+1)}{2b}R\right]^2(t-\tau)\right]\cos\left[\frac{(2n+1)\pi}{2b}\mathrm{y}\right]\cos\left[\frac{(2n+1)\pi}{2b}\xi\right]. \tag{12}$$

Then solution of (11) will acquire the form

$$V(y,t)=\int_0^t d\tau\int_0^b G(y,\xi,t-\tau)\,Pe^{-\gamma(b-\xi)}\mathrm{d}\xi. \tag{13}$$

Substituting (13) into (5), upon integration we obtain final expression for the unsteady–state temperature distribution

$$\begin{aligned}T(y,t)=T_0+\frac{2q_0\gamma}{b\chi_{22}}\exp\left[-\gamma_1 b_1\right]\sum_{n=0}^{\infty}\frac{\left[(-1)^n\frac{(2n+1)\pi}{2b}-\gamma e^{-\gamma b}\right]}{\left[\gamma^2+\left(\frac{(2n+1)\pi}{2b}\right)^2\right]\left(\frac{(2n+1)\pi}{2b}\right)^2}\times\\ \times\left[1-\exp\left(\frac{-(2n+1)^2\pi^2}{4}\frac{\chi_{22}}{C_0\,d\,b^2}\mathrm{t}\right)\right]\cos\left(\frac{(2n+1)\pi}{2b}y\right).\end{aligned} \tag{14}$$

From relation (14) it is seen that unsteady-state temperature distribution of AOT depends on the absorption coefficients of thermostat and thermoelement materials and the anisotropy of thermal conductivity coefficient. Moreover, temperature distribution is a function of thermoelement and thermostat geometry and has a complex nonlinear dependence on coordinate *y* and time *t*.

Fig. 2 shows a plot of temperature field distribution $\Delta T(y,t)=T(y,t)-T_0$ for AOT made of $ZnAs_2$ in the case of optical transmission ($\gamma b<<1$), and Fig. 3 – in the case of surface absorption ($\gamma b>>1$).

Components of thermoelectric field intensity E_i^T are defined by the following relation

$$E_i^T = \sum_{k=1}^{3} \alpha_{ik} \frac{\partial T}{\partial x_k}, \quad (i = 1, 2, 3). \tag{15}$$

Substituting (14) into (15) we obtain

$$E_x^T = -\frac{2 q_0 \gamma \alpha_{12}}{b \chi_{22}} \exp\left[-\gamma_1 b_1\right] \sum_{n=0}^{\infty} \frac{\left[(-1)^n \frac{(2n+1)\pi}{2b} - \gamma e^{-\gamma b}\right]}{\left[\gamma^2 + \left(\frac{(2n+1)\pi}{2b}\right)^2\right] \frac{(2n+1)\pi}{2b}} \times$$
$$\times \left[1 - \exp\left(\frac{-(2n+1)^2 \pi^2}{4} \frac{\chi_{22}}{C_0 d b^2} t\right)\right] \sin\left(\frac{(2n+1)\pi}{2b} y\right). \tag{16}$$

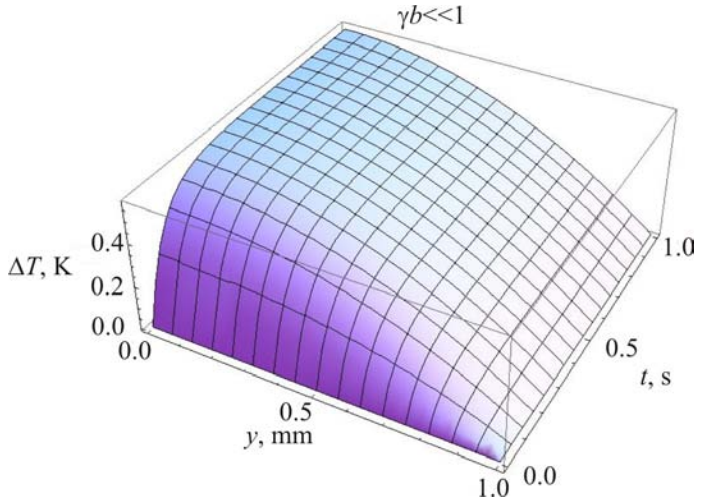

Fig.2. Temperature field distribution of $ZnAs_2$ AOT in the case of optical transmission.

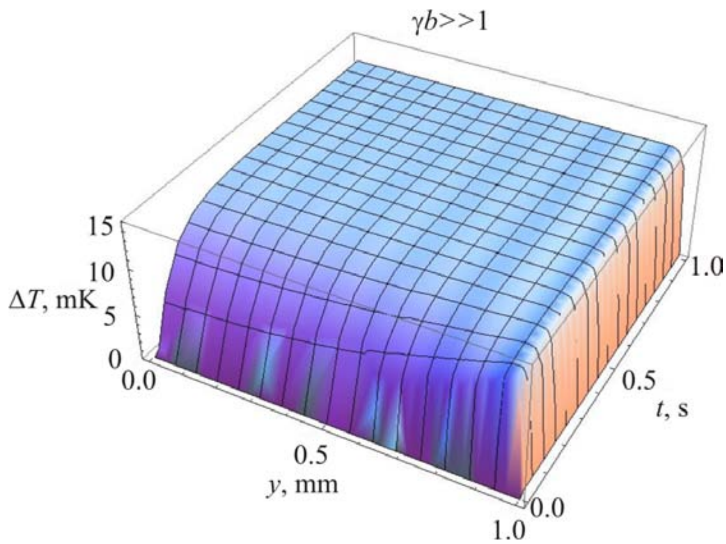

Fig.3. Temperature field distribution of $ZnAs_2$ AOT in the case of surface absorption.

According to [10], the transverse thermopower ε_x is defined by the following relation

$$\varepsilon_x = \frac{1}{bc}\int_0^b dy\int_0^c dz\int_0^a E_x^T dx\,. \tag{17}$$

Substituting (16) into (17), upon integration we obtain the expression for the transverse thermoelectromotive force ε_x of AOT at hand in the form

$$\varepsilon_x(t) = -\frac{2q_0\gamma a\alpha_{12}}{\chi_{22}}\exp\left[-\gamma_1 b_1\right]\sum_{n=0}^{\infty}\frac{\left[(-1)^n\frac{(2n+1)\pi}{2b} - \gamma e^{-\gamma b}\right]}{\left[\gamma^2 + \left(\frac{(2n+1)\pi}{2b}\right)^2\right]\left(\frac{(2n+1)\pi}{2b}\right)^2}\times$$
$$\times\left[1 - \exp\left(\frac{-(2n+1)^2\pi^2}{4}\frac{\chi_{22}}{C_0 d b^2}\mathrm{t}\right)\right]. \tag{18}$$

Figs. 4 and 5 show the plots of transverse thermopower ε_x versus b of AOT and time t for optical transmission ($\gamma b << 1$) and surface absorption ($\gamma b >> 1$), respectively.

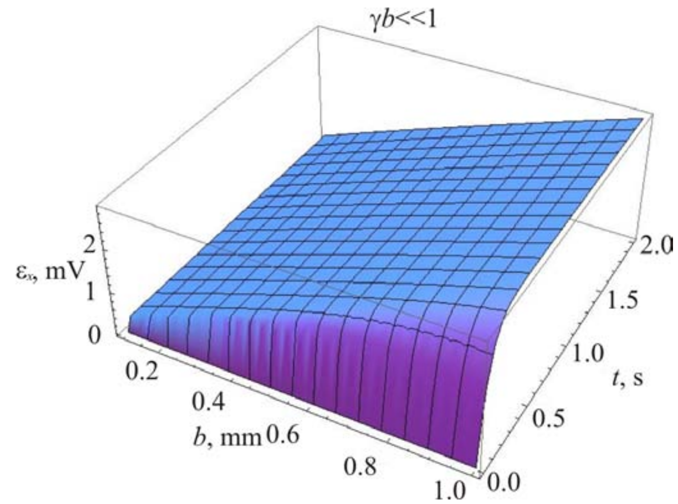

Fig.4. Transverse thermopower ε_x of $ZnAs_2$ AOT in the case of optical transmission versus time t and height b of thermoelement at thermostating of the upper working face.

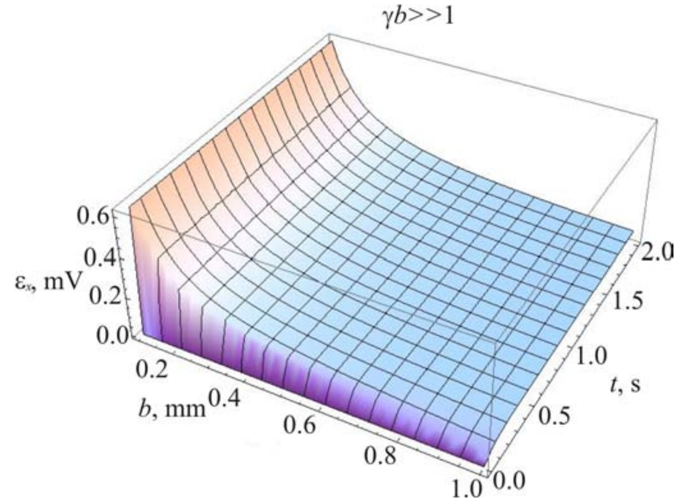

Fig.5. Transverse thermopower ε_x of $ZnAs_2$ AOT in the case of surface absorption versus time t and height b of thermoelement at thermostating of the upper working face.

It has been established that with increasing time *t*, both in the case of optical transmission and surface absorption the transverse thermopower grows and at certain value of *t* it becomes constant. From the plots it is seen that with increasing AOT height, the transverse thermopower has a complex nonlinear dependence at the initial time instants both for optical transmission and surface absorption. At later time instants, the complex nonlinear height dependence is retained for surface absorption and for optical transmission there is a quasilinear dependence of the transverse thermopower on AOT height *b*.

Conclusions

The expressions for the transverse thermopower of $ZnAs_2$ AOT under thermostating of the upper working face for the cases of optical transmission and surface absorption have been studied. The AOT considered make it possible to record and control radiant fluxes in a wide spectral range.

References

1. W.Thomson, On Thermoelectric Currents in Linear Conduction Crystalline Substance, *Math. Phys. Pap.* – 1882. – № 1. – P. 266-273.
2. I.M.Pilat, A.G.Samoilovivh, and L.I.Anatychuk, Thermoelement, USSR *Certificate of Authorship 230915,* Publ.in Bulletin of Inventions, №3, 1970.
3. L.I.Anatychuk, Thermoelements and Thermoelectric Devices (Kyiv: Naukova Dumka, 1979), 767p.
4. L.I.Anatychuk, O.J.Luste, Eddy Thermoelectric Currents and Eddy Thermoelements, *Semiconductors* **10**(5), 817-831(1976).
5. A.A.Snarskii, A.M.Palti, A.A.Ashcheulov, Anisotropic Thermoelements (Review), *Semiconductors* **31**(11), 1281-1298 (1997).
6. A.A.Ashcheulov, I.M.Rarenko, and N.K.Voronka, Optimized Materials Based on Cadmium Antimonide and their Application, *J. Thermoelectricity* – 1995. – № 3. – P. 73-88.
7. A.A.Ashcheulov, I.V.Gutsul, Anisotropic Optical Thermoelectric Elements, *J. Thermoelectricity* – 1997. – № 3. – P. 81-89.
8. V.N.Slipchenko, A.A.Snarskii, Effect of Thermal Conductivity Anisotropy on Transverse Thermopower, *Semiconductors* **8**(10), 2010-2012 (1974).
9. A.V.Lykov, Theory of Thermal Conductivity (Moscow: Vysshaya Shkola, 1967), 600 p.
10. A.A.Snarskii, EMF of Thermoelements Using Thermopower Anisotropy. I. Rectangular Anisotropic Thermoelements, *Semiconductors* **11**(10), 2053-2055 (1977).

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