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**PREDICTION OF THE VALUES OF NATURAL OSCILLATION  
FREQUENCIES IN THE DESIGN OF MULTI-STAGE STACKED  
THERMOELECTRIC STRUCTURES**

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*The paper deals with the possibility of theoretical determination during design phase of natural oscillation frequencies of thermoelectric coolers with different number of stages and different configuration. The natural oscillation frequencies were determined in parallel for products with regard to damping created by a number of structural members, as well as without regard to damping capacity of solder layers along thermoelement ends. Analysis and comparison of the obtained theoretical results was performed. The results of determination of natural oscillation frequencies of thermoelectric coolers (TEC) by using the system of Lagrange's equations of the second kind were compared with the calculated values obtained by the method of electrodynamic analogies.*

**Key words:** resonance, natural oscillation frequencies, damping.

### **Formulation of the problem**

For the purpose of cooling electronic products and radio equipment in order to assure their normal operating conditions, reduce their weight and dimensions, electronic devices and systems employ thermoelectric coolers (TEC) based on the well known Peltier principle. Depending on the purpose, specificity and parameters of electronic devices, their power and the amount of thermal energy released, TEC with different number of stages are used.

### **Analysis of recent research**

As is known, the operation of any type of carrier is characterized by certain frequency and amplitude of oscillations that are forced for the equipment installed thereupon. Provision of normal operation of TEC, their mechanical integrity is related to prevention of resonance effect on product exposure to certain frequencies of forced oscillations.

### **Singling out parts of general problem unsolved before**

Thus, to determine natural oscillation frequencies of products as early as during their design and development phase is an important engineering, technical and scientific problem.

For this purpose, apart from experimental methods, use is made of mathematical simulation which permits with assigned precision to calculate the required parameters (dimensions, shape of structural elements, corresponding materials) with the use of TEC models of different design and complexity [1]. For instance, the general view of a 3-stage TEC is represented in Fig. 1.

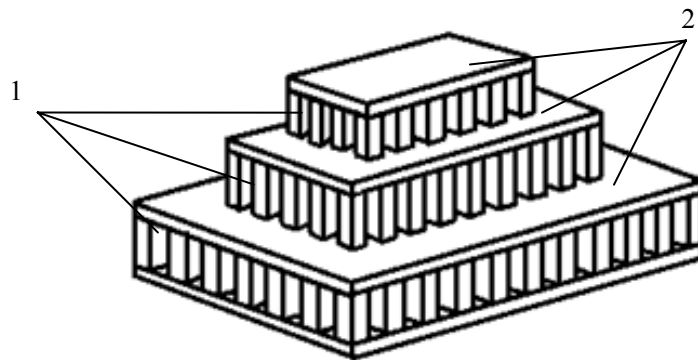


Fig 1. General view of a 3-stage construction:  
1 – thermoelements; 2 – heat spreaders.

**Formulation of the goal of research.** The paper is concerned with a calculation of a 4-stage TEC structure the parameters of which are given below.

The number of thermoelements in each stage of devices studied in the paper was:  $n_1 = 12$ ,  $n_2 = 22$ ,  $n_3 = 52$ ,  $n_4 = 128$  (as an example, Fig. 1 shows a 3-stage cooler). The length of each thermoelement was  $l = 1.4$  mm; cross-section:  $0.7 \times 0.7$  mm. The thermoelements were made of bismuth telluride  $Bi_2Te_3$  with the elasticity modulus  $E = 6 \cdot 10^5$  kg/cm<sup>2</sup>.

Heat spreaders 2 (Fig. 1) were made of glass ceramic plates 0.1 mm thick with copper current-conducting strips deposited thereupon. For attachment of thermoelements to current-conducting strip, solder was used on the lateral surface of heat spreaders with the thickness  $\Delta = 1 \cdot 10^{-1}$  mm, the elasticity modulus was assumed to be  $E_{II} = 1.2 \cdot 10^5$  kg/cm<sup>2</sup>.

The masses of heat spreaders with the above number of thermoelements were:  $m_1 = 10.92896 \cdot 10^{-5}$  kg;  $m_2 = 4.55961 \cdot 10^{-5}$  kg;  $m_3 = 1.95753 \cdot 10^{-5}$  kg;  $m_4 = 1.31891 \cdot 10^{-5}$  kg. Such products, despite sufficiently small dimensions and mass, allow achieving temperature difference up to 20 °C on each stage, cooling essentially the electronic devices and products in operation. Mounting of TEC on mobile carriers (terrestrial, underwater, aeronautic, space) allows reducing considerably total weight of radioelectronic equipment and the dimensions of the entire installation.

### Formulation of the goal of research

The natural oscillation frequencies of stacked-type TEC were determined with the aid of developed mathematical model based on the use of differential Lagrange's equations of the second kind [2]:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} = - \frac{\partial \Pi}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}, \tag{1}$$

where:  $T$  is kinetic energy of the system;

$\Pi$  is potential energy of the system;

$i$  is generalized coordinate;

$x_i$  is motion of  $i$ -th stage,  $i = 1, 2, 3, 4$ ;

$\Phi$  is dissipative function,

$t$  is time.

Kinetic energy of the system:

$$T = \sum_{i=1}^4 T_i. \tag{2}$$

Potential energy of the system:

$$\Pi = \sum_{i=1}^4 \Pi_i. \tag{3}$$

Dissipative function is proportional to the velocity of motion of system masses:

$$\Phi = \sum_{i=1}^4 \Phi_i, \tag{4}$$

$$\Phi_i = \frac{1}{2} \beta_i \dot{x}_i^2, \tag{5}$$

where  $\beta_i$  is damping coefficient.

The paper is concerned with the case when the structure is composed of four stages of thermoelements which can be schematically shown as follows (Fig. 2):

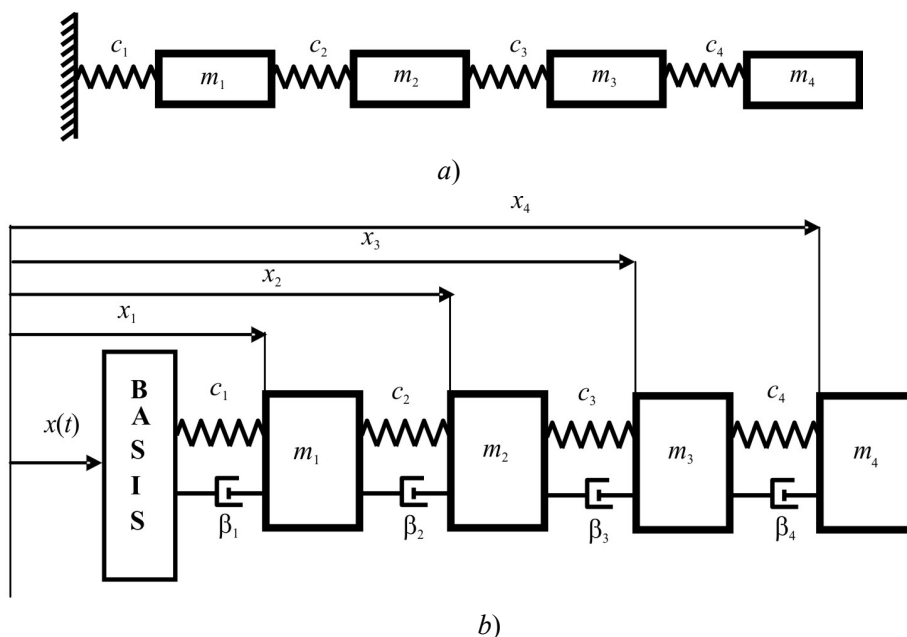


Fig. 2. Design circuit of a 4-stage TEC; a) without regard to damping; b) with regard to damping.

### The main results of the investigation

This paper is concerned with investigation of a specific 4-stage TEC, i.e.  $i = 1, 2, 3, 4$ . Here,  $m_i$  is the masses of respective heat spreaders, and  $c_i$  – the rigidity of thermoelements in each stage, respectively [1].

Solder layers along thermoelement ends have essentially lower values of elasticity modulus than the similar parameter of thermoelements material, i.e. they are characterized by a much lower rigidity. Therefore, it can be considered that solder layers along thermoelement ends in each stage serve as peculiar dampers reducing the values of product natural oscillations.

It is known that the dissipative function is nonlinear [4]. Account for this nonlinearity in complex technical systems is related to considerable, including mathematical, difficulties. Their account in many cases can introduce considerable specifications into the values of required parameter values with regard to products specificity and their operating conditions.

Due to small size of products under study, their dimensions and specificity of their use, it is assumed here that during the design phase of investigations the value of dissipative forces can be disregarded. (This assumption has been verified by calculation. The discrepancy in the results made 5 %).

The obtained results permit to foresee as early as during the design and development of future products possible cases of system resonance origination, take measures on “taking away” the values of natural oscillations from the values of resonance frequencies. It is particularly important in the development of responsible and expensive systems working on mobile carriers that have certain values of forced oscillations.

To verify the validity of calculated results, natural oscillation frequencies of the above TEC designs have been determined by independent approach, namely the method of electrodynamic analogies (EDA) known in theoretical mechanics.

For this purpose, the mechanical system under study is replaced by the respective electric analogies [3].

The electrical circuit of a 4-stage thermoelectric cooler will look as follows (Fig. 3):

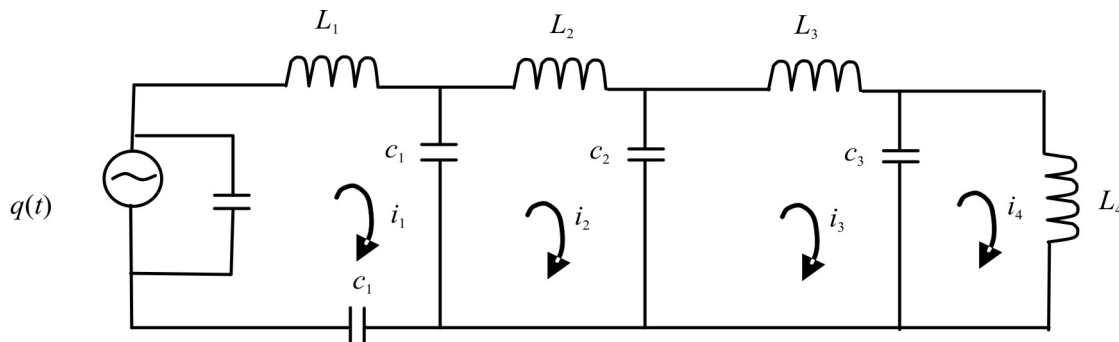


Fig. 3. Electric schematic of a 4-stage TEC.

The results obtained by using two independent design procedures for convenience of analysis and comparison are listed in Tables 1 – 4.

*Table 1*

*The values of natural oscillation frequencies of a 3-stage product (Hz)  
for  $n_1 = 124, n_2 = 54, n_3 = 24$  with regard to damping*

Number of thermoelements in a stage	124	54	24
Differential Lagrange's equations of the second kind	$2.177 \cdot 10^4$	$4.027 \cdot 10^4$	$5.50 \cdot 10^4$
EDA	$2.22 \cdot 10^4$	$4.234 \cdot 10^4$	$5.76 \cdot 10^4$

*Table 2*

*The values of natural oscillation frequencies of a 3-stage product (Hz)  
for  $n_1 = 124, n_2 = 54, n_3 = 24$  without regard to damping*

Number of thermoelements in a stage	124	54	24
Differential Lagrange's equations of the second kind	$2.949 \cdot 10^6$	$5.455 \cdot 10^6$	$7.45 \cdot 10^6$
EDA	$3.113 \cdot 10^6$	$5.878 \cdot 10^6$	$7.821 \cdot 10^6$

Analysis of the data in Tables 1 and 2 testifies that the spread in the values of natural frequencies is a function of the number of elements in a stage and varies within 1.94 % – 4.51 % (for products with regard to damping). At calculation without regard to damping this spread lies accordingly within 5.27 % – 5.19 %. With a reduction of the number of thermoelements in the stage, in both cases the spread in the obtained results is increased. Account of damping yields the values of natural frequencies, like in the previous cases, two orders lower, which is attributable to considerable effect of dampers on the values of natural vibration frequencies of the product. This makes it possible to “control” the values of natural vibration frequencies, varying them as required by the designer.

*Table 3*

*The values of natural oscillation frequencies (Hz) for a system  
with 4 degrees of freedom with regard to damping*

Number of thermoelements in a stage	124	54	24	12
Differential Lagrange's equations of the second kind	$1.797 \cdot 10^4$	$3.187 \cdot 10^4$	$4.682 \cdot 10^4$	$5.741 \cdot 10^4$
EDA	$1.842 \cdot 10^4$	$3.324 \cdot 10^4$	$4.791 \cdot 10^4$	$5.913 \cdot 10^4$

*Table 4*

*The values of natural oscillation frequencies (Hz) for a system  
with 4 degrees of freedom without regard to damping*

Number of thermoelements in a stage	124	54	24	12
Differential Lagrange's equations of the second kind	$2.434 \cdot 10^6$	$4.318 \cdot 10^6$	$6.344 \cdot 10^6$	$7.774 \cdot 10^6$
EDA	$2.033 \cdot 10^6$	$4.93616 \cdot 10^6$	$6.872 \cdot 10^6$	$8.234 \cdot 10^6$

Analysis of the data in Tables 2, 3 testifies that the spread in the values of natural oscillation frequencies obtained by two independent methods with regard to damping lies within 2.44 % – 2.91 %, and on condition that damping is disregarded by calculation procedure, these values will be 6.49 % – 5.58 %.

## **Conclusions**

The results obtained by two independent methods showed the repeatability of 3 – 8 %, which confirms the quality and validity of mathematical models used in the paper.

Analysis and comparison of calculated results with the use of two independent procedures make it possible to estimate as early as during design phase possible values of natural oscillation frequencies of specific products used in practice, as well as those which are planned to be produced and used.

The elaborated algorithms and calculation programs on personal computers make it possible to determine natural oscillation frequencies of various TEC configurations and design, to estimate the advisability of using different embodiments of designed products, to determine the most acceptable variant for each type of equipment carrier. Permanent development and the advent of new structural materials used in electronics allows prompt selection during design phase of necessary material combinations for each specific device, assuring quality and functionality of equipment and complicated technical systems.

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