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OPTIMAL CONTROL OF TIME DEPENDENCE OF COOLING TEMPERATURE IN THERMOELECTRIC DEVICES

We consider a physical model of thermoelement in the unsteady-state cooling mode, on the cold surface of which account is taken of the volumetric heat capacity of connecting and insulating plates, heat release of the object, heat exchange with the environment, the Joule heat release on the contact resistances between thermoelectric material and metal interconnects, as well as the influence of the Thomson effect in the bulk of thermoelement legs. A method for calculation of optimal time dependences of thermoelement supply current ensuring the prescribed time dependences of cooling temperature is described. Examples of computer simulation results of optimal current control functions for given continuous, piecewise continuous and periodic functions of cooling temperature versus time are provided.

Key words: thermoelement, unsteady-state thermoelectric cooling process, time dependences of temperature, current control functions.

Introduction

The thermoelectric cooling method is widely used to provide thermal modes for a variety of electronic, medical and measuring devices. In some cases, such devices operate in dynamic modes, which require changing the temperature of cooling (heating) the object in accordance with the prescribed time law. The thermoelectric cooler makes it easy to realize such dynamic modes by controlling the temperature of cooling (heating) due to time variation of thermoelement supply current.

The actual problem of thermoelectric cooling or heating control lies in the establishment of optimal time dependence of supply current that would ensure the prescribed time dependence of the operating surface temperature of thermoelements. This problem relates to the problems of control synthesis of the process of unsteady-state thermoelectric cooling or heating.

Analysis of scientific information [1 – 10] shows that published results of theoretical research on the unsteady-state cooling process mainly concern determination of temperature behavior in thermoelement powered from current pulses of certain prescribed shape. As regards control synthesis problems related to search for optimal time functions of current for the unsteady-state cooling and heating processes, ways to solve them have not been adequately studied.

The purpose of the work was to develop the algorithm and computer means for the calculation of optimal time dependence of thermoelectric converter supply current ensuring in accordance with the prescribed law a change in the temperature of object which is cooled or heated by this converter.

The problem of cooling temperature control synthesis in the unsteady-state mode is related to solving two questions. First of all, it is necessary to be able to determine whether or not the prescribed time dependence of cooling temperature can be realized, since the possibility of achieving the prescribed temperature within certain time is related to restricted fast response of thermoelectric cooler. Answer to this question is given by solving optimization problem of finding minimum cooling temperature during certain time interval. Computer methods of solving this problem were proposed in [11, 12]. In these works it is shown which minimal temperature is achieved in the unsteady-state cooling mode during different time intervals with the aid of single-stage and two-stage thermoelectric coolers.

The second question lies in finding the algorithm according to which for the prescribed time function of cooling temperature $T_c(t)$ the shape of control current pulse $I(t)$ is determined. The methods of solving such problem were considered in [13, 14]. The results were obtained for the approximate physical models of thermoelectric cooler in the unsteady-state mode. Only one thermoelectric leg was considered, whose material parameters, namely the Seebeck coefficient α, coefficient of resistivity ρ, coefficient of thermal conductivity κ and volumetric heat capacity c, do not depend on temperature. In [13], on the cold surface of leg, only the Peltier heat absorption is taken into account, disregarding the heat capacity of cooled object, the Joule heat release on the contact resistance between thermoelectric material of the leg and metal interconnects which have a significant impact on cooling temperature in a dynamic mode [15]. In [14], the influence of these factors is taken into account, but the intensity of active heat release of cooled object in the process of unsteady-state cooling is disregarded, which is also important in practical application. Therefore, to find a relationship between given temperature function $T_c(t)$ and control current $I(t)$, we will use the specified physical model of thermoelement.

Physical model of thermoelement in the unsteady-state mode and its mathematical description

Schematic of thermoelement in the unsteady-state cooling mode is shown in Fig. 1. The thermoelement legs of height l and cross-section s are made of n- and p-type conductivity materials. Characteristics of leg materials, namely the Seebeck coefficient $\alpha_{n,p}(T)$ and coefficient of resistivity $\rho_{n,p}(T)$ are temperature-dependent, and coefficient of thermal conductivity $\kappa_{n,p}$ and coefficient of heat capacity $c_{n,p}$ will be assumed to be constants due to their inessential dependence on temperature in thermoelectric materials for coolers.

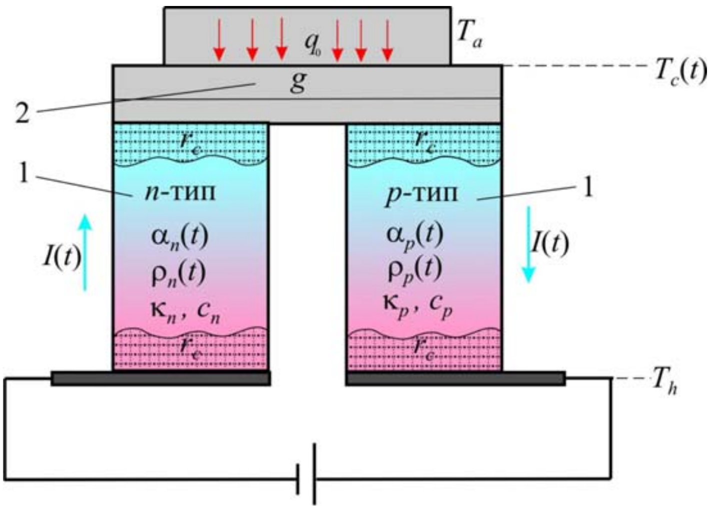

Fig. 1. Schematic of thermoelement in the unsteady-state cooling mode.
1 – thermoelement legs, 2 – lumped volumetric heat capacity
g of connecting and insulating plates of cooled object.

It is assumed that heat-releasing surface of thermoelement is maintained at fixed temperature T_h, the lateral surface of legs is adiabatically insulated. At the cold junction of thermoelement we take into account absorption of the Peltier heat, release of the Joule heat on the junction contacts with contact resistance r_c, lumped volumetric heat capacity g of connecting and insulating plates and cooled object, heat exchange between the cold surface and the environment with temperature T_a, as well as active heat release of cooled object of power q_0.

For such a model temperature distribution in thermoelement legs is assigned by a system of one-dimensional equations of unsteady-state heat conduction given by

$$\begin{cases} c_n \dfrac{\partial T_n}{\partial t} = \kappa_n \dfrac{\partial^2 T_n}{\partial x^2} + \rho_n(T)\dfrac{I^2(t)}{s^2} - T_n \dfrac{\partial \alpha_n(T)}{\partial T_n}\dfrac{I(t)}{s}\dfrac{\partial T_n}{\partial x} \\ c_p \dfrac{\partial T_p}{\partial t} = \kappa_p \dfrac{\partial^2 T_p}{\partial x^2} + \rho_p(T)\dfrac{I^2(t)}{s^2} - T_p \dfrac{\partial \alpha_p(T)}{\partial T_p}\dfrac{I(t)}{s}\dfrac{\partial T_p}{\partial x} \end{cases}, \tag{1}$$

where $x \in [0, l]$, $t \in [0, t_{\max}]$. $I(t)$ is current in thermoelement legs which is a function of time. Eqs.(1) take into account the Thomson effect which arises in the bulk of thermoelement legs due to temperature dependence of the Seebeck coefficient $\alpha_{n,p}(T)$.

The boundary conditions for these equations are of the form:

$$\left[\kappa_n s \frac{\partial T_n}{\partial x} + \kappa_p s \frac{\partial T_p}{\partial x}\right]_{x=0} - \left[\alpha_p(T_c(t)) + \left|\alpha_n(T_c(t))\right|\right] I(t) T_c(t) -$$

$$- g\frac{\partial T_c(t)}{\partial t} - 2sH\left(T_c(t) - T_a\right) + 2\frac{r_c}{s}I^2 + q_0 = 0 \tag{2}$$

$$T_n(0,t) = T_p(0,t) \equiv T_c(t)$$

$$T_n(l,t) = T_p(l,t) \equiv T_h$$

where $T_c(t)$, the temperature of thermoelement cold surface, is the prescribed function of time, H – coefficient of convective heat exchange in the environment.

Initial temperature distribution in the legs corresponds to steady-state distribution with the initial value of current I_0 and is given as a function

$$T_{n,p}(x,0) = C_0 I_0^2 x^2 + C_1 x + C_2 . \tag{3}$$

where C_0, C_1 and C_2 are constants determined by solutions of a steady-state problem of thermal conductivity in thermoelement legs at constant current I_0.

Under conditions $I_0 = 0$ A, $T_h = T_a$ the initial conditions of the boundary problem (1) – (2) will have a simple form

$$T_n(x,0) = T_p(x,0) \equiv T_a . \tag{4}$$

The problem is to find current control function $I(t)$ such that would ensure the prescribed time dependence of cold temperature $T_c(t)$.

Solving the problem of optimal control synthesis

To solve the formulated problem, we use the averaged parameter values of thermoelement leg materials, namely

$$\alpha = (\overline{\alpha_p} + \left|\overline{\alpha_n}\right|)/2;\ \ \rho = (\overline{\rho_p} + \overline{\rho_n})/2;\ \ \kappa = (\kappa_p + \kappa_n)/2;\ \ c = (c_p + c_n)/2\,. \tag{5}$$

Note that in the first approximation the influence of the Thomson effect in Eqs.(1) can be taken into account by the arithmetic averaging of the Seebeck coefficients in the operating temperature range [16], while for coefficients of resistivity it is advisable to use the integral averaging [16]. Then the average values of these coefficients in (5) are determined as follows:

$$\overline{\alpha}_{n,p} = (\alpha_{n,p}(T_h) + \alpha_{n,p}(T_c(t))/2\,, \tag{6}$$

$$\overline{\rho}_{n,p} = \frac{1}{(T_h - T_c(t))}\int_{T_c(t)}^{T_h} \rho_{n,p}\, dT\,. \tag{7}$$

Such approximations allow solving instead of problem (1) – (3) for thermoelement a similar initial boundary problem of unsteady-state thermal conduction for a single leg with averaged according to (5) – (7) thermoelectric parameters α, ρ, κ and c.

Using the Laplace transform method, as it was proposed in [14], we obtain the relation between given function of cooling temperature $T_c(t)$ and current control function $I(t)$ in the form

$$I(t) = \frac{1}{\alpha T_c(t)}\left[\frac{r_c}{s}I^2(t) + \frac{\kappa}{c}\frac{\rho}{sl}\int_0^{at} K(t-\tau)I^2(\tau)d\tau + \Phi(t, T_c(t))\right], \tag{8}$$

where

$$\Phi(t, T_c(t)) = -g\frac{dT_c(t)}{dt} - HsT_c(t) - \frac{r_c}{s}I_0^2 + A - \frac{\kappa s}{l}\int_0^{at} \vartheta_1(t-\tau)\frac{dT_c(\tau)}{d\tau}d\tau - \frac{\kappa}{c}\frac{\rho}{sl}I_0^2\int_0^{at} K(\tau)d\tau\,,$$

$$A = (\alpha I_0 + Hs)\frac{\frac{\kappa s}{l}T_h + HsT_a + q_0 + (0.5 + {}^{r_c}\!/_{\rho l})\frac{\rho l}{s}I_0^2}{\alpha I_0 + Hs + \frac{\kappa s}{l}}\,,$$

$$a = \frac{\kappa}{cl^2}\,,\quad K(t) = \vartheta_1(t) - \vartheta_0(t)\,,\quad \vartheta_1(t) = 1 + 2\sum_{k=1}^{\infty}\exp(-\pi^2 k^2 at)\,,\quad \vartheta_0(t) = 1 + 2\sum_{k=1}^{\infty}(-1)^k\exp(-\pi^2 k^2 at)$$

Relation (8) is a nonlinear integral equation which is solved by numerical method of successive approximations. A fairly complex algorithm for solving such an equation is realized with the help of computer simulation software developed in the MathLab environment.

Computer simulation results

Simulation of current control functions ensuring the prescribed time dependences of cooling temperature was performed for thermoelements the legs of which are made of *Bi-Te* based materials of *n*- and *p*-type conductivity with standard thermoelectric characteristics $\alpha_{n,p}$, $\rho_{n,p}$, $\kappa_{n,p}$, $c_{n,p}$ [17]. Calculations were performed for the legs of height $l = 0.14$ cm, cross-section area $s = 0.1 \times 0.1\text{cm}^2$, with contact resistance value $r_c = 5\cdot10^{-6}$ Ohm·cm^2. Account was taken of heat exchange between the heat-absorbing surface of thermoelements and the environment of temperature $T_a = 300$ K, with the heat exchange coefficient $H = 10^{-3}$ W/cm^{2}K. The lumped volumetric heat capacity of connecting and insulating plates and cooled object was equal to $g = 0.002$ J/K. Considered were variants of modules operation in the mode without thermal load, that is, the value of heat release of cooled object was assumed equal to $q_0 = 0$, and with the load per one leg $q_0 = 0.03$ W.

Figs. 2 – 4 show examples of the prescribed time dependences of cooling temperature $T_c(t)$ and the respective control functions of thermoelement supply current $I(t)$ calculated for them. Fig. 2 gives dependences $I(t)$ ensuring continuous temperature reduction to the prescribed value within the prescribed time interval.

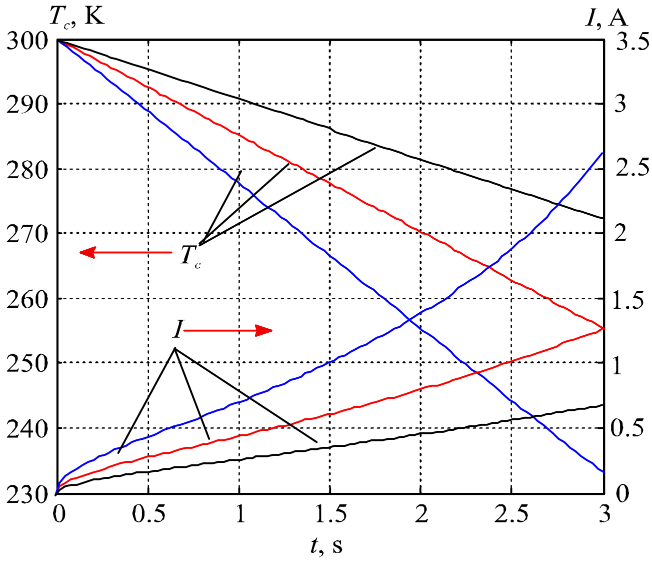

Fig. 2. The prescribed time dependences of cooling temperature $T_c(t)$ and the respective control functions of thermoelement supply current I(t). $q_0 = 0$.

Fig. 3 shows examples of piecewise linear function of temperature versus time $T_c(t)$ and their respective controls $I(t)$. It is apparent that the choice of such optimal control makes it possible to reduce the time required by thermoelectric device for reaching steady-state mode.

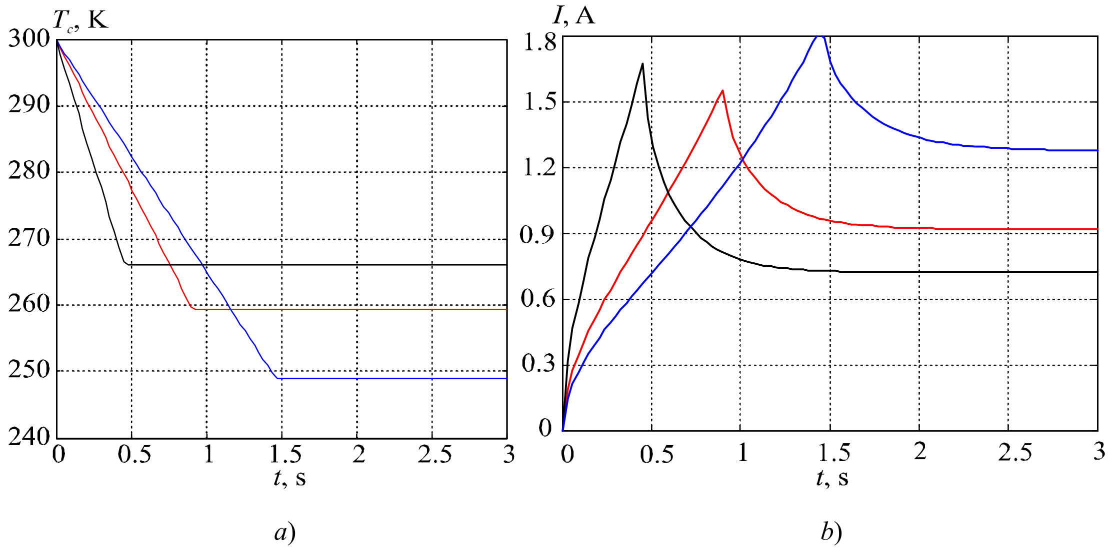

Fig. 3. a) The prescribed time dependences of cooling temperature $T_c(t)$ and b) the respective control functions of thermoelement supply current I(t). $q_0 = 0$.

Fig. 4 shows a periodic change in cooling temperature which is ensured by the respective periodic control function of thermoelement supply current.

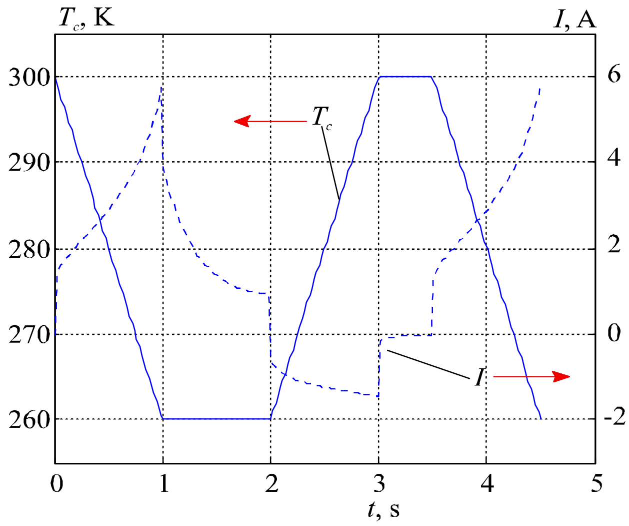

Fig. 4. The prescribed time dependences of cooling temperature $T_c(t)$ and the respective control functions of thermoelement supply current $I(t)$. q_0 = 0.03 W.

Conclusions

1. The developed computer simulation method allows determination of optimal time functions of thermoelement supply current control ensuring the prescribed dependences of change in cooling temperature with time. It should be noted that there are practically no experimental methods which would allow finding similar functions for thermoelement current control.
2. Establishment of the time dependences of current optimal for specific purposes is of important practical significance. Such functions are used for the design and autocalibration of special electronic regulators which are necessary to ensure the operation of system for automatic control of unsteady-state cooling process in thermoelectric devices of various applications.

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