Calculation of heat transfer in fluid around gas-vapour bubbles

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Abstract
Marine gas hydrates are considered the most probable alternative fuel in many countries. Their exploration actively engages specialists from France, Germany, the USA, Canada and Japan [1-9]. The Japanese plan to start commercial production of methane from the "Ice fuel" around their islands in the basin Nyanhay in 2016. However, effective technology acquisition, storage and transport of methane gas hydrates is still in the development stage [10-15]
Key words: HEAT AND MASS TRANSFER, MATHEMATICAL MODEL, THERMAL CONDITIVITY, HEAT INSULATION

Introduction

There are a number of technological processes which are based on the use of gas-vapor bubbles, surface cleaning, cavitation, homogenization fuel mixing colloidal solutions, water degassing, distillation of petroleum products, foaming in the food industry and in the manufacture of thermal insulation materials, transportation of natural gas as a hydrate and many others. Typically the formation and existence of gas-vapor bubbles are accompanied by intense heat and mass transfer processes at phase interface. The small size surfaces and significant speed of these processes led to the widespread use of mathematical modeling for their research. Mathematical models can determine the most influential factors and optimize technological processes. For an exact description of the various hydrodynamic, heat exchange and mass transfer processes, a mathematical model must take into account the heat transfer in the fluid that surrounds the gas-vapor bubble.

Overview of recent sources of research and publications

For consideration of heat exchange processes on the boundary of separation medium mathematical model of gas-vapor bubbles should contain equations describing the heat transfer of fluid in the environment. In [1] the temperature of the liquid is described by an exponential function, which is independent of the time direction of the wall of bubbles. Some authors take the temperature of the liquid constant [2, 3], and thermodynamic processes in gas-vapor environment of bubbles of adiabatic. However, these assumptions are possible only to a very limited group of tasks. In works [4, 5] the analytical solution of the problem of unsteady heat conduction in the layer of fluid that surrounds oscillating bubbles. For obtaining solutions, the author suggested the following simplifying assumptions: finite liquid layer in which there heat exchange processes are of parabolic nature of the temperature distribution in the thickness of this layer. Due to these assumptions, in the resulting solution of this problem, the heat transfer is not dependent on thermal and physical characteristics of the fluid. In work [6] process of heat transfer in the liquid is not considered, and its temperature is determined only on the basis of the combined boundary conditions. Some authors [7] consider heat transfer in such a thin layer of fluid that the curvature of the surface of the bubble can not be ignored.

Selection of not solved earlier parts of the general problem

To improve the accuracy of mathematical models of gas-vapor, bubbles must be considered in the process of heat transfer fluid surrounding the gas-vapor bubble. The feature of this problem is the movement of the walls of bubbles, the rate of which, in certain moments of time can be up to several hundred meters per second. Temperature mode of gas inside the bubbles also varies widely. Consequently, the thermophysical characteristics of fluid on the boundary of the bubble can also significantly vary.

Problem statement

The aim of this work is to create a digital mathematical model of heat transfer in the fluid around the vapor bubbles, which change its size. The liquid should have variable thermophysical characteristics.

Basic material and results

For the development of mathematical models of heat transfer fluid, the following simplifying assumptions are used:

- gas-vapor bubble has a spherical shape;
- at the surface of bubbles boundary conditions of the second kind are considered.

To determine the temperature on the inner surface of the bubbles, the process of heat transfer fluid should be considered. It is done by heat conduction and convection. For calculation of heat transfer by heat conduction, Fourier heat equation is usually used. In order to take account of convection, effective coefficient of thermal conductivity can be used.

Let us denote *«x»* coordinate, at which the radius of bubbles changes. To determine the unknown temperature on the surface, bubbles can apply nonlinear heat equation Fourier bullet considering the mobility of its walls [1]

$$\frac{\partial\left(\rho_r c_r T_{(x,\tau)}\right)}{\partial\tau}+\dot{x}\frac{\partial\left(\rho_r c_r T_{(x,\tau)}\right)}{\partial x}=\frac{1}{x^2}\frac{\partial}{\partial x}\left(\lambda_r x^2\frac{\partial T_{(x,\tau)}}{\partial x}\right), \quad (1)$$

where $\dot{x}-$ the rate of change of the radius bubbles, m/s; $\lambda_r -$ effective coefficient of thermal conductivity of fluid, W/(m°C); $c_r -$ heat capacity of fluid, J/(kg°C); $\rho_r -$ density, kg/m³. Considering the continuity of flow conditions when the same mass of liquid $\rho_r \dot{x} 4\pi x^2=\rho_r \dot{R} 4\pi R^2=const$ flows through any surface of bullet of radius $\boldsymbol{x}$ per unit time; equation (1) can be written as

$$\frac{\partial T_{(x,\tau)}}{\partial\tau}=\frac{1}{x^2}\frac{\partial}{\partial x}\left(\frac{\lambda_r}{\rho_r c_r}x^2\frac{\partial T_{(x,\tau)}}{\partial x}-\dot{R}R^2 T_{(x,\tau)}\right), \quad (2)$$

As a result of heat exchange processes in the border of bubbles liquid may change its thermophysical characteristics, so the problem will be solved as nonlinear.

Considering the fact that near the bubbles surface, specific heat flux (q) is known, let us write the boundary condition of the second kind

$$-\lambda_r\frac{\partial T}{\partial x}(x=R,\tau)=q \quad (3)$$

In order to describe the thermal conductivity in the liquid around the bubbles, let us divide layer of liquid that surrounds bubble to a number of concentric shells. Let us define mass distribution of each shell

$$m_{r(2)}=2K_r m_{r(1)}$$

$$m_{r(i)}=K_r m_{r(i-1)}$$

where $m_{r(1)}-$ mass of shell of the 1st (inner) layer; $m_{r(i)}$ – mass of each one of the next shell; K_r – coefficient of proportionality. This coefficient is used to optimize its calculations and its typical values are within 1.5÷2. It is chosen so as to achieve maximum speed values for a given accuracy.

Let us determine the temperature on the inner surface of the first (inner) shell. Then, in the inner shell, which has a boundary condition differential equation will be of the form

$$m_{r(1)}c_{r(1)}\frac{dT_{(R,\tau)}}{d\tau}=-F_Rq-\frac{4\pi\lambda_{r(1)}\left(T_{(R,\tau)}-T_{(R+\delta 1)}\right)}{\frac{1}{r_{(R)}}-\frac{1}{r_{(R+\delta 1)}}}. \qquad (4)$$

For ease of calculation, let us denote

$$K_1=\frac{\lambda_{r(1)}}{\frac{1}{r_{(R)}}-\frac{1}{r_{(R+\delta 1)}}}=\frac{\lambda_{r(1)}}{\frac{1}{R}-\frac{1}{R+\delta 1}}. \qquad (5)$$

Now the differential equation that determines the temperature on the surface of the inner layer of bubbles is of the form

$$\frac{dT_{(R,\tau)}}{d\tau}=\frac{4\pi}{m_{r(1)}c_{r(1)}}\left(-R^2q-K_1\left(T_{(R,\tau)}-T_{(R+\delta 1)}\right)\right). \qquad (6)$$

In the absence of mass transfer processes mass of 1st layer remains intact, because

$$m_{r(1)}=\frac{4}{3}\pi\rho_{r1}\left[\left(R+\delta 1\right)^3-R^3\right]=const$$

Where the outer radius of the 1st shell can be defined

$$R+\delta 1=\sqrt[3]{r_R^3+\frac{3m_{r(1)}}{4\pi\rho_{r1}}}=\sqrt[3]{R^3+\frac{3m_{r(1)}}{4\pi\rho_{r1}}}. \qquad (7)$$

For all next shells, the following equation can be written

$$m_{r(i)}c_{r(i)}\frac{dT_{(x,\tau)}}{d\tau}=\frac{4\pi\lambda_r\left(T_{(i-1)}-T_{(i)}\right)}{\frac{1}{r_{(i-1)}}-\frac{1}{r_{(i)}}}-\frac{4\pi\lambda_r\left(T_{(i)}-T_{(i+1)}\right)}{\frac{1}{r_{(i-1)}}-\frac{1}{r_{(i)}}}. \qquad (8)$$

In this task, let us replace the temperature of the conditioned layers by difference between the mean temperature of the layer and its edges (borders).

$$\frac{dT_{(r_i,\tau)}}{d\tau}=\frac{4\pi}{m_{r(i)}c_{r(i)}}\cdot\left(K_3\left(T_{(r_i-\delta i,\tau)}-T_{(r_i,\tau)}\right)-K_1\left(T_{(r_i,\tau)}-T_{(r_i+\delta i,\tau)}\right)\right). \qquad (9)$$

The temperature at the external borders of i-th shell

$$T_{(r_i+\delta i,\tau)}=\frac{K_1T_i+K_2T_{i+1}}{K_1+K_2}. \qquad (10)$$

The temperature on the inner border

$$T_{(r_i-\delta i,\tau)}=\frac{K_4T_{i-1}+K_3T_i}{K_4+K_3}. \qquad (11)$$

The coefficients K_1, K_2, K_3, K_4 are determined by the following formulas:

$$K_1 = \frac{\lambda_{r(i)}}{\dfrac{1}{r_{(i)}} - \dfrac{1}{r_{(i)} + \delta_{(i)}}}, \qquad (12)$$

$$K_2 = \frac{\lambda_{r(i+1)}}{\dfrac{1}{r_{(i)} + \delta_i} - \dfrac{1}{r_{(i+1)}}}, \qquad (13)$$

$$K_3 = \frac{\lambda_{r(i)}}{\dfrac{1}{r_{(i)} - \delta_{(i)}} - \dfrac{1}{r_{(i)}}}, \qquad (14)$$

$$K_4 = \frac{\lambda_{r(i-1)}}{\dfrac{1}{r_{(i-1)}} - \dfrac{1}{r_{(i)} - \delta_i}}. \qquad (15)$$

Medium-radius of i-th shell is given by

$$r_{(i)} = \sqrt[3]{r_{(i-1)}^3 + \frac{3}{8\pi}\left(\frac{m_{r(i-1)}}{\rho_{r(i-1)}} + \frac{m_{r(i)}}{\rho_{r(i)}}\right)}. \qquad (16)$$

Since the mass of the first shell is not divided in half, for the 2nd shell radius, mass is determined by the following formula

$$r_{(2)} = \sqrt[3]{R^3 + \frac{3}{4\pi}\frac{m_{r(1)}}{\rho_{r1}} + \frac{3}{8\pi}\frac{m_{r(2)}}{\rho_{r2}}}. \qquad (17)$$

The outer radius of i-th shell

$$r_{(i)} + \delta_i = \sqrt[3]{r_{(i)}^3 + \frac{3m_{r(i)}}{8\pi\rho_{r(i)}}}. \qquad (18)$$

The inner radius of i-th shell

$$r_{(i)} - \delta_i = \sqrt[3]{r_{(i)}^3 - \frac{3m_{r(i)}}{8\pi\rho_{r(i)}}}. \qquad (19)$$

The differential equations (6) and (9) were solved by the method of Runge-Kutta 4th order. In order to assess the adequacy of the developed mathematical model a computer program was written and a series of mathematical experiments were performed.

Output data

The duration of estimated time interval is 100 ns (nanoseconds). Time step - 0,001 ns, specific heat flux - 10 MWt/m². The initial diameter of the bubbles - 0.1 mm, initial temperature of the water - +5 °C. 10 layers were calculated, coefficient - K_r= 1.5. Thermal conductivity, density and heat capacity of water were held constant at temperature +5°C.

Figures 1 and 2 show the results of calculation of cooling wall bubbles under constant specific heat flow conditions for increasing its radius at speeds $\dot{R} = 100$ m/s. In Fig. 3 and 4, the speed was 50 m/s. Obtained results show that during the compression of bubbles changing temperature conditions are changed just closest to the interfacial boundary of layers of fluid. “Depth” heat wave penetration is about 0.1 of the initial radius of bubbles. When expanding bubbles, temperature conditions surrounding the layer at a distance of more than 3 values of the initial radius of the bubble are changed. When reducing the size of bubbles, the total heat flow decreases, resulting in slower heat exchange processes (Figure 3).

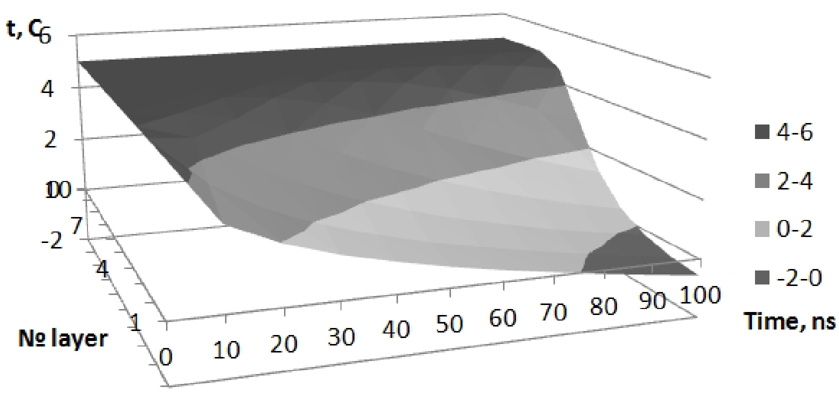

Figure.1. Diagram of temperature fields in the water that surrounds bubble (phase transition in water is not considered). The calculation results for $\dot{R} = 100$ m/s

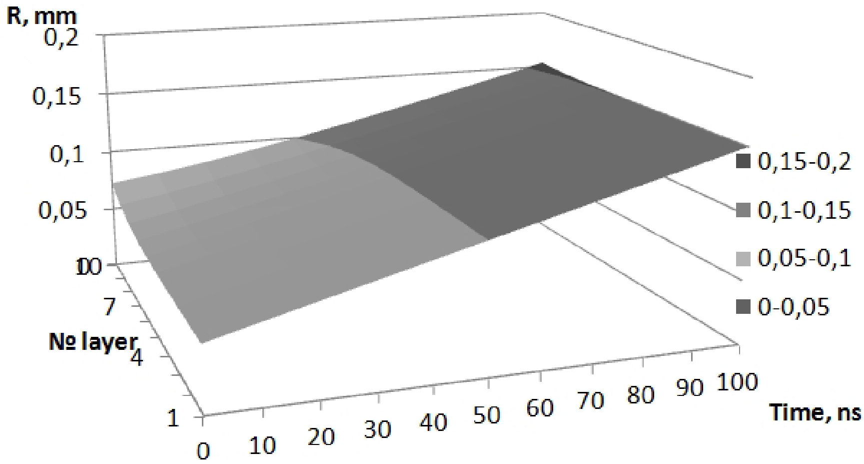

Figure 2. The diagram of radius changes in the calculation of layers at $\dot{R} = 100$ m/s

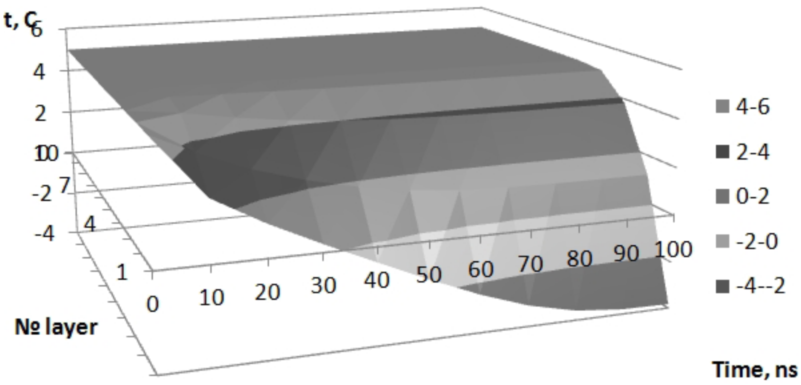

Figure 3. The diagram of temperature fields in the water that surrounds bubble (phase transition in water is not considered). The calculation results for $\dot{R} =$ -50 m/s

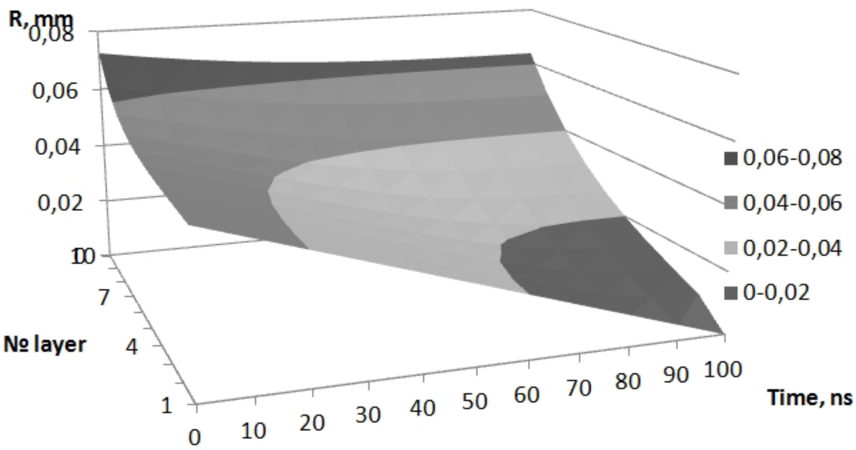

Figure 4. The diagram of radius changes in the calculation of layers at $\dot{R} =$ -50 m/s

Conclusions

The mathematical model for calculating heat transfer in the fluid that surrounds the oscillating gas-steam bubble was developed. The model takes into account the changing thermal and physical characteristics of the liquid, changing the size of bubbles, heat exchange processes at its border. Computer program for the calculation of this mathematical model was created. Distribution of temperature fields in the liquid during the transition was obtained. The proposed calculation method can be used to determine the thermal and physical characteristics of fluid and vapor in a variety of technological processes related to boiling fluid cavitation and the formation of gas hydrates.

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