MULTIPURPOSE MEASUREMENT MODELS FOR ADJUSTMENT BY THE LEAST-SQUARES METHOD

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Abstract. The development of multipurpose measurement models is the precondition for software development for simultaneous adjustment of the large scope and complicated combinations of the measurement results by the least-squares method. Multipurpose measurement models for software can be a helpful tool for processing the final measurement results provided by different measurement methods applying the mentioned software; processing the measurement results of measurement standards comparisons, interlaboratory comparison, and calibration procedures; estimating the additive and multiplicative systematic components of measurement errors and their uncertainty; processing complicated combinations by binding or linking up of the interlaboratory comparison and calibration results in the time; simultaneous processing of the measurement results obtained by various methods e.g. by the method of direct measurements and comparisons; fast-changing the multipurpose measurement models from linear to non-linear type. Processing of the results by software based on the multipurpose measurement model algorithm can help to established a comprehensive measurement traceability network by pooling the single traceability chains.

Key words: Multipurpose measurement models; Least-squares method; Measurement subjects and objects; Comprehensive measurement traceability network; Uncertainty.

1. Introduction

Processing of the measurement results during comparisons, interlaboratory comparisons, and calibrations is rather a labor-intensive process. Authors are strongly confident that the low automatization level of the measurement results processing deters the quantity increase for example of the measurement standards maintained in the laboratory and traveling measurement standards, that can be involved in the comparisons. Microsoft Excel processing software considerably simplifies the results processing, but it is not suitable for solving a large system of equations (with the number of equations above 100). Special software for data bulk of measurement results processing by the least-squares method should be developed to overcome this problem.

The results of any measurements are the quantity values and their uncertainty. Notwithstanding the procedure according to which these results are provided – measurement standards comparison, interlaboratory comparisons, or calibration. Results provided by almost any measurement method or submethod can be represented by several measurement models. These models are called here multipurpose. Software development based on these multipurpose models is rather important.

2. The rationale for multipurpose measurement models development

Vocabulaire International de Metrologie (VIM) [1] provides a general definition of the measurement model: "2.48 Mathematical relation among all quantities known to be involved in a measurement". It is necessary to clarify here that the multipurpose measurement model does not depend on the measured quantity type but depends on the measurement method.

The main and widespread methods are direct quantity values measurement, direct measurements of increment quantity values, comparisons, etc.

In [2–6], the model of direct quantity value measurement during the measurement standards comparison is defined. The defined model includes only additive measurement standards degree of equivalence (though it is not called so in the explicit form) as the estimation of systematic component of the measurements error. There is a detailed description of the least-squares method estimation model of more than only one additive component in [7]. A multiplicative degree of equivalence of the measurement standard is added to the model as an estimation of the relative systematic measurement error component. It means, that this model, estimates the systematic component of the error linearly dependent on the measured quantity value.

Proposed in [2] algorithm of the comparison's measurement results processing is used for evaluation of the measured quantity value obtained for the single artifact in the certain point by several measurement standards. The measurement results uncertainty estimation of the certain value according to [8] is a partial case of adjustment according to the least-squares method.

It is considered in [9] the adjustment by the least-squares method of the length measurement results using electronic tachometer by increment quantity value measurement model. This model does not include systematic measurement components. For the first time, the model with additive and multiplicative systematic measurement error components of the length measurement using an electronic tachometer was introduced in [10].

The adjustment by the least-squares method of the mass measurement results using the comparison model is described in detail in [11]. This measurement model [11] does not include additive and multiplicative systematic

error components analysis of the involved mass comparators.

Another special measurement model was used in [12] for processing the direct measurement results comparison of two measurement standards or devices. It is assigned for the exclusion of the additive systematic error component of the measurement by these devices for the evaluation of the multiplicative component.

Processing by the least-squares method using multipurpose measurement models introduced below is similar to the multivariable model [13] regarding the estimation of the measurements uncertainty and processing results in general. Multipurpose measurement models are not considered in [13] from the same point of view as introduced below.

In the draft [14], there are very detailed recommendations with various examples on how to form the specific (basic) measurement models for their methods and sub-methods. However, there is no reference to the multipurpose measurement models, general for various measurement methods and sub-methods, which can be used for the final processing stage. This draft refers to recommendations regarding the measurement models development for the previous stages, i.e. before the final measurement value and its uncertainties were obtained.

The source analysis demonstrates that the additive measurement standards degree of equivalence (though it is not called so in the explicit form) is used for the measurement standards characteristics. The general method for obtaining additive and multiplicative measurement standards degree of equivalence and related uncertainties according to the least-squares method is described in detail in [7]. As shown below, exactly these two the most important values (not only additive) form the basis of all multipurpose models.

3. The Aim of the Paper

The current research aims to develop multipurpose measurement models as the precondition for special software development for simultaneous adjustment of the large number and combinations of the measurement results.

4. The Measurement Models Consideration

To reach this aim the developed models should be able to:

– process the final measurement value received using various measurement methods and sub-methods by the same software;

– process the measurement results of the measurement standards comparison, interlaboratory comparisons, and calibrations provided in different periods;

– estimate additive and multiplicative systematic components of the measurement error and their uncertainties using every measurement standard or device;

– process any complicated combinations by binding or linking up the measurement results of the interlaboratory comparisons and calibrations;

– the process simultaneously the measurement results obtained by various methods e.g. by the method of direct measurements and comparisons;

– change multipurpose measurement models from linear to non-linear type quickly.

Without the multipurpose measurement models, the software will not be able to simultaneously process the measurement results for example performed on hundreds of the measurement standards maintained in the different laboratories, with hundreds of the traveling measurement standards plying among them in various combinations. The number of the simultaneously processed measurement results may reach tens of thousands and even hundreds of thousands. Such a procedure of simultaneous processing of the measurement results is called adjustment by the least-squares method. This procedure appears when the measurement model equation during the processing (adjustment) results together with the measurement results include both parameters of the measurement standard, which is constantly maintained in the laboratory, and the traveling measurement standard. For example, in the case of the measurement standards comparison, the measurement model should include the reference value of the artifact and the additive or/and multiplicative measurement standards degree of equivalence.

It is hard to enumerate all the advantages of this method. Nevertheless, some general arguments in its favor are set below.

The absence of the special software for measurement results processing automatization compels to limit the number of measurements and the number of measurement standards, which are used for the measurement comparisons, correspondingly.

Measurement results should be processed considering the correlation links, which appear during the measurement process. This leads to the complication of the measurement models, as well as the expansion in the number of linear equations. Only special software based on the multipurpose measurement model can process them. The software's absence compels to simplify the measurement models often limited to calculation of the mean or the weighted mean and bias from it. Due to that fact, valuable information concerning estimations of systematic measurement errors is often lost.

The systematic measurement error components estimation is one of the most important tasks of the comparison, interlaboratory comparisons, and calibrations. They should be reliably estimated utilizing processing the large scope of the measurement results provided by a large number of devices. The measurement results in

biases as systematic error estimations should be included in the equation in the process of adjustment as the additional unknown quantity. The necessity of their estimation increases the number of unknown quantities in the equations. Reliable estimation of the systematic components of measurement errors for a large set of measurements performed by homogeneous instruments becomes very problematic without special software.

The most important task is to provide measurement traceability. But who can guarantee its confidence and reliability if the results are processed in the manual "paper-and-pencil" way? Can human faults and misunderstandings be prevented when the results of comparisons and calibrations, in the form of corrections, are used manually for subsequent comparisons and calibrations? Only structured databases and software for its population and updating can solve this task. The database will be automatically populated by the software using the measurement comparison results on the initial stages of the adjustments results processing. These results will be automatically used as a reference for the next processing and the results of adjustment will be re-entered into the database for further use.

For the software to be used by as many laboratories as possible, it must be based on certain simple principles.

The first of the basic principles is that the measurement models embedded in the software should be **multipurpose**. These models should be suitable for the final measurement results processing by different methods and sub-methods using various measurement equpment. The final result means the quantity value corrected by the necessary adjustments and its uncertainty.

The second of the main principles is that the measurement models and their implementation in the software should be **flexible**. This means easily configuring software-based for the measurement and computational task to be solved. It is also described below.

5. Multipurpose Linear Measurement Models

5.1 Rationale for the terms "measurement subject" and "measurement object"

In the process of the research results preparation, there was a need to introduce some new terms and their definitions. The main document by which the authors of the article were guided was VIM [1].

Generally accepted terms appeared to be not fully appropriate for the considered models' description. Thus, the generally used term "artifact" (for example [4, 5, 7]) is too general and unspecific and is absent in [1]. It means everything that is somehow human-made. This term usually refers to a traveling measurement standard. However, measurement standard, which is constantly kept in the laboratory and is used for measurement is also an artifact.

The term "traveling measurement standard" (5.8 VIM [1–2]) is too narrow on the contrary for the presented below research. It does not define how the measurements were performed and what the measurement result will be related to – to a mobile standard or to one that is stationarily maintained in the laboratory. For example, when comparing interferometers, they are standards – "measuring instruments" (3.1 VIM [1]). There is a set of gauges that circulates between the institutes. These gauges are the traveling measurement standards (in general terms, they are artifacts). Another example, during the comparisons of the electronic distance meters or tachometers, both in this case are the traveling measurement standards used as measuring devices. The measurements are performed on the stationary field comparator [10]. In this case, the field linear comparator is the measure (artifact) rather than the traveling standard.

To describe the multipurpose measurement models, it is necessary to define some new general terms such as "the subject of measurements" and "the object of measurements".

According to VIM [1], "2.1 measurement is the process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity".

For this study, it would be appropriate to supplement this definition with an additional one that does not contradict the main one: measurement is the process of interaction between the subject of measurement and the object of measurement, which results in a quantity value.

In the proposed definition:

Measurement subject is that one that realizes or reproduces the measurement value during measurements. Measurement subject corresponds to measuring instrument, measuring system, comparator, sensor, meter, etc.

Measurement object is one to which the quantity value based on the results of measurements is reasonably assigned. Measurement object corresponds to the single-valued or multivalued measure, material or material measurement standard, gaseous mixture measurement standard, measurement standard installation, for example, force machine (dead-weight force machine or the lever one), field linear comparator, environment, quantity parameters of which are measured, etc.

Based on the introduced terms and their definitions measurement subjects as well as objects should be both traveling and maintained in the laboratory. Thus, in the process of the adjustments and the interlaboratory comparisons and calibrations measurement subjects as well as objects may circulate between the laboratories. It depends on the particular measurement method and sub-method.

Based on the principles of the multipurpose measurement models and software, the authors do not take into account under which procedure measurements

were performed, either during measurement standards comparison, interlaboratory comparisons, and calibrations or in other ways. The main thing is that the independent measurement results become interdependent (correlated) with each other through the general parameters that are determined in the final result of processing. They become such when several measurement subjects perform measurements on the several same measurement objects, but the parameters of the measurement subjects should be included in the measurement model for the correlation to occur.

Taking into account the introduced terms, the ”vertical” metrological traceability chain (VIM 2.42[1]) through calibration can be defined in the generalized form as subject-object – subject’ – object’ – subject” (s – o – s’ – o’ – s”). For example, for the length measurements this chain can be as follows: interferometer – set of reference standard gauge blocks– gauge blocks comparator – set of working standard gauge blocks – micrometer. Another example of the traceability chain through calibration: object-subject – object’ – subject’ – object” (o – s – o’ – s’ – o”). In the case of force measurement this chain can be as follows: reference standard machine (dead-weight machine) – reference standard force transducer – working standard force machine (lever amplification or jockey-weight force machine) – working standard force transducer – compression testing machine.

The software, compiled according to universal measurement models, will allow to process of the measurement results of the homogeneous values and similar traceability chains regardless of the measurement method and sub-method. Nevertheless, the result at any stage of the traceability chain will be divided into two sets of numbers – a set of the measured value quantities and a set of uncertainties corresponding to these values.

Schematically, the similar, but “horizontal”, measurement traceability chains can implement the procedures of the measurement standards comparison and the interlaboratory measurement results comparisons. For example, in the chain s – o – s’ – o’ – s” or o – s – o’ – s’ – o” the measurement subjects can be the measurement standards or installations and the objects are measures or homogenous measurement standard samples.

That is, the strategical task of such software is particularly accurate mathematical binding or integration through the measurement results processing of any complicated combinations of key, regional and additional comparisons, interlaboratory comparisons, and calibrations into a unified **Comprehensive measurement traceability network.**

5.2 Direct quantity values measurement model

The easiest way to explain the principles of universality and flexibility on the example by an elementary and common model of direct measurements of a quantity (1). For the assigned task solution, it is necessary to clarify several important points. First of all, based on the mentioned principles, authors abstracted from the concrete procedures accepted in metrology such as comparison, interlaboratory comparison, calibration, validation, etc., and accepted in this sphere terminology.

The direct quantity value measurement model has the following form:

$$x_j^i = y^i + d_j + x_j^i \cdot b_j \,. \quad (1)$$

Here x_j^i is the quantity value measured by the measurement subject; y^i is the quantity value reproduced by the measurement object with the number $i = 1...i...n$ (measurement object parameter); d_j is an additive measurement subject parameter $j = 1...j...k$; b_j is the multiplicative measurement subject parameter $j = 1...j...k$.

Note. In equation (1) and the following measurement models equations, the upper indexes relate to the measurement objects and the lower indexes relate to the measurement subjects. This was made for the programming calculation convenience by these formulas.

Measurement object parameter y^i may be either key comparison reference value, the reference value for the lower level comparisons, assigned value in the process of the interlaboratory comparisons, or the result of the calibration of the measure.

Measurement subject parameters d_j and b_j may be either additive and multiplicative degrees of equivalence in the measurement process in [7], functional characteristics in the process of the interlaboratory comparisons in [15, 16, 17], additive and multiplicative measurement biases (2.18 VIM [1]) in the calibration process according to [18], or the corresponding corrections (2.53 VIM [1]) in the measurement process.

It is necessary to mention that d_j parameter can be interpreted as an estimate of the additive systematic error (bias) of the measured value at the zero points of scale reproduced by the measurement subject. Correspondingly, b_j the parameter can be interpreted as an estimate of the multiplicative systematic error (bias) of the measured quantity value, which is realized or reproduced by the measurement subject.

For the models considered, it is assumed that the parameters of the measurement objects are discrete character and the parameters of the measurement subjects are continuous and are described by the linear function (second and the third terms in the right part of the equation (1)). Using the multipurpose and flexibility principles, in the future, these measurement models can be modified by adding, in justified cases, new parameters of the measurement subjects.

Coming over from the equation (1) to correction equations can be obtained:

$$v(x_j^i) = \delta y^i + d_j + x_j^i \cdot b_j + l_j^i \quad (2)$$

or

$$v(x_j^i) = \delta y^i + \delta d_j + x_j^i \cdot \delta b_j + l_j^i,$$

where $v(x_j^i)$ is the measured quantity values correction; δy^i are the initial quantity values corrections, which are maintained by the measurement objects; δd_j, δb_j are the initial quantity values corrections of the measurement subjects; l_j^i are the constant terms of the correction equations.

The influence of measurement results on the results of adjustment is regulated by the weights coefficients (weights) of the measurement results:

$$w(x_j^i) = \frac{\sigma_0^2}{u^2(x_j^i)}, \quad (3)$$

where σ_0 is the standard deviation of the measurement, the weight of which equals 1; $u(x_j^i)$ is the standard measurement uncertainty.

The weight coefficient of the measurement result is a dimensionless quantity that is the relative measure of the accuracy of the measurement. Constant equation terms in (2) are calculated by the formulas:

$$l_j^i = -x_j^i \quad \text{or} \quad (4)$$

$$l_j^i = y^{0i} - x_j^i + d_j^0 + x_j^i \cdot b_j^0,$$

where y^{0i} are the initial measurement parameters of the measurement objects before the adjustment; d_j^0; b_j^0 are the initial measurement parameters of the measurement subjects before the adjustment.

If $d_j^0 = 0$; $b_j^0 = 0$, then for many equations of correction will be $l_j^i = y^{0i} - x_j^i$. If the biases from the nominal values are used as the parameters of the measurement objects and the measured quantities, then $l_j^i = -x_j^i$. After the adjustment of the measurement results, values and parameters are estimated by:

$$x_j^i = x_j^i + v(x_j^i); \quad (5)$$

$$y^i = y^{0i} + \delta y; \quad (6)$$

$$d_j = d_j^0 + \delta d_j; \quad b_j = b_j^0 + \delta b_j. \quad (7)$$

The equations (1) – (4) are the basic ones. Several common measurement models can be developed based on these equations. These models are defined below.

It should be noted that for the analysis of a particular measurement model, the software should be able to exclude certain parameters from the complete measurement model (1). For example, only measurement objects parameters can be left in the mode

This is the case when the parameters of the object are found as the average or weighted average of the measurement results. Either additive or multiplicative parameters may remain in the model.

5.3 Model of direct measurements of increment quantity values

The model of direct measurements of increment quantity values can be easily explained on the example of the electronic distance meters or tachometer comparisons using the field linear comparator. Such comparator is a multivalued length end gage, which includes several very stable columns for electronic distance meters or tachometer to be fixed on them [10]. Columns are set into a range it means that they are almost in a line. For the measurement results of the field linear comparator line length to be comparable, the distances measured in space should be reduced to the same surface, such as a geoid.

If there are no columns range, then the measured line length should be projected on one vertical plane.

On such a field linear comparator it is possible to measure the length of lines not only from the first column to all others but from other columns to any. That is, perform measurements in all or many combinations. This approach gives a wide range of opportunities for determining the best estimates for the parameters of objects and subjects of measurements, as well as reducing their uncertainty by the means of the adjustment.

The model of direct measurements of increment quantity values can be represented by the following equation:

$$\Delta x_j^{ti} = y^i - y^t + d_j + \Delta x_j^{ti} \cdot b_j, \quad (8)$$

where Δx_j^{ti} is the measured increment quantity value between the measuring object points with numbers i and t; y^i and y^t are the quantity values maintained by the measurement object at the i and t points (for the field linear comparator it is a distance between the first column and any other one).

The equation of the corrections corresponds to the model (8) is as follows:

$$v(\Delta x_j^{ti}) = \delta y^i - \delta y^t + \delta d_j + \Delta x_j^{ti} \cdot \delta b_j + l_j^{ti}. \quad (9)$$

This model is intended for cases when a certain number of measurement subjects are compared with one or more measurement objects – multivalued measures. Measurements are performed between the point of the multivalued measure in many or all combinations, similarly to a field linear comparator. In this case, it is possible to estimate both additive and multiplicative parameters of the measurement subjects.

The measurement object in such a model can also be a plane angle measure, for example, a polygonal prism. The measurement subject in this case can be a

goniometer. The goniometer can be used to measure the angles between the normals to the specular prism faces in a plane parallel to its basic surface. The measurements can be made sequentially from the first face to the last one with or without closing to the first face. Besides, measurement can be started from different faces using various methods. Some goniometers are sensitive to the measurement period. Therefore, rapid measurements can be made on the pairs of faces in the various combinations using a particular software. The angle measurements of the arbitrary face groups can also be made.

Anyway, all these measurements should be processed simultaneously to find the best estimations of angles between the faces. The number of the estimated values in the equations should not be larger than the number of the prism faces. Certainly, according to the flexibility principle, the results of the adjustment can be described by a linear model using only the additive parameter in the model.

5.4 Measurement model of the measurement objects comparison

The very common case of such a model is when the measurement subject reproduces the measurement unit when measuring the difference between the quantity values, maintained by the two measurement objects of the same nominal value. This measurement subject is a well-known comparator. If one of the measurement objects has assigned quantity value from previous comparisons or calibrations, then the value for the other measurement object can be calculated as it takes place during the calibration.

Comparisons can also be described by a measurement model (8) if the increment value between two measurement objects of the same nominal value is measured. This measurement model is singled out due to the fact, that the increments between the sums of quantity values may be measured. For example, this takes place when comparing the mass standards [11]. In such a case the measurement model is as follows:

$$\Delta x_j^{ti} = y^i + y^{i+1} - y^t + y^{t+1} + d_j + \Delta x_j^{ti} \cdot b_j . \quad (10)$$

In the most general case, measurement model (10) takes the following form:

$$\Delta x_j^{ti} = \sum_{i=1}^{I} y^i - \sum_{t=1}^{T} y^t + d_j + \Delta x_j^{ti} \cdot b_j . \quad (11)$$

The equation of the corrections corresponded to the model (11) is as follows:

$$v(\Delta x_j^{ti}) = \sum_{i=1}^{I} \delta y^i - \sum_{t=1}^{T} \delta y^t + \delta d_j + \Delta x_j^{ti} \cdot \delta b_j + l_j^{ti} , \quad (12)$$

where I is the number of the measurement objects in the first group; T is the number of the measurement objects in the second group.

Measurement model (11) is applicable for the case of the increment quantity value direct measurements Δx_j^{ti}, including the case described by the model (8). Model (8) has the independent matter since it describes several important measurement cases, where is unnecessary to apply (11). That is, in the general case (12) the sum of the quantity values assigned to the certain group of measurement objects group is compared with the sum of quantity values assigned to the other group of the same kind through a certain measurement subject – the comparator. In the partial case (8), a pair of two measurement objects are compared or the increment of the quantity between two points of the measurement object is measured.

In subsection 7.5 [11], an example of the unit of mass disseminating from the reference standard with nominal mass 1 kg to seven unknown other weights. The nominal mass of the unknown weights was 1 kg (one weight), 0.5 kg (two weights), 0.2 kg (two weights) and 0.1 kg (two weights). Two mass comparators were used for dissemination by the comparison method. The equation (11) for this particular case is as follows:

$$\begin{aligned}
\Delta x_1^{12} &= y_R^1 - y^2 + d_1 + \Delta x_1^{12} \cdot b_1 ;\\
\Delta x_1^{134} &= y_R^1 - y^3 - y^4 + d_1 + \Delta x_1^{134} \cdot b_1 ;\\
\Delta x_1^{234} &= y^2 - y^3 - y^4 + d_1 + \Delta x_1^{234} \cdot b_1 ;\\
\Delta x_1^{34} &= y^3 - y^4 + d_1 + \Delta x_1^{34} \cdot b_1 ;\\
\Delta x_1^{3567} &= y^3 - y^5 - y^6 - y^7 + d_1 + \Delta x_1^{3567} \cdot b_1 ;\\
\Delta x_1^{4568} &= y^4 - y^5 - y^6 - y^8 + d_1 + \Delta x_1^{4568} \cdot b_1 ;\\
\Delta x_2^{56} &= y^5 - y^6 + d_2 + \Delta x_2^{56} \cdot b_2 ;\\
\Delta x_2^{578} &= y^5 - y^7 - y^8 + d_2 + \Delta x_2^{578} \cdot b_2 ;\\
\Delta x_2^{678} &= y^6 - y^7 - y^8 + d_2 + \Delta x_2^{678} \cdot b_2 ;\\
\Delta x_2^{78} &= y^7 - y^8 + d_2 + \Delta x_2^{78} \cdot b_2 , \qquad (13)\\
&\dots
\end{aligned}$$

where $\Delta x_1^{12}, \dots, \Delta x_2^{78}$ are the measured mass differences (the lower case is the comparator's number and the upper case is the compared weights number); y_R^1 is a mass of the reference weight; $y^2 \dots y^8$ are the masses of the unknown weights; d_1 and d_2 are the additive components of the systematic measurement error of the comparators, and b_2 are the multiplicative components of the systematic measurement error of the comparators.

The conceptual difference between the defined equations (13) and [11] is that, according to the proposed method, both the mass of the unknown weights and the systematic error components of the mass difference measured by comparators can be estimated.

However, it should be mentioned, that there are only 10 equations and 11 unknown quantities (seven weight mass quantities, two additive, and two multiplicative parameters) in the equation (13). That is, the number of measurements should be increased to estimate

all the unknown values in (13). If only additive systematic error components of the comparators have to be estimated, then there will be only 9 unknown parameters and the system of equations (13) can be solved without additional measurements.

5.5 Measurement model for direct comparisons of the measurement subjects

To build up the measurement model for direct comparisons of the measurements subjects, it can be assumed that the same quantity value is simultaneously measured by two measurement subjects j and m ($j \neq m$):

$$x_j^i - x_m^i = d_j - d_m + x_j^i \cdot b_j - x_m^i \cdot b_m , \quad (14)$$

where x_j^i and x_m^i are the measurement results of the measurement subjects j and m; d_j and d_m are the additive parameters of the measurement subjects j and m; b_j and b_m are the multiplicative parameters of the measurement subjects j and m. If there are absent the systematic and random measurement uncertainty components, the difference is defined as:

$$x_j^i - x_m^i = 0 . \quad (15)$$

Taking (15) into account, the equation (14) takes the following form:

$$d_j - d_m + x_j^i \cdot b_j - x_m^i \cdot b_m = 0 . \quad (16)$$

In practice, between the quantity values measured by different measuring instruments, there exists a small difference – the quantity value increment:

$$\Delta x_{mj}^i = x_j^i - x_m^i . \quad (17)$$

Then the equation of the corrections is as follows:

$$v\left(\Delta x_{mj}^i\right) = \delta d_j - \delta d_m + x_j^i \cdot \delta b_j - x_m^i \cdot \delta b_m + l_{mj}^i , \quad (18)$$

The constant term l_{mj}^i in the equation (18) equals:

$$l_{mj}^i = x_j^i - x_m^i \quad (19)$$

on condition that $d_j^0 = d_m^0 = 0$ and $b_j^0 = b_m^0 = 0$.

One of the examples of a given model realization is the comparison of two meters (water meters, gas meters, etc.) that are sequentially installed on a pipeline. In this case, it is possible to compare the readings simultaneously taken from the meters. For further processing purposes, the difference between the meters' simultaneous readings is taken.

It is also possible to install sequentially several such pairs of meters and take their readings in the measuring range. After the adjustments of such measurement results, the parameters of each meter can be obtained concerning the general mean of all measurements.

The measurement results based on this measurement model can be simultaneously adjusted with the results of the direct measurement model, provided by the measurement standard hydrodynamic test facility (see the model description in paragraph 5.2).

In the (14) and (18) equations there are no reference quantity values, that are usually assigned to the measurement object in one or other measurement range point in other models. They are not used in the direct measurement results comparison because the measurement object serves only to provide equal conditions for measurement subjects when measuring the random (in a certain range) objects parameters. For the example above, the measurement object is a substance passing through the pipeline. The estimated parameter of the object is the same volume or mass of the substance, passing through the pipe.

This measurement model can also be used to define the comparison of the simultaneous measurement results by the pair of interferometers. The adjustment of the measurement results provided by the three pairs of the interferometers with numbers 1 and 2, 2 and 3, 3 and 1 is described in [12]. The measuring object in this research was the measuring machine. The reflectors were installed on the machine's carriage. Interferometers provided measurements on the reflectors simultaneously. A measuring tool was only used to provide the smooth movement of the carriage with reflectors and its fixed positioning in the particular point of the measurement range.

As an unknown in [12] was a certain constant $C_{mj} = d_j - d_m$ for each pair of the interferometers and each individual set of measurements. If this had not been done, the multiplicative degree of equivalence of the interferometers could be strongly distorted. The constant C_{mj} does not make any practical sense in the further operation of the interferometers, as it is only for a specific series of interferometers comparisons. It is necessary to exclude the additive systematic component of the error so that it does not affect the result of determining the multiplicative degree of equivalence.

5.6 Model of direct simultaneous quantity value measurements by two subjects

This model is more complicated than the one described in paragraph 5.3. This model aims to perform a more deep analysis of the systematic error sources in the measurement results. This model is based on the interaction of two subjects, that provide simultaneous quantity value measurement of the object. At the same time, one of the subjects is active and provides measurement results. The second one is passive. It only takes part in the measurements, but the measurements cannot be performed without it. Active and passive measurement subjects each have their additive and a multiplicative component of the measurement error. The

measurement model that integrates both subjects' parameters is as follows:

$$x_{mj}^{i} = y^{i} + d_{j}^{i} - d_{m}^{i} + x_{mj}^{i} \cdot b_{j}^{i} - x_{mj}^{i} \cdot b_{m}^{i}, \quad (20)$$

where x_{mj}^{i} is the quantity value measured by the active measurement subject at the i point; y^{i} is the quantity value, maintained by the measurement object at the i; d_{j}^{i} and d_{m}^{i} are the additive parameters of the active and the passive measurement subjects with numbers j and m; b_{j}^{i} and b_{m}^{i} are the multiplicative parameters of the active and the passive measurement subjects with numbers j and m. The equation of corrections corresponds to (20), is as follows:

$$v\left(x_{mj}^{i}\right) = \delta y^{i} + \delta d_{j}^{i} - \delta d_{m}^{i} + x_{mj}^{i} \cdot \delta b_{j}^{i} - x_{mj}^{i} \cdot \delta b_{m}^{i} + l_{mj}^{i} \quad (21)$$

An example of this model application is the calibration on an interferometer of the gauge blocks wrung to the flat glass plates. The interferometer is an active measurement subject and the flat glass plate is the passive one. The experiment aims to estimate systematic bias provided by different flat glass plates in the length measurements of the gage blocks. To do this, it is necessary to provide the wringing of the gage blocks consecutively to different flat glass plates with their identification when and which of gage blocks is wrung to which plate and measure it by the interferometer. The measurement results adjustment using the measurement model (20) and the equation of the corrections (21) allows estimation of the systematic bias from wrilling the gage blocks to each flat glass plate by their measured length.

5.7 Model of direct simultaneous quantity value increment measurements by two subjects

This model is more complicated than the one described in paragraphs 5.3 and 5.6. This model can be used to perform a more deep analysis of the systematic error sources in the measurement results. This model is based on the interaction of two subjects, which provide simultaneous quantity value increment measurement of the object. As it is mentioned above in 5.6, one of the subjects is active and the second one is passive. The measurement model, which unites them, is as following:

$$\Delta x_{mj}^{ti} = y^{i} - y^{t} + d_{j}^{i} - d_{m}^{t} + \Delta x_{mj}^{ti} \cdot b_{j}^{i} - \Delta x_{mj}^{ti} \cdot b_{m}^{t}, \quad (22)$$

where Δx_{mj}^{ti} is the quantity value increment measured by the active measurement subject between the i and t points; y^{i} and y^{t} are the quantity values, which is maintained by the measurement object at the i and t points; d_{j}^{i} and d_{m}^{t} are the additive parameters of the active and passive measurement subjects j and m; b_{j}^{i} and b_{m}^{t} are the multiplicative parameters of the active and passive measurement subjects j and m. The equation of corrections corresponds to (22), is as follows:

$$v\left(\Delta x_{mj}^{ti}\right) = \delta y^{i} - \delta y^{t} + \delta d_{j}^{i} - \delta d_{m}^{t} + \\ + \Delta x_{mj}^{ti} \cdot \delta b_{j}^{i} - \Delta x_{mj}^{ti} \cdot \delta b_{m}^{t} + l_{mj}^{ti} \quad (23)$$

An illustrative example of this model application is the comparison or calibration of the tachometers on the field linear comparator. In model 5.3, the tachometer with bundled reflector is described as a single measurement subject. The measurement model, which determines the additive and multiplicative parameter of this set, is described in 5.3. However, if the measurements are performed by several tachometers on several reflectors when comparing, each tachometer will have its systematic measurement component on each reflector. Thus, if the additive parameter needs to be determined separately for each tachometer and each reflector, it is better to use a measurement model (22). To do this, it is necessary to perform the distance measurements of the field linear comparator in various combinations of tachometers and reflectors.

If only one reflector is used with each tachometer, then this model usage has no sense and the model in 3.3 will be appropriate. This measurement model can be also used for the tachometer's calibration results estimation on the field linear comparator when several different reflectors are used. The example of the measurement processing according to this model is described in [10].

6. Results Adjustment According to the Multipurpose Measurement Models

The aim of the software based on the multipurpose measurement models is not only to overcome the limitations number on the number of measurements processed, but also the limitations on the number of objects and subjects, the parameters of which can be determined by the results of the adjustment. The software should be compiled in such a way that the results of measurement performed on different models should be adjusted simultaneously.

The general equation in matrix form corresponds to the above measurement models and their correction equations are as follows:

$$V_{x} = \overline{A}_{y} \cdot \delta y + \overline{A}_{d} \cdot \delta d + \overline{A}_{b} \cdot \delta b + l. \quad (24)$$

where $\overline{A}_{y}$ is the matrix of the equations correction coefficients related to the measurement objects parameters, for example, key comparison reference values; $\overline{A}_{d}$ is the matrix of the equations correction coefficients related to the additive measurement subjects parameters, for example, additive measurement stan-

dards degrees of equivalence; $\overline{A}_b$ is the matrix of the equations correction coefficients related to multiplicative measurement subjects parameters, for example, multiplicative measurement standards degrees of equivalence; δy is the column vector of the correction coefficients to initial parameters values of the measurement objects; δd is the column vector of the correction coefficients to additive initial parameters values of the measurement subjects; δb is the column vector of the correction coefficients to multiplicative initial parameters values of the measurement subjects; l is the column vector of the correction equations constant terms; V_x is the diagonal matrix of the correction coefficients to measured quantity values.

The further results adjustment, which consists of setting up and solving the normal equations, as well as calculating the corresponding corrections and estimating accuracy, is performed similarly to [3, 7, 9–12.

7. Conclusions

1. Various measurement results are obtained by appropriate procedures for different measurement methods and sub-methods. One of the ways to solve the processing of increased volume of the measurement information from the interlaboratory comparisons and calibrations powering over time is the development of flexible software. The measurement results adjustment by the least-squares method has to be developed based on the multipurpose measurement models.
2. New general terms characterizing the developed multipurpose measurement models such as "measurement subject" that somehow realize the quantity value during the measurements and "measurement object" to which the quantity value is reasonably assigned according to the measurement results are introduced.
3. The proposed multipurpose measurement models are inherent in a common structure. Each model includes the quantity values maintained by the measurement objects as well as the additive and multiplicative parameters of the measurement subjects. The proposed models should be the basis for the software development aiming the accurate mathematical binding or integration through the measurement results processing of the complicated combinations of key, regional and additional, interlaboratory comparisons, and the calibrations within the unified comprehensive measurement traceability network. The benefit of the software development based on the multipurpose models depends on the support and coordinated cooperation of the entire metrology community.

8. Conflict of interest

There is no financial or other potential conflict within the current paper.

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