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RAYLEIGH WAVES ARISING IN THE COLLISION OF ELASTIC BODIES, AND GENERALIZATION OF THE THEORY OF HERTZ IMPACT

Ua Метою цієї роботи є розвиток та деяке узагальнення локальної теорії деформування пружних тіл при їх прямому центральному співударі. Розглядається вплив поверхневих хвиль, що спричиняються ударом, на процес удару. Як відомо, ще Релей відзначав, що цей вплив може бути суттєвим. Е роботі розглядаються сили інерції, на разі використання інтегрального перетворення Лапласа-Карсона до рівнянь еластодинаміки, у результаті чого динамічна задача теорії пружності перетворюється на квазистатичну задачу щодо зображень шуканих величин.

Дослідження мінімальних властивостей контактного тиску виявило, що під час удару виникає спектр частот поверхневих хвиль, та найбільший вплив на процес удару надають хвилі найнижчої частоти. Коефіцієнт відновлення у цьому випадку наближається до класичного.

Ru Целью этой работы является развитие и некоторое обобщение локальной теории деформирования упругих тел при их прямом центральном соударении. Рассматривается влияние поверхностных волн, вызванных ударом, на процесс удара. Как известно, еще Релей отмечал, что влияние это может быть существенным. В работе рассматриваются силы инерции, посредством применения к уравнениям эластодинамики интегрального преобразования Лапласа-Карсона, в результате которого динамическая задача теории упругости сводится к квазистатической задаче относительно изображений искомых величин.

Исследование минимальных свойств контактного давления показало, что при ударе возникает спектр частот поверхностных волн, и наибольшее влияние на процесс удара оказывают волны самой низкой частоты. Коэффициент восстановления в этом случае близок к классическому.

Introduction

The purpose of this work is the development and some generalization of the local theory of deformation of elastic bodies in their direct central collision. There is considered an issue of influence of surface waves caused by impact on

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the impact process. As is known, Rayleigh also noted that this influence could be significant.

Problem statement and method solutions

Let us turn to the theory of collisions of solids and consider the theory of H. Hertz of local deformation of bodies under impact (1). In the theory of Hertz only local deformations are taken into account, the rest deformations are ignored. Meanwhile, it is far from obvious that the transverse and longitudinal oscillations that occur when the bodies collide do not significantly affect the propagation of local stresses and deformations. Hertz proceeded from a simplified mechanical model of a system consisting of undeformed solids separated by a deformable "gasket." Compression "gasket" α determines the convergence of the centers of inertia of colliding bodies. The function α satisfies equation

$$m\ddot{\alpha} = -P; \qquad m = \frac{m_1 m_2}{m_1 + m_2}, \tag{1}$$

where P – contact force, m_i – solids' masses.

The dependence between α and P is established from the solution of the static contact problem, i.e. with the use of the Lame equations with excluded inertia forces. We have:

$$\alpha = k\mathrm{P}^{\frac{2}{3}}. \tag{2}$$

Hertz did not take into account the inertia forces of the elements of the deformed bodies. However, there are sufficiently convincing proofs of the need to consider the local forces of inertia that arise when colliding elastic bodies (2). We consider the inertia forces by applying the Laplace-Carson integral transformation to the equations of elastodynamics, as a result of which the dynamic problem of the theory of elasticity reduces to a quasistatic problem with respect to the images of the unknown quantities.

According to the hypothesis of Hertz in the Lame equation

$$\mu\nabla^2\vec{U} + (\lambda+\mu)\,\mathrm{graddiv}\vec{U} + \rho\vec{F} = \rho\frac{\partial^2\vec{U}}{\partial t^2}. \tag{3}$$

the forces of inertia $\rho\frac{\partial^2\vec{U}}{\partial t^2}$ and mass forces $\rho\vec{F}$ are not taken into account. In the image space, this equation corresponds to

$$\mu\nabla^2\vec{U}^* + (\lambda+\mu)\,graddiv\vec{U}^* = 0. \tag{4}$$

The mechanism of integral transformation makes it possible to find a different meaning in this transition. We transform the equation (3) into the image space

$$\mu\nabla^2\vec{U}^* + (\lambda+\mu)\,\mathrm{graddiv}\vec{U}^* + \rho(\vec{F}^* + p\dot{\vec{U}}_0 + p^2\vec{U}_0 - p^2\vec{U}^*) = 0. \tag{5}$$

From the equation (5), we can obtain the equation (4) by setting

$$\vec{F}^* + p\dot{\vec{U}}_0 + p^2\vec{U}_0 - p^2\vec{U}^* = 0\,. \qquad (6)$$

In the space of originals, this corresponds to the introduction of fictitious forces $\vec{F} = \rho\dfrac{\partial^2\vec{U}}{\partial t^2}$. These forces are an additional "source of energy". This becomes clear when determining the recovery factor: in this case it exceeds unity (3), which is obviously impossible.

Equation (4) is identical in form to the Lamé elastostatic equation. Applying the integral transformation to the kinematic contact condition and solving the equations of the quasi-static contact problem in the image space, we get the correlation

$$\alpha^* = k\mathrm{P}^{*\frac{2}{3}}\,. \qquad (7)$$

It should be emphasized that the equation (7) is not the result of the Laplace-Carson transformation of the nonlinear correlation (2), but is a nonlinear consequence of the linear equations of the theory of elasticity in image space.

It is necessary to clarify the analytical formulation of the problem. For this, a well-known method of integrating the equations of the theory of elasticity, indicated by Somilyan (the theorem on the reciprocity of work) can be applied.

The first or real state of the body is defined by the equation

$$\mu\nabla^2\vec{V}^* + (\lambda+\mu)\,\mathrm{graddiv}\vec{\mathrm{V}}^* - \rho p^2\vec{V}^* = 0\,, \qquad (8)$$

obtained on the basis of (4) under the assumption

$$\vec{F}^* = 0\,, \quad U_0 = 0\,, \quad \vec{U}^* = \frac{1}{p}\vec{U}_0 - \vec{V}^*\,. \qquad (9)$$

The second or auxiliary state is determined by the equation

$$\mu\nabla^2\vec{W}^* + (\lambda+\mu)\,\mathrm{graddiv}\vec{W}^* = 0 \qquad (10)$$

with excluded forces of inertia, as well as boundary and initial conditions.

The integral equations obtained on the basis of the reciprocity theorem are as follows

$$V_k^*(\mathrm{M},p) = W_k(\mathrm{M},\mathrm{p}) - \rho\,\mathrm{p}^2\iiint\limits_{(v)}\sum_j G_{(k)\,j}(Q,\mathrm{M})V_j^*(\mathrm{Q},\mathrm{p})\,\mathrm{dV}_Q\,,\; k = 1, 2, 3. \qquad (11)$$

An approximate solution of this system allows us to find the local compression image $\alpha^*_{(p)}$ in the form.

$$\alpha^*(p)=\left[\frac{\vartheta_1}{\vartheta_1+\vartheta_2}\left(1-\frac{\chi_1^* p^2}{\overset{(1)}{n^2}+p^2}\right)+\frac{\vartheta_2}{\vartheta_1+\vartheta_2}\left(1-\frac{\chi_2^* p^2}{\overset{(2)}{n^2}+p^2}\right)\right]\alpha_0^*(p)+ \qquad (12)$$

$$+\frac{1}{p}\left(\dot U_{30}^{(1)}+\dot U_{30}^{(2)}\right),$$

Let us put the dimensionless parameters

$$p=q\frac{V_0^{\frac{1}{5}}}{m^{\frac{2}{5}}k^{\frac{3}{5}}};\qquad \overset{(i)}{n}=\overset{(i)}{\nu}\frac{V_0^{\frac{1}{5}}}{m^{\frac{2}{5}}k^{\frac{3}{5}}}. \qquad (13)$$

In the space of originals, this corresponds to the introduction of dimensionless time

$$\tau=t\frac{V_0^{\frac{1}{5}}}{m^{\frac{2}{5}}k^{\frac{3}{5}}}.$$

Then

$$\alpha^*(q)=k^*(q)\mathrm{P}^{*\frac{2}{3}}+k\left(\frac{mV_0^2}{k}\right)^{\frac{2}{5}}\frac{1}{q}, \qquad (14)$$

where

$$k^*(q)=k\left[1-\frac{\vartheta_1}{\vartheta_1+\vartheta_2}\frac{\chi_1^* q^2}{\overset{(1)}{\nu^2}+q^2}-\frac{\vartheta_2}{\vartheta_1+\vartheta_2}\frac{\chi_2^* q^2}{\overset{(2)}{\nu^2}+q^2}\right]. \qquad (15)$$

The function $k^*(q)$ contains two undefined functions χ_i^* and the oscillation frequencies that occur upon impact. These indefinite elements show that formula (15) covers a significant part of the consequences of the Lame equations. To find one of the possible choices of the function $k^*(q)$, let us turn to the theory of surface waves caused by the collision of elastic bodies (4).

For bodies bounded by surfaces of revolution, the kinematic condition for contact of bodies has the form

$$\overset{(1)}{W}(\mathrm{M},t)+\overset{(2)}{W}(\mathrm{M},t)=\alpha(t)-\left[A+\frac{1}{2}\left(\frac{\partial^2 V}{\partial r^2}\right)_{r=0}\right]r^2. \qquad (16)$$

V – the vertical component of the displacement vector is determined by the equation

$$V=\left(\mathrm{M}\xi e^{-\xi z}+\mathrm{N}\kappa e^{-\eta z}\right)I_{0(\kappa r)}\begin{cases}\cos\omega t\\ \sin\omega t\end{cases}, \qquad (17)$$

containing unknown parameters M, N, ξ, η, κ, ω (amplitude and frequency characteristics arising from the impact of Rayleigh waves). From the conditions for the absence of additional stresses on the surfaces of bodies, the correlations between these parameters follow (4). Solving the equation (16) in the space of images, we find

$$\alpha_s^*(q)=k\left[1-\frac{\overset{(1)}{B}}{A}\frac{q^2}{\overset{(1)}{\Omega}{}^2+q^2}-\frac{\overset{(2)}{B}}{A}\frac{q^2}{\overset{(2)}{\Omega}{}^2+q^2}\right]^{\frac{1}{3}}P^{*\frac{2}{3}}\ . \tag{18}$$

Here

$$\overset{(i)}{B}=\frac{1}{4}\overset{(i)}{\kappa}{}^2\left(\overset{(i)}{M}\overset{(i)}{\xi}+\overset{(i)}{N}\overset{(i)}{\kappa}\right).$$

Thus, from the theory of surface waves we have

$$k_s^*(q)=k\left[1-\frac{\overset{(1)}{B}}{A}\frac{q^2}{\overset{(1)}{\Omega}{}^2+q^2}-\frac{\overset{(2)}{B}}{A}\frac{q^2}{\overset{(2)}{\Omega}{}^2+q^2}\right]^{\frac{1}{3}}. \tag{19}$$

Identifying the indefinite elements in the equations (15) and (19) $\dfrac{\vartheta_i}{\vartheta_1+\vartheta_2}\chi_i^*$ with $\dfrac{\overset{(1)}{B}}{A}$ and $\overset{(i)}{\nu}{}^2$ with $\overset{(i)}{\Omega}{}^2$, we find

$$\alpha_R^*(q)=k_R^*P^{*\frac{2}{3}}, \tag{20}$$

where

$$k_R^*(q)=k\left[1-\frac{\overset{(1)}{B}}{A}\frac{q^2}{\overset{(1)}{\Omega}{}^2+q^2}-\frac{\overset{(2)}{B}}{A}\frac{q^2}{\overset{(2)}{\Omega}{}^2+q^2}\right]^{\frac{4}{3}}. \tag{21}$$

Here the index *R* shows the presence of Rayleigh waves caused by impact. If this identification is not made, then we get

$$\alpha^*(q)=k^*(q)P^{*\frac{2}{3}}, \tag{22}$$

where

$$k^*(q)=k\left[1-\frac{\vartheta_1}{\vartheta_1+\vartheta_2}\frac{\chi_1^*q^2}{\overset{(1)}{\nu}{}^2+q^2}-\frac{\vartheta_2}{\vartheta_1+\vartheta_2}\frac{\chi_2^*q^2}{\overset{(2)}{\nu}{}^2+q^2}\right]\times$$
$$\times\left[1-\frac{\overset{(1)}{B}}{A}\frac{q^2}{\overset{(1)}{\Omega}{}^2+q^2}-\frac{\overset{(2)}{B}}{A}\frac{q^2}{\overset{(2)}{\Omega}{}^2+q^2}\right]^{\frac{1}{3}}. \tag{23}$$

Conclusions

The research of the minimum properties of the contact pressure showed (4) that upon impact a spectrum of surface wave frequencies arises, and waves of the lowest frequency have the greatest impact on the impact process. The recovery coefficient in this case is close to the classical one, $\varepsilon \cong 0,68$ (5).

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