FOURIER COEFFICIENTS FOR THE LIGHT CURVES OF 62 MIRA–TYPE STARS

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ABSTRACT. For 62 Mira–type stars the values of periods have been redetermined; the mean curves have been fitted by a trigonometric polynomial with an optimal number of harmonics; a number of the parameters of the light curve have been calculated (amplitude of the present harmonic in stellar magnitudes, phase of the maximum of the amplitude of the present harmonic, ratio of the amplitude of the harmonic to the amplitude of the main harmonic wave, shift of the phase of the present harmonic relatively to the main wave, shift of the phase of the present harmonic relatively to the maximum of brightness). The parameters of the sharpness of the both branches of the light curve are obtained: the time of the brightness changes by 1 mag, the ratio between the maximal slope of the incline to the derivative for sinusoidal signal with present amplitude and period. All these parameters are tabulated.

Key words: Stars, Mira, pulsating, correlations

The Mira–type stars are long period variables of AGB. Their light curves are generally regular, but have sometimes strong distortions changing the shape of the light curve.

Earlier it was considered that the light curve, especially in the visual range, is not the fundamental characteristic of Mira, because it is affected by molecular bands. However, the researches appeared in the eighties indicative, that the visual light curve is doubtless directly dependent on the physical properties of Mira–star (e.g. Vardya, 1987).

Many correlations have been detected between the visual, IR– and maser variations for Miras (Le Bertre, 1987; Keping et al., 1984). For example, the correlations were found between the period, amplitude at 2.2 μ, color index [3.5μ] - [10μ], OH velocity emission peaks and the presence of H_2O emission. The three kinds of the circumstellar masers possess the variability of the radiation, caused by a variability of the star itself. The maximum of the maser flux occurs later than the maximum of the visual brightness for OH and SiO by $0\overset{p}{.}1$ and for H_2O on $0\overset{p}{.}2$ (Rudnitskij, 1983).

Thus, it is very important to obtain the detailed visual light curve and accurately determined extrema. It is not easy to obtain such a light curve. The Miras have amplitudes about 5^m–7^m (and sometimes more), the periods of about 1 year, and the curve is unstable itself. We have to obtain a mean light curve to describe the star. And this curve has to contain all characteristic changes and at the same time this light curve must not to be disturbed too strongly. It is possible, if the light curve of the Mira–type star is investigated during not too long time interval, for example, 1000 days. One may note that the Miras with the periods from 100 to 500 days pass during this time from 10 to 2 cycles of variation. But we know that the Miras with small value of the period have more regular light curves, than Miras with large value of the period, which have distortion almost at the every cycle of the variation.

The observations of 62 Mira–type stars obtained by the AAVSO members (Mattei, 1979) have been used for the analysis. The initial values were converted to obtain the 10^d means, which were fitted by a trigonometric polynomial

$$m(t) = a_0 - \sum_{k=1}^{s} r_k \cos(2\pi k \cdot (t - t_k)/P), \qquad (1)$$

where r_k are semiamplitudes and t_k are initial epochs for the brightness maximum (minimum magnitude) of the wave with a period $P_k = P/k$. The value of the main period was corrected by using the method of differential corrections. The computer program FOUR–M (Andronov, 1994) was used. The phases of the individual waves are computed as ϕ_k =FLOAT($k(t_k - T_0)/P$, where T_0 is the initial epoch and FLOAT(x) = x–INT(x) is a decimal part of x defined in the range from 0 to 1.

The next parameters are calculated:

the fundamental characteristics:

T_0 – the epoch of the maximum;

P – the period of the light variation;

Δm – the amplitude of the light variation;

"$M - m$" – the asymmetry of the light curve defined as the ratio of the duration of the ascending branch to the period;

Table 1. The characteristics of the mean light curve.

Star	T_0	P	$M-m$	Δm	s	r_1	r_2	r_3	r_4	ϕ_1	ϕ_2	ϕ_3
RT Cyg	42882.9	191.3	0.484	4.60	2	2.29	0.20			0.010	-0.436	0.000
R Vir	42808.7	146.0	0.555	4.14	2	2.03	0.24			-0.036	0.308	
R Vul	42712.1	136.9	0.500	4.19	1	2.09				0.000		
RZ Sct	42819.3	160.2	0.500	3.19	1	1.59				0.000		
S Car	42718.9	151.1	0.564	2.68	2	1.33	0.36			-0.063	0.372	
SS Cas	42810.4	140.4	0.426	3.49	2	1.71	0.36			0.011	-0.027	
SS Her	42778.6	104.4	0.500	3.68	1	1.84				0.000		
T Her	42824.9	164.3	0.500	5.03	1	2.51				0.000		
V Tau	42782.0	169.8	0.500	4.88	1	2.43				0.000		
W Lyr	42705.8	197.9	0.477	3.93	2	1.95	0.20			0.017	-0.420	
X Cam	42801.4	142.6	0.438	4.45	2	2.18	0.26			0.039	-0.271	
R Ari	42827.7	186.9	0.500	4.21	1	2.10				0.000		
RR Boo	42857.0	194.3	0.432	4.09	2	1.99	0.23			0.033	-0.174	
X Aur	42811.5	163.9	0.580	4.25	2	2.06	0.40			-0.061	0.291	
SY Her	42892.0	118.2	0.500	3.36	1	1.68				0.000		
W Pup	42798.5	119.9	0.500	3.73	1	1.86				0.000		
BG Cyg	42894.7	292.2	0.529	2.55	2	1.26	0.07			-0.012	0.108	
R Aql	42888.1	284.2	0.418	4.69	2	2.25	0.33			0.040	-0.167	
R Boo	42743.1	222.6	0.500	5.18	1	2.58				0.000		
R Dra	42808.4	245.2	0.450	5.25	2	2.59	0.24			0.020	-0.121	
RS Her	42777.1	220.6	0.500	4.83	1	2.41				0.000		
RS UMa	42884.4	261.1	0.354	5.76	3	2.74	0.67	0.16		0.044	-0.032	-0.237
R Tri	42814.5	268.5	0.481	5.56	3	2.58	0.13	0.18		0.002	-0.176	0.058
S Boo	42770.0	266.9	0.529	4.69	2	2.33	0.13			-0.012	0.119	
S UMa	42848.0	228.2	0.502	3.74	3	1.80	0.32	0.15		-0.033	0.493	0.129
TU Cyg	42788.4	220.6	0.435	5.27	2	2.57	0.28			0.033	-0.192	
T UMa	42778.6	257.0	0.339	5.41	3	2.71	0.43	0.30		0.070	-0.048	-0.201
V Cas	42746.8	229.4	0.449	4.33	2	2.13	0.18			0.027	-0.215	
W Her	42846.6	278.7	0.466	5.19	3	2.66	0.12	0.08		0.013	-0.134	-0.469
Y Per	42750.6	247.1	0.479	2.01	3	0.94	0.16	0.06		0.004	-0.428	0.100
o Cet	42822.7	333.5	0.300	5.52	3	2.66	0.50	0.46		0.085	-0.042	-0.141
R Leo	42918.7	313.7	0.450	4.01	4	1.85	0.19	0.13	0.08	0.039	-0.029	-0.023
R Ser	42676.5	353.6	0.434	6.81	3	3.23	0.49	0.28		0.059	-0.132	-0.086
R UMa	42890.2	301.5	0.419	5.46	4	2.64	0.62	0.27	0.22	0.080	0.016	-0.110
S CrB	42809.7	357.8	0.395	5.27	2	2.45	0.50			0.051	-0.138	
T Cam	42697.9	374.8	0.465	5.74	4	2.61	0.69	0.27	0.13	0.002	-0.444	0.142
TU And	42865.0	318.4	0.483	4.87	4	2.23	0.16	0.22	0.13	-0.009	0.397	0.080
U Per	42821.9	327.6	0.500	3.54	1	1.76				0.000		
U UMi	42769.3	327.6	0.487	3.96	3	1.91	0.07	0.11		-0.015	-0.105	0.152
V Aur	42727.1	353.7	0.538	2.40	3	1.06	0.13	0.12		-0.013	0.238	-0.079
W And	42714.2	399.2	0.459	11.0	4	4.37	0.78	1.13	0.53	0.055	-0.420	0.068
W Peg	42795.8	344.3	0.457	4.46	4	1.92	0.18	0.29	0.09	0.035	-0.167	0.002
X Oph	42763.8	327.7	0.500	1.34	1	0.66				0.000		
R Gem	42980.5	368.9	0.396	6.46	2	3.01	0.61			0.050	-0.135	
T UMi	42707.7	316.7	0.482	5.63	3	2.45	0.52	0.36		0.019	-0.031	-0.012
U Ori	42656.9	385.6	0.337	5.51	3	2.56	0.64	0.22		0.067	-0.055	-0.173
S UMi	42959.4	326.0	0.519	4.24	3	2.02	0.15	0.15		-0.034	0.380	0.082
V Cam	42885.4	503.5	0.376	5.37	5	2.12	0.68	0.37	0.29	0.033	-0.052	0.025
χ Cyg	42974.3	421.5	0.423	8.99	4	3.93	0.61	0.33	0.32	0.080	-0.048	-0.190
R And	42631.3	426.2	0.316	7.78	3	3.54	0.95	0.40		-0.036	0.314	0.105
R Aur	42728.9	448.0	0.516	6.14	5	2.57	0.69	0.59	0.40	0.104	-0.037	0.008
R Cas	42898.8	443.9	0.369	6.25	5	2.82	0.55	0.43	0.16	0.033	-0.064	
R Cyg	42808.5	443.8	0.382	6.73	2	3.10	0.82			-0.034	0.268	-0.174

Table 1 (continued).

Star	T_0	P	$M-m$	Δm	s	r_1	r_2	r_3	r_4	ϕ_1	ϕ_2	ϕ_3
S Cep	42834.3	503.5	0.568	2.77	3	1.21	0.29	0.12		-0.075	0.227	-0.073
T Cas	42763.3	465.3	0.590	4.69	3	2.08	0.58	0.15		0.067	-0.035	-0.111
U Aur	42738.4	410.5	0.316	5.66	3	2.62	0.67	0.39		-0.082	0.252	-0.047
U CMi	42704.7	422.0	0.579	4.69	3	2.04	0.61	0.25		0.005	-0.144	0.024
U Cyg	42959.4	484.5	0.487	3.30	3	1.48	0.07	0.15		0.071	0.018	-0.007
U Her	42885.4	414.9	0.409	4.45	5	2.00	0.37	0.28	0.16	0.063	0.016	0.023
V Cyg	42756.1	423.9	0.427	3.46	4	1.63	0.23	0.15	0.10	0.073	-0.025	-0.083
W Cas	42976.2	417.7	0.470	2.81	2	1.40	0.12			0.019	-0.383	
X Cas	42660.9	441.9	0.621	1.71	2	0.81	0.26			-0.111	0.243	

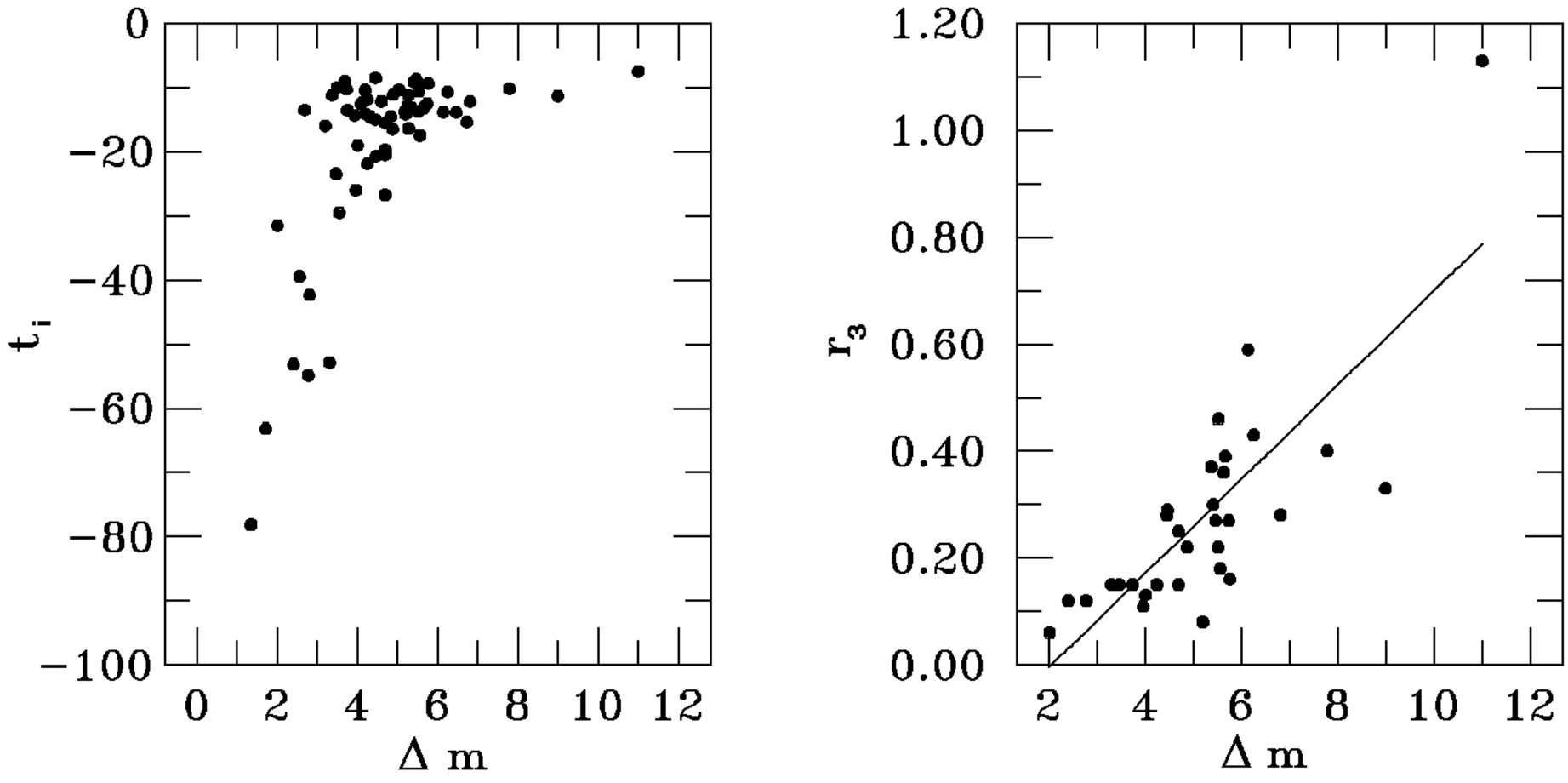

Figure 1. Several correlations between the parameters of the light curves of Mira–type stars.

the additional characteristics:
s – degree of the trigonometric polynomial which is obtained by using the Fischer's criterion and a "false alarm" probability $P_r = 10^{-3}$;
r_k – the amplitude of the wave with a frequency $k \cdot f_1$;
ϕ_k – the phase of the maximum of the wave in respect to the brightness maximum;
$\phi_{k1} = \phi_k - k \cdot \phi_1$ – the shift of the phase relative the maximum of the main wave;

the parameters of the sharpness of light curve:
m_{is} – $(dm/dt)_{curve}/(dm/dt)_{sin}$, where $(dm/dt)_{sin} = \pi(m_{min} - m_{max})/P$ – for ascending branch;
m_{ds} – the same for the descending branch;
t_i – dt/dm – the characteristic time of the increase of brightness (ascending branch) by 1^m;
t_d – dt/dm – the characteristic time of the decrease of brightness (descending branch) by 1^m.

The correlation analysis has been carried out between all pairs of these parameters. The correlation coefficients for 63 pairs from 325 exceed 3σ and were discussed in more detail by Kudashkina and Andronov (1996). Some of these correlations are expected ones (e.g. P and $1/P$). The Figure 1 shows two sample diagrams with nearly linear and non-linear dependencies. The diagram "$\Delta m - t_i$" shows the increasing $|t_i|$ with the decreasing amplitude. In the figure the sign of $t_i = dt/dm$ is negative, because during the incline the magnitude decreases. This figure shows in fact two clusters of the stars with different slopes $dt_i/d(\Delta m)$ which cross at a point $\Delta m \approx 5^m$, $|t_i| \approx 16$ days/mag. Another dependence $r_3 - \Delta m$ shows no significant change of the slope.

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Table 2. The Relationships between the Phases.

Star	r_2/r_1	r_3/r_1	ϕ_{21}	ϕ_{31}	Star	r_2/r_1	r_3/r_1	r_4/r_1	ϕ_{21}	ϕ_{31}	ϕ_{41}
RT Cyg	0.080		-0.450		R Leo	0.100	0.070	0.040	-0.100	-0.140	-0.350
R Vir	0.120		0.380		R Ser	0.150	0.080		-0.250	-0.270	
R Vul					R UMa	0.230	0.100	0.080	-0.140	-0.350	0.450
RZ Sct					S CrB	0.200			-0.240		
S Car	0.270		0.500		T Cam	0.260	0.100	0.040	-0.450	0.130	-0.070
SS Cas	0.210		-0.050		TU And	0.070	0.090	0.050	0.420	0.110	-0.090
SS Her					U Per						
T Her					U UMi	0.030	0.060		-0.070	0.190	
V Tau					V Aur	0.120	0.110		0.260	-0.050	
W Lyr	0.100		-0.460		W And	0.170	0.250	0.120	0.470	-0.100	0.500
X Cam	0.120		-0.340		W Peg	0.090	0.150	0.040	-0.230	-0.100	-0.180
R Ari					X Oph						
RR Boo	0.110		-0.240		R Gem	0.200			-0.240		
X Aur	0.190		0.410		T UMi	0.210	0.150		-0.060	-0.060	
SY Her					U Ori	0.250	0.080		-0.190	-0.380	
W Pup					S UMi	0.070	0.070	0.080	0.080	0.450	0.190
BG Cyg	0.060		0.140		V Cam	0.150	0.080		-0.110	-0.070	-0.200
R Aql	0.140		-0.240		χ Cyg	0.260	0.110	0.150	0.150	-0.200	-0.430
R Boo					R And	0.270	0.230	0.050	0.050	0.380	0.210
R Dra	0.090		-0.160		R Aur	0.190	0.150		-0.240	-0.300	0.450
RS Her					R Cas	0.260			-0.130		
RS UMa	0.240	0.050	-0.120	-0.360	R Cyg	0.240	0.100		0.330	-0.070	
R Tri	0.050	0.070	-0.180	0.050	S Cep	0.280	0.070		0.370	0.150	
S Boo	0.050		0.140		T Cas	0.250	0.140		-0.160	-0.300	
S UMa	0.180	0.080	-0.440	0.230	U Aur	0.300	0.120		0.410	0.190	
TU Cyg	0.110		-0.250		U CMi	0.040	0.100	0.080	0.080	-0.160	0.000
T UMa	0.150	0.110	-0.180	-0.400	U Cyg	0.180	0.130	0.130	0.130	-0.120	-0.220
V Cas	0.080		-0.270		U Her	0.320	0.170	0.060	0.060	-0.110	-0.160
W Her	0.040	0.030	-0.160	0.490	V Cyg	0.140	0.090		-0.170	-0.300	-0.490
Y Per	0.170	0.070	-0.440	0.090	W Cas	0.090			-0.420		
o Cet	0.190	0.170	-0.210	-0.390	X Cas	0.320			0.460		

Table 3. The parameters of sharpness of the light curve.

Star	m_{is}	m_{ds}	t_i	t_d	Star	m_{is}	m_{ds}	t_i	t_d	Star	m_{is}	m_{ds}	t_i	t_d
RT Cyg	1.088	1.012	12	13	RS UMa	1.541	0.852	9	17	X Oph	1.000	1.000	78	78
R Vir	0.923	1.183	12	9	R Tri	0.881	0.887	17	17	R Gem	1.314	0.703	14	29
R Vul	1.000	1.000	10	10	S Boo	0.921	1.094	20	17	T UMi	1.351	1.177	13	15
RZ Sct	1.000	1.000	16	16	S UMa	1.435	0.869	14	22	U Ori	1.620	0.704	14	32
S Car	1.332	1.334	13	13	TU Cyg	1.194	0.764	11	17	S UMi	1.118	0.972	22	25
SS Cas	1.284	1.137	10	11	T UMa	1.649	1.012	9	15	V Cam	1.322	0.920	13	26
SS Her	1.000	1.000	10	9	V Cas	1.153	0.822	15	20	χ Cyg	1.705	0.725	11	16
T Her	1.000	1.000	10	10	W Her	1.206	1.050	14	16	R And	1.685	1.257	10	24
V Tau	1.000	1.000	11	11	Y Per	1.240	1.006	31	39	R Aur	2.108	1.130	14	18
W Lyr	1.120	1.023	14	16	o Cet	1.818	1.088	11	18	R Cas	1.370	0.988	11	20
X Cam	1.196	0.873	9	12	R Leo	1.315	0.829	19	30	R Cyg	1.055	1.257	15	21
R Ari	1.000	1.000	14	14	R Ser	1.357	0.924	12	18	S Cep	1.182	1.141	55	46
RR Boo	1.201	0.753	13	20	R UMa	2.004	0.992	9	18	T Cas	1.762	0.893	27	28
X Aur	1.036	1.273	12	10	S CrB	1.317	0.689	16	31	U Aur	1.395	1.040	13	26
SY Her	1.000	1.000	11	11	T Cam	1.667	1.017	12	20	U CMi	0.885	0.905	21	28
W Pup	1.000	1.000	10	10	TU And	1.268	0.949	16	22	U Cyg	1.973	0.744	53	52
BG Cyg	0.925	1.096	39	33	U Per	1.000	1.000	29	29	U Her	2.252	1.132	15	40
R Aql	1.244	0.694	16	28	U UMi	1.012	1.057	26	25	V Cyg	1.663	0.880	23	44
R Boo	1.000	1.000	14	14	V Aur	0.881	0.971	53	48	W Cas	1.117	0.974	42	49
R Dra	1.156	0.870	13	17	W And	1.544	1.524	7	8	X Cas	1.302	1.413	63	58
RS Her	1.000	1.000	15	15	W Peg	1.186	0.920	21	27					