

# ON THE PERIODS OF THE $\beta$ CEPHEI STARS

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**ABSTRACT.** On the base of GCVS' data the analysis of periods distributions for  $\beta$  Cep stars (BCEP and BCEPS) has been carried out. Identifications for all peaks on the histogram have been performed. There are some overtones and their harmonics. These are related with multiplicity ratios. In addition to GCVS classification it is proposed to extend the range of periods for short-period group (BCEPS) to (0.02 - 0.13) days instead of (0.02 - 0.04) days. We propose a new classification of these stars according to their mode identifications.

**Key words:** Stars:  $\beta$  Cep,  $\delta$  Scuti, RR Lyrae, Cepheids, histogram of periods distribution, mode identifications

## 1. Introduction

The matter of modal content for the pulsating stars has only been solved satisfactory for double-mode Cepheids. The failure to establish the driving mechanism in  $\beta$  Cephei stars is one of the most serious problems in our understanding of stellar pulsations.

We look at the matter the other way round using: 1) statistical data from the fourth edition of the General Catalogue of Variable Stars (Volumes 1-3, hereafter GCVS), considering distribution of periods for these stars; 2) histogram of periods distribution and dependencies of masses, radii and radial pulsation constants,  $Q$ , upon period (from Shobbrook's data, 1985); 3) the results of periodogram analysis of individual  $\beta$  Cep stars in NGC 3293 (Engelbrecht, 1986).

## 2. The analysis of periods distributions from GCVS' data

On the base of GCVS' sampling (75  $\beta$  Cep stars with the known periods) has been carried out the analysis of periods of these stars. We have constructed a histogram of periods distribution of  $\beta$  Cep variables with interval of periods  $\Delta P = 0.02$  days. It is given in Table 2 and shown in Figure 1.

On the base of our analysis of this histogram (Table 2 and Figure 1), we have 10 peaks (maxima) and some steps near to them (see Table 1). The main peak at the mean period of 0.170 days (interval of periods  $\Delta P=0.160-0.180$  days) has amplitude  $n=12$  stars. If

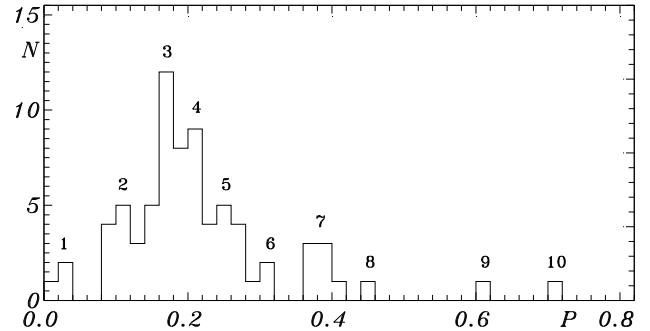


Figure 1: Periods distribution of  $\beta$  Cep stars according to GCVS, total number of stars  $N=75$ )

Table 1. Results of periods identifications for GCVS' data ( $m$  is the number of the peak)

$m$	$i$	$P_i$	$n$	$P_{theor}$	Ident.	$k_i$
1	1	0.0196	1	0.0188	$P_g/4$	1.043
1	2	0.028	2	0.028	$P_f/4$	1.000
2	3	0.08	2	0.084	$P_{1H}$	0.952
2	4	0.0896	2	0.090	$P_e$	0.996
2	5	0.101	2	0.100	$P_r$	1.01
2	6	0.108	1	0.1081	$P^*$	0.999
2	7	0.11267	3	0.11264	$P_f$	1.000
2	8	0.12377	1	0.120	$2P_s$	1.031
3	9	0.1378	3	0.135	$2P_{2H}$	1.021
3	10	0.153	4	0.150	$2P_g$	1.02
3	11	0.162	5	0.162	$2P^*_{1H}$	1.000
3	12	0.173	7	0.1728	$2P^*_e$	1.001
3	13	0.1684	12	0.169	$2P_{1H}$	0.996
3	14	0.1846	2	0.180	$2P_e$	1.026
3	15	0.1909	5	0.192	$2P^*_{r}$	0.994
4	16	0.201	6	0.200	$2P_r$	1.005
4	17	0.203	1	0.203	$3P_{2H}$	1.000
4	18	0.2156	4	0.216	$2P^*_{f}$	0.998
5	19	0.2374	4	0.2400	$4P_s$	0.989
5	20	0.2538	6	0.2535	$3P_{1H}$	1.001
5	21	0.2758	4	0.270	$4P_{2H}$	1.021
6	22	0.305	2	0.300	$3P_r = 4P_g$	1.017
7	23	0.3768	3	0.3755	$5P_g$	1.003
7	24	0.3971	4	0.400	$4P_r$	0.993
8	25	0.4504	1	0.4506	$4P_f = 6P_g$	0.9996
9	26	0.6096	1	0.609	$9P_{2H}$	1.001
10	27	0.7015	1	0.701	$7P_r$	1.001

Table 2. The histogram of  $\beta$  Cep stars' periods distribution ( $\Delta P=0.02$  days)

$\Delta P$ (days)	$N_{stars}$	$\Delta P$ (days)	$N_{stars}$	$\Delta P$ (days)	$N_{stars}$	$\Delta P$ (days)	$N_{stars}$
0.00-0.02	1	0.20-0.22	9	0.40-0.42	1	0.60-0.62	1
0.02-0.04	2	0.22-0.24	4	0.42-0.44	0	0.62-0.64	0
0.04-0.06	0	0.24-0.26	5	0.44-0.46	1	0.64-0.66	0
0.06-0.08	0	0.26-0.28	4	0.46-0.48	0	0.66-0.68	0
0.08-0.10	4	0.28-0.30	1	0.48-0.50	0	0.68-0.70	0
0.10-0.12	5	0.30-0.32	2	0.50-0.52	0	0.70-0.72	1
0.12-0.14	3	0.32-0.34	0	0.52-0.54	0	0.72-0.74	0
0.14-0.16	5	0.34-0.36	0	0.54-0.56	0	0.74-0.76	0
0.16-0.18	12	0.36-0.38	3	0.56-0.58	0	0.76-0.78	0
0.18-0.20	8	0.38-0.40	3	0.58-0.60	0		

we compare it with individual periods in this step, it gives us the mean period 0.1684 days. Its identification (see Table 1) is a double period of the first overtone  $2P_{1H}=0.169$  days for a fundamental period  $P_f=0.11267$  days (the mean period for three stars) in peak 2. This period is equal to the fundamental period of radial pulsation of the sun  $P_f=0.11264$  days. The ratio of these two periods  $P_i$  to  $P_{theor}$  is  $k_i=1.0003!$  The ratio of the period 0.1684 days to the theoretical one  $2P_{1H}=0.169$  days is 0.996. The analysis of periods of individual stars in the peak 2 gives us else four mean periods. We identify period 0.08 days (for two stars with equal periods) as the first overtone  $P_{1H}=0.084$  days of fundamental period  $P_f=0.11264$  days. The ratio of the observed and theoretical periods is equal 0.952. Period 0.0896 days is close to theoretical one  $P_e=0.90$  days (the ratio is 0.996) in our extended set of radial modes (see Bezdenezhnyi, 1994a, 1994b, 1997a, 1997b). Period 0.101 days (for two stars with close periods) is identified as period  $P_r=0.100$  days (the ratio is 1.01). And period 0.12377 days for one individual star is close to double period  $P_s$ :  $2P_s=0.120$  days (the ratio is 1.031). For the first peak we have one individual period 0.0196 days near the theoretical one  $P_g/4=0.0188$  days (the ratio is 1.04) and one the mean period about 0.028 days (for two stars) identified as  $P_f/4=0.028$  days. The main (third) peak has some more three mean periods in its wings (steps on the left and on the right). Period 0.1378 days (for three stars) is close to the theoretical one  $2P_{2H}=0.135$  days (the ratio is 1.021), period 0.153 days (for four stars) is identified as  $2P_g=0.150$  days (the ratio is 1.02) and period 0.1846 days (for two close periods) - as  $2P_e=0.180$  days (the ratio is 1.026). Period 0.1909 days (for five close periods) is not commensurable with  $P_f=0.11264$  days but it is so with period 0.108 days (we denote it as  $P^*$ ). The latter is a minimal of ones in the mean period  $P_f$ . Their ratio ( $P^*$  to 0.1909) is equal 0.566 (close to the theoretical ratio 0.5625 for double period  $P_r$  if we consider  $P^*$  as a fundamental one). There is one more

case like that for the mean period 0.2156 days in the next (the fourth) peak. One can identify this period as a double of  $P^*$ . We have a splitting of the period  $P_f$  and its overtones accordingly. In the fourth peak we have some more two mean periods: 0.201 days (for five close periods) near the theoretical period  $2P_r=0.200$  days (the ratio is 1.005) and 0.203 days (an individual period) equal to  $3P_{2H}$  (the ratio is 1). These two and the next periods are commensurable with  $P_f$ .

The fifth peak has three mean periods. Period 0.2374 days (for four close periods) identified as  $4P_s=0.240$  days (the ratio is 0.989), 0.2538 days (for six periods) near the triple  $P_{1H}$  ( $3P_{1H}=0.2535$  days, the ratio is 1.001), and 0.2758 days (for four periods) near  $4P_{2H}=0.270$  days (the ratio is 1.021). The sixth peak has only one mean period 0.305 days (for two close individual periods). It may be identified as the following equal overtones:  $3P_r=4P_g=5P_s=0.300$  days (the ratio is 1.017). The seventh peak has two mean periods: 0.3768 days (for three close periods) near  $5P_g=0.3755$  days (the ratio is 1.003) and 0.3971 days (for four close periods) near  $4P_r=0.400$  days (the ratio is 0.993). Further, three small peaks with single periods follow. Peak 8 has period 0.4504 days near  $4P_f=6P_g=0.4506$  days (the ratio is 0.9996). Peak 9 has period 0.6096 days near  $9P_{2H}=0.609$  days (ratio is 1.001), and peak 10 has period 0.7015 days near  $7P_r=0.701$  days (the ratio is 1.001).

So, we have explained all peaks in the histogram of periods distribution for  $\beta$  Cep stars by means of commensurability with the fundamental period for these stars coincides with the fundamental period of radial pulsation of the sun. Period  $P^*$  (with its sequence) is found in other pulsating stars types:  $\delta$  Sct and SX Phe, for example (from our unpublished data).

### 3. Identifications of $\beta$ Cep stars' periods in NGC 3293

Engelbrecht (1986) has studied ten  $\beta$  Cep stars in NGC 3293. All stars are multiperiodic, some showing

Table 3. Results of identifications of  $\beta$  Cep stars' periods in NGC 3293

<i>star</i>	<i>freq</i>	$P_i$	<i>A, mag</i>	<i>ident</i>	$k_i$	<i>star</i>	<i>freq</i>	$P_i$	<i>A, mag</i>	<i>ident</i>	$k_i$
5	5.64	0.177	.0074	$2P_e$	0.983	16	3.99	0.251	.0267	$3P_{1H}$	0.990
	6.66	0.150	.0050	$2P_g$	1.000		4.92	0.203	.0023	$3P_{2H}$	1.000
	7.17	0.139	.0028	$2P_{2H}$	1.030	18	5.66	0.177	.0136	$2P_e$	0.983
10	5.92	0.169	.0100	$2P_{1H}$	1.000		5.74	0.174	.0094	$2P_{*e}$	1.006
	5.68	0.176	.0061	$2P_{*e}$	1.019	6.58	0.152	.0046	$2P_g$	1.013	
	4.76	0.210	.0043	$2P_{*f}$	0.972	5.77	0.173	.0046	$2P_{1H}$	1.024	
11	6.11	0.164	.0034	$2P_{*1H}$	1.012	23	6.17	0.162	.0063	$2P_{*1H}$	1.000
	5.46	0.183	.0027	$2P_e$	1.017		5.74	0.174	.0043	$2P_{*e}$	1.007
	6.86	0.146	.0065	$2P_{*g}$	1.014	6.64	0.151	.0036	$2P_g$	1.004	
14	6.69	0.149	.0039	$2P_g$	0.993	24	4.85	0.206	.0065	$3P_{2H}$	1.015
	7.22	0.138	.0034	$2P_{2H}$	1.021		6.25	0.160	.0062	$2P_{*1H}$	0.988
	7.06	0.142	.0028	$2P_{*g}$	0.986	5.86	0.171	.0052	$2P_{1H}$	1.012	
14	6.61	0.151	.0023	$2P_g$	1.007	27	5.65	0.177	.0034	$2P_e$	0.983
	6.56	0.152	.0045	$2P_g$	1.013		4.40	0.227	.0103	$2P_f$	1.008
	6.33	0.158	.0040	$2P_{*1H}$	0.975	4.33	0.231	.0022	$4P_{*s}$	1.002	
65	5.90	0.169	.0029	$2P_{1H}$	1.000	8.81	0.114	.0030	$P_f$	1.012	
						9.97	0.100	.0022	$P_r$	1.000	

from two to five periods. He has found a range of possible non-radial modes identifications. "A roughly equal distribution of radial, dipole and quadrupole modes appears in these stars."

Our identifications give that all ten stars from Table 3 of this work maybe have radial modes (and their harmonics) in our extended system: F (2 times), 1H (5), 2H (4) and the modes R (1), G (6), E (4) introduced by the author (Bezdenzhnyi, 1994a, 1994b, 1997a, 1997b) earlier for RR Lyrae,  $\delta$  Scuti and for other radial pulsating stars. The sequence of period  $P^*=0.1081$  days:  $2P_{*1H}$ ,  $2P_{*f}$ ,  $2P_{*e}$ ,  $2P_{*g}$ , and  $4P_{*s}$  is present too (see Table 3). All the periods we may explain as radial modes for these two main fundamental periods  $P_f=0.11264$  and  $P_{*f}=0.1081$  days. The presence of these two sequences is the reason that we have so many close frequencies.

We know (GCVS) that the majority of  $\beta$  Cep stars probably show radial pulsations, but some (V649 Per) display non-radial pulsations. Multiperiodicity is characteristic of many of these stars. In our identifications we don't need to attract non-radial pulsations for ten  $\beta$  Cep stars as in original work (Engelbrecht, 1986).

#### 4. The analysis of periods distributions from Shobbrook's (1985) data

On the base of Shobbrook's (1985) data for 47  $\beta$  Cep variables we have constructed a histogram of periods distribution of these stars with the interval of periods  $\Delta P=0.01$  days (Fig.2). Using distributions of individual periods into close groups, we have the following 23 mean periods (see Table 4). 11 periods from 23 ones

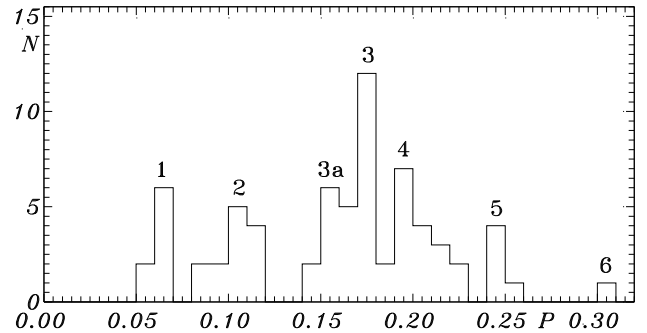


Figure 2: Periods distribution of  $\beta$  Cep stars (Shobbrook, N=47)

are commensurable with period  $P^*=0.1081$  days, and the rest 12 periods follow  $P_f$ -sequence.

This border at period of 0.133 days has astrophysical bases as it follows from Shobbrook's (1985) data. The mean pulsation constant, Q, has a linear correlation with period and a sharp bend at this period of 0.133 days (see Fig. 3). After this period the slope of the second line is less. Using the pulsation ratio  $Q = P \sqrt{\frac{\langle \rho^* \rangle}{\langle \rho \rangle}}$ , we estimate mean densities  $\langle \rho \rangle_1=0.068$  and  $\langle \rho \rangle_2=0.038 \frac{g}{cm^3}$  for the mean lines (before and after period 0.133 days of this sharp bend). This border coincides with a gap between two peaks (2 and 3, see Fig 1 and 2). This gap lies between theoretical periods  $2P_{*2H}=0.130$  days and  $2P_{2H}=0.135$  days. Mean masses and radii distributions from period have a gap (or minimum) near this period. They change synchronously. So is behavior of  $\lg T_e$  and  $M_{bol}$  with pe-

Table 4. Periods of  $\beta$  Cep stars' from Shobbrook's data histogram (47 stars)

$P_i$	<i>ident</i>	$k_i$	$P_i$	<i>ident</i>	$k_i$	$P_i$	<i>ident</i>	$k_i$	$P_i$	<i>ident</i>	$k_i$
0.055	$P_f/2$	0.977	0.115	$2P^*_s$	0.997	0.169	$2P_{1H}$	1.000	0.212	$2P^*_f$	0.981
0.061	$P_s$	1.017	0.118	$2P_s$	0.983	0.174	$2P^*_e$	1.006	0.229	$4P^*_s$	0.993
0.067	$P_{2H}$	0.991	0.130	$2P^*_{2H}$	theor	0.178	$2P_e$	0.989	0.246	$4P_s$	0.984
0.088	$P^*_e$	1.017	0.141	$2P^*_g$	0.979	0.191	$2P^*_r$	0.995	0.250	$3P_{1H}$	0.984
0.100	$P_r$	0.999	0.153	$2P_g$	1.020	0.201	$2P_r$	1.005	0.258	$4P^*_{2H}$	0.995
0.108	$P^*_f$	0.999	0.161	$2P^*_{1H}$	0.993	0.203	$3P_{2H}$	1.002	0.302	$4P_g$	1.007

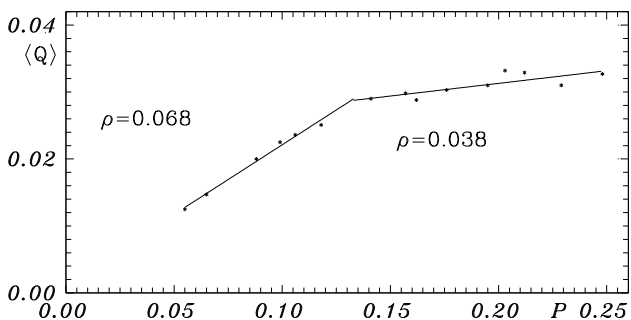


Figure 3: Mean pulsation constant correlation with period

riod as minimum  $M_{bol}$  coincides with one for  $T_e$ . That behavior of physical parameters before, after and at period of 0.133 days let us to pick out two groups of  $\beta$  Cep stars. It is possible one of them is BCEPS stars group.

In addition to GCVS' classification it is proposed to extend the range of periods for short-period group (BCEPS) to (0.02-0.13) days instead of (0.02-0.04)

days. It is proposed a new classification of these stars according to their mode identifications.

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