MODELING OF REGIONS OF ASTEROID POSSIBLE MOTION

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ABSTRACT. Peculiar properties of construction of initial regions of asteroids' possible motion from observational data in the form of probabilistic ellipsoids are investigated. For the objects which observed only in one appearance, this problem may be essentially nonlinear, and the usual method for their construction with the help of linear estimations of covariance matrices may became unacceptable. In order to make possible application of the linear estimation methods which has been developing in mathematical statistics the problem of decreasing nonlinearity is discussed. The solution of this problem with the help of appropriate system of initial parameters of asteroid orbits choice, as well as initial time and weighting matrices of observational errors is proposed. Efficiency of such technique had justified by numerical experiments with the usage of model and real observations.

Key words: asteroid, probabilistic motion, initial region, weighting matrix, nonlinearity.

The number of asteroids known today (e.g. from Bowell catalogue) is about 375000. About a quarter of them is observed only at one appearance, and the sixth part is observed on small measured intervals (less than 40 days). For such objects, the regions of possible motion defined with traditional methods of linear estimation, may be inauthentic. Determination of asteroid's probabilistic motion contained two basic stages: the forming of the initial region C_q^0 in space of the motion parameters $q = (q_1, ..., q_m)$ and its reflection $C_q^0 \to C_p^t$ to space of the parameters $p = (p_1, ..., p_s)$, on any given time t (Chernitsov et al., 1998). The display statement is differential equation system of the asteroid motion, as well as equations joined with each other spaces $\{q\}_t$ and $\{p\}_t$. The reflection is realized by means compact ensemble of trajectories from the whole of initial region or from its boundary surface. The methods of solution this problem is well developed and so basic our efforts were turned to the problem of construction of initial region.

The problem of determination of the initial region is reduced to finding estimations initial parameters $\hat{q}$ and their covariance error matrix $\hat{D}$ for construction of probabilistic ellipsoids which enveloped the regions of possible motion of investigated asteroid. For objects, which were observed only in one appearance, this problem may be essentially nonlinear, and usage of covariance matrices become impossible. But, if in some way or other one reduces initial nonlinearity to permissible level, the application of well-known technique of linear estimation on basis of using covariance matrices becomes rightful (Chernitsov et al., 2006). With that end in view we fulfilled great number of numerical experiments based on real and model observations. Simulation has been realized in the following way:

1. The initial parameters of the asteroid orbit are taken as "true";
2. On basis of these parameters the "true" values of measured parameters have been determined on given times of observations;
3. In these "true" values one introduced errors so that model observations were not uniformly precise. For that random number generators and given weighting matrices of observation errors have been used;
4. After that nonlinear least squares problem have been applied for finding estimations of initial parameters and their covariance error matrix.

For construction of error ellipsoids in nonlinear problems one use covariance matrices, although that is, strictly speaking, acceptable only within the framework of the theory of linear estimation.

The least squares problem is reduced to minimization of target function

$$\Phi(q) = [d - d^*]^T P [d - d^*] = min,$$

where d is N - dimensional vector function measured parameters, d^* is N - dimensional vector measurements, P is the weighting matrix. In linear formulation $d = Aq$, where q is m-dimensional vector defined parameters, and solution evaluated by relations

$$\hat{q} = \left(A^T P A\right)^{-1} A^T P d^*,$$

$$\hat{D} = \sigma_0^2 \left(A^T P A\right)^{-1}.$$

Here $\sigma_0 = \sqrt{\Phi(\hat{q})/(N-m)}$ is root-mean-square error of unit weight. In nonlinear case connection between measured and defined parameters is nonlinear, $d = d(q)$, hence the problem may be solved only by numerical iterative methods (e.g. by method of differential correction data)

$$q^{n+1} = q^n - \left\{ \left[R^T(q) P R(q)\right]^{-1} R^T(q) \left[d(q) - d^*\right] \right\}_{q=q^n}.$$

Here $R(q) = (\partial d / \partial q)$ is the matrix of $(N \times m)$ - dimension; $n = 0, 1, ...$ is the number of iteration. Covariance matrix one calculated by means of the same relation as in linear case

$$\hat{D} = \sigma_0^2 \left(R^T P R\right)^{-1}_{q=\hat{q}}.$$

These solutions defined probabilistic regions for error's dispersion. In linear case their isosurfaces will be ellipsoidal, while in nonlinear case they will be non ellipsoidal. Distinction of the level surface from ellipsoidal ones is given by remainder term $\Delta\Phi$ in expansion of target function of nonlinear least-squares problem

$$\Phi(q) = \Phi(\hat{q}) + \Delta q^T \left(R^T R\right) \Delta q + \Delta\Phi.$$

Subject to value of that term the initial region may be found either weakly or strongly nonlinear. The degree of nonlinearity is depended on size of the region probabilistic spread of estimations for initial parameters and on degree of nonlinearity functional connection between measured and defined parameters. In many problems of estimation the influence of the last factor on legitimacy for application of the linear methods is negligible because of relatively small-sized regions of probabilistic solution spread (e.g. the problem of positioning with the help of the satellite system NAVSTAR). In the case of asteroids, which have been observed at one appearance, regions of probabilistic solution spread will be large, and degree of nonlinearity of the functional connection between measured and defined parameters will produce the strongly nonlinear initial region.

To obtain how often the strong nonlinearity of estimation problem occurred for such objects, we investigated a few hundred asteroids which were observed in one appearance. As parameter of nonlinearity we have used the quantity

$$\chi = \frac{\sigma_{max} - \sigma_{min}}{\sigma_{min} - \sigma_0},$$

where σ_{max} and σ_{min} is maximal and minimal root-mean-square residuals in vertices of an error ellipsoid. This quantity gives deviation of the level surface of probabilistic solution dispersion from ellipsoidal ones. We have considered the influence of some factors on value of this parameter. These factors were the space of used phase variables, disposition of initial time with respect to measuring interval, the insertion of suitable weighting matrices in estimation algorithms and possibility of decreasing nonlinearity with the help of screening observations.

In the table 1 are given asteroids of some hundred ones, for which the estimations of nonlinearity factor were obtained by construction of initial domain in spaces of Cartesian and Keplerian variables. About one third part of these asteroids has low nonlinearity in Cartesian variables, whereas in Keplerian variables the nonlinearity is strong for all objects. In this case that due to the fact that functional connection between rectangular coordinates and measured parameters α, δ is less nonlinear than connection with them Keplerian elements.

Table 1: The nonlinearity coefficients in spaces of Cartesian (χ_{dec}) and Keplerian (χ_{kep}) variables. ΔT is the measured interval (in days), N is the number of observations.

Object	ΔT	N	χ_{dec}	χ_{kep}
2001XN254	48	47	0.000	0.690
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2004GE2	12	94	0.000	245.7
2003WN188	24	75	0.009	1805.9
2004RZ164	22	15	0.580	17.13
1998QA62	21	44	3.584	7097.9
2003WV172	25	09	12.56	25995.3
2003UN284	31	06	62.97	44194.3
2003XB22	09	16	168.84	20369.0
2003QZ30	49	55	441.5	44288.1
2002PQ142	14	18	822.0	70990.0

The estimations of nonlinearity factor for some asteroids from Bowell catalogue, whose initial moments lie outside of measured observations interval are given in table 2. One can see that by choice of initial moments from Bowell catalogue the problem of construction initial probabilistic regions of motion is strongly nonlinear. Displacement of initial moment to inside of measured interval essentially decreases nonlinearity for all objects and make estimation problem weakly nonlinear for four asteroids.

In the table 3 and figure 1 we presented experimental results of decreasing nonlinearity of least-squares problem with the help of appropriate screening observations. Here $n = 0$ corresponds to initial sample of observations and $n = 1, 2, 3$ corresponds to samples of observations after first, second and third screening.

The insertion of suitable weighting matrix in algorithm of construction of initial domain may significantly decrease it, but we do not know for the time

Table 2: The nonlinearity of initial region by various choice of initial time. Here χ is nonlinearity factor where initial moment lie outside of measured interval, $\chi^{'}$ is nonlinearity factor where initial moment is arithmetical mean of observational moments, Δt is the value of initial moment shift (in day).

Object	Δt	χ	$\chi^{'}$
2003WC168	+11	69.93	7.03
2003WU172	+04	12.56	12.52
2003WX153	+09	44.55	3.94
2003XB22	+09	168.8	13.79
2003XG	+02	5.07	1.56
2003YQ94	+15	515.1	4.61
2004XO63	-44	1.42	0.05
2004XM130	+14	344.4	24.45
2004YA	-44	158.1	0.71
2004YC	-43	8.28	0.05
2004YD	-43	35.83	0.24
2004YD5	-40	12.52	0.01
2004YE	-43	30.95	0.63
2004YG1	+17	13.50	2.52
2004YR	+17	371.7	15.21
2004YZ23	+02	50.51	45.53
2005AB	+11	68.96	17.80
2005AC	+11	94.31	29.80

present how made that in real case. However, in case of modeling unequal precise observations we specified weighting matrix for observational errors, which may be used later on as exact weighting matrix for solution of the least-squares problem. Numerical experiments have showed that exact weighting coefficients distinctly decreases nonlinearity of least-squares problem and probabilistic region, if there are more than one group of observations with the same numbers, the mean accuracy of which differed from each other more than three times. Such result was received both for observability of asteroid in one appearance and in several ones. The last result is especially important, since for many asteroids which were observed in several appearances in 80 – 90-th of the past century take place just such distinction in mean accuracy of observations in different appearances. For example, observational accuracy of asteroid Geographos has been increased from 2" up to 0.5", i.e. about in 4 times. For such cases of observability of asteroids sufficiently simple method of weighting matrices construction is realizable. We named such matrices "averages". They represent diagonal matrices, whose elements are constant for observations of single appearance, and they are defined by the rule

$$p_i = \frac{\sigma^2}{\sigma_{0i}^2},$$

Table 3: The estimations of nonlinearity factors by screening observations of asteroid 2004XM130. Here n is the sample number, N is the number of observations, ΔT is the measured interval (in days), χ is the nonlinearity factor.

n	N	ΔT	χ
0	36	5	20.06
1	33	5	18.42
2	30	5	15.95
3	28	5	13.42

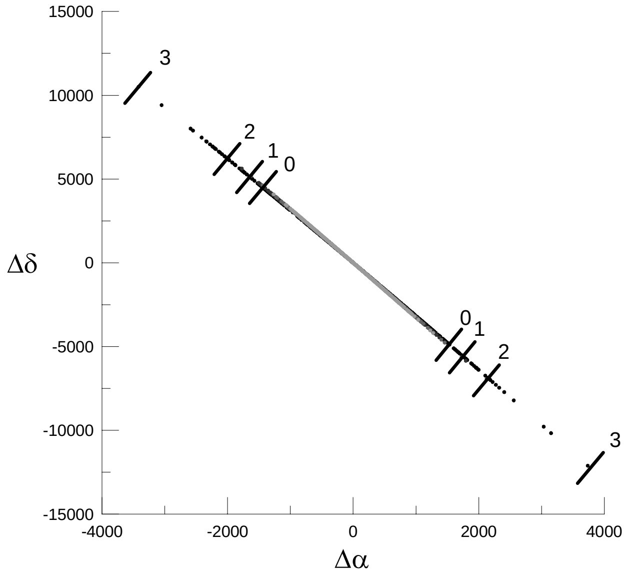

Figure 1: The regions of probabilistic motion of asteroid 2004XM130 obtained by successive screening observations.

where σ_{0i}^2 is the mean-square error for representation of observations of i-th appearance.

In the figure 2 we presented the probabilistic dispersion of estimations, which were obtained by using "exact", "average" and identity weighting matrices. One can see that utilization of "average" weighting coefficient in the least-squares problem allows us to reduce essentially the region of probabilistic displacements of initial parameter estimations in comparison with generally accepted variant, when weighting matrix is considered as identity. Similar picture is for probabilistic regions defined by means of covariance matrices of initial parameter estimations (figure 3).

The comparison of domain size supplements the numerical estimations of their volumes (table 4). These estimations represent the ratio of volumes of probabilistic error ellipsoids, defined with usage of "average" ($V_{p\sigma}$) and "exact" (V_{pt}) weighting coefficients, to vol-

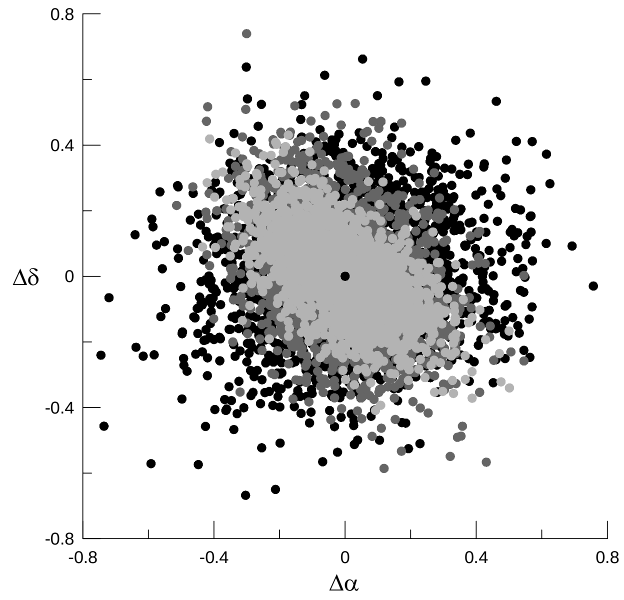

Figure 2: The probabilistic dispersion of estimations, which were obtained with usage of "exact" (light-grey colour), "average" (dark-grey colour) and identity (black colour) weighting matrices. $\Delta\alpha$ and $\Delta\delta$ are given in second of arc, and represented the deviation of estimation from the "true" solution.

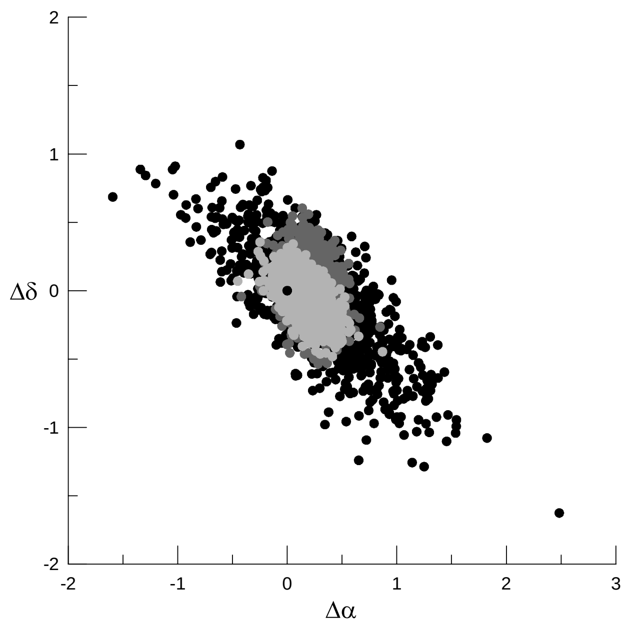

Figure 3: The probabilistic regions defined by means of covariance matrices of initial parameter estimations with different weighting matrices.

ume of error ellipsoid obtained under the assumption of equal precision for all observations (V_I). One can see that usage of "average" weighting coefficients in case of observability of asteroid in two appearances with different accuracy, distinguished on average in 4 times, allows us noticeably to decrease both regions of probabilistic dispersion of initial parameter estimations itself and computational probabilistic error regions of these estimations, which are defined by their covariance matrices.

Table 4: The ratio of volumes of probabilistic error ellipsoids, defined with different weighting coefficients.

	min	max	average	dispersion
$V_{p\sigma}/V_I$	0.0023	0.197	0.0377	0.0279
V_{pt}/V_I	0.0007	0.044	0.0055	0.0049

So, the computations carried out allows us to make conclusions as follow:

1. Utilization of rectangular coordinate system makes the problem of construction of initial probabilistic region practically linear on numerous occasions. Properties of differential correction method in space of Cartesian coordinates (convergence domain and rate of convergence) are also better than in spaces of other variables.

2. The estimation problem has the least nonlinearity, if initial moment is equal to arithmetical mean of observational moments.

3. In some cases the nonlinearity of problem may be decreased by means of screening of observations.

4. Insertion of suitable weighting matrix in estimation algorithms also decreases nonlinearity of problem and size of probabilistic regions. If observations of asteroid cover more than one appearance, in a number of cases there may be an easy method of construction of such weighting matrices.

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References

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