ABOUT INFLUENCE OF LATERAL HETEROGENEITIES IN THE EARTH UPPER MANTLE ON THE LOVE NUMBERS FOR DIURNAL TIDES

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ABSTRACT. On the base of the combined method, consisting of the Sasao, Okubo, Saito approach with the iteration procedure, allowing the coupling of the finite element method with the solution of the Laplace equation, it was defined the influence of the horizontal heterogeneities in the upper mantle on the diurnal Love and Shida numbers of the second order. It was determined, that maximum deviations due to such heterogeneities can reach 0.5 percent for Love numbers k and h, for Shida number - l the deviations don't depend from the tidal wave frequency and approximately make up 0.2 percent. For evaluation of the accuracy of the suggested method it was carried out the comparison of the Love numbers k for the main diurnal tides, obtained on the base of the PREM - earth model, with respective results of Mathews, Buffet, Shapiro (1995) and Dehant (1987).

Key words: Diurnal Love numbers : lateral heterogeneities of the upper mantle: finite element method.

1. Introduction

It is known about availability of the lateral heterogeneities in the earth upper mantle for a long time. The investigations of the earth surface wave propagation, started at the beginning of the last century, had permitted to discover horizontal heterogeneities of the upper mantle. The existence of this heterogeneities is the main factor, which makes difficulty on the detailed study of the upper part of the earth structure by the surface wave methods. So it's not wonder, that so far quantitative investigations of the horizontal heterogeneities of the earth upper part, so it's rest part, are on the initial state. The grave seismic investigation of horizontal heterogeneities of the upper mantle in the Tibet plate region was made by Woodward, Molnar (1995) [1]. The authors went to conclusion, that Tibet plate is the most significant by the depth and stretch of the horizontal heterogeneities of the upper mantle. At this region the width of continental lithosphere approximately two ones large then normal continental lithosphere width, stretches out more then 2000 km. It tends to the forming of the lateral heterogeneities with depth from 250 to 300 km in the upper mantle. It's arise the question, how much similar anomalies influence on the such earth global characteristics, as the Love and Shida numbers. Because it's very difficult to give strict answer on this question, so we have modeled the influence of horizontal upper mantle heterogeneities similar that, described in the work [1], on the diurnal Love and Shida numbers of the second order. For the evaluation of the accuracy of the suggested method it was carried out the comparison of the Love numbers k for the main diurnal waves, obtained on the base of spherical symmetric earth model PREM, with respective results of Mathews, Buffet, Shapiro (1995) [2] and Dehant (1987) [3].

2. Combined method resolving of the problem

As in further we shall consider the Love and Shida numbers of the second order in the diurnal range of tidal waves, so we can neglect by influences of the inner core dynamics [2] and use the approximate Saso, Okubo, Saito (1980) approach [4]. Following this approach, we shall define the complex compliances of the earth, consisting on the elastic lateral heterogeneity upper mantle, liquid outer core and inner solid core in the spherical quasi-statical approximation. In this case, we have two quasi-statical equations for elastic mantle and liquid core, presented in the Tisserand reference system (X, Y, Z):

$$0 = grad(V_c + \phi_n + V_1 u_R g(R)) -$$

$$-div\vec{u}gradW_g + \frac{1}{\rho}div\hat{P}_1;$$

$$0 = grad(V_e + \phi_n + \phi_c + V_1) + \frac{1}{\rho}div\hat{P}_2. \qquad (1)$$

Here $\vec{u}$ - displacement vector; $V_e = \gamma\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - tesseral part of tidal wave potential, γ - it's amplitude; $\phi_n = -\epsilon\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - changing of centrifugal potential due to nutation; ϵ - polar motion radius of tidal wave; $V_1 = \gamma_1\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - changing of gravitation potential due to earth deforming, γ_1 - it's amplitude; $\phi_c = -\beta\Omega^2(xzcos\sigma t + yzsin\sigma t)$ - changing of centrifugal potential due to earth core rotation; β - polar motion radius of core rotation; Ω - angle earth velocity; W_g - self gravitation potential; ρ - density; σ - frequency of the tidal wave; R - earth point radius; u_R - radial displacement; $g(R)$ - gravity acceleration; $\hat{P}_1, \hat{P}_2$ - changing of stress tensors in the elastic mantle and liquid core respectively. Suggesting, that oscillations into the liquid core are happening with the tidal wave frequency σ, also suggesting the absence of the earth surface loads, make up Lagrange functionals. The functionals express the full energy of the elastic mantle and liquid core in the cylindric coordinate system (z, r, φ), here axis r coincide with the Tisserand axis Z:

$$\begin{aligned} E_m = \pi \int_{F_s} & [B_1(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 4B_2\varepsilon_{zr}^2 + \\ & + 2B_3(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})]rdzdr - \\ & - \pi \int_{F_s} [2(\gamma - \epsilon + \gamma_1)\Omega^2 rw + w(\frac{\partial w}{\partial z}cos\alpha + \\ & + 2\frac{\partial u}{\partial z}sin\alpha)g(R) + 2w(wcos\alpha + \\ & + 2usin\alpha)g'_R cos\alpha - 2w(\frac{\partial w}{\partial z} + \\ & + 2(\frac{\partial u}{\partial r}))g(R)cos\alpha + 2(\gamma - \epsilon + \gamma_1)\Omega^2 zu + \\ & + u(2\frac{\partial w}{\partial r}cos\alpha + \frac{\partial u}{\partial r}sin\alpha)g(R) + 2u(2wcos\alpha + \\ & + usin\alpha)g'_R sin\alpha - 2u(2\frac{\partial w}{\partial z} + \\ & + \frac{\partial u}{\partial r} + \frac{u}{r})g(R)sin\alpha]\rho rdzdr; \end{aligned} \quad (2)$$

$$\begin{aligned} E_f = \pi \int_{F_s} & [B_4(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2) + 2B_4(\varepsilon_{zz}\varepsilon_{rr} + \\ & + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi})]rdzdr - \\ & - \pi \int_{F_s} [2(\gamma - \epsilon - \beta + \gamma_1)\Omega^2 rw + \\ & + 2(\gamma - \epsilon - \beta + \gamma_1)\Omega^2 zu]\rho rdzdr. \end{aligned} \quad (3)$$

Where $B_1 = K + \frac{4\mu}{3}$; $B_2 = \mu$; $B_3 = K - \frac{2\mu}{3}$; $B_4 = K_f$; μ, K - shear and compress modules of the elastic mantle; K_f - compressibility of the liquid core; ε_{ij} - deformation components; w, u - displacement components along z, r respectively; F_s - meridional cross section area of the earth; $cos\alpha = \frac{z}{R}, sin\alpha = \frac{r}{R}$. For resolving the system equations (1), without calculation of the surface loads, the finite element method is using. This method is based on the variational Lagrange principal, expressing the minimum of the full system energy, tends to resolving system of the variational equations:

$$\delta E_m = 0; \delta E_f = 0. \quad (4)$$

The detailed description of the finite element resolving problem is presented in the publications [5,6]. Another hand, the finite element only, doesn't allow to calculate the earth gravity field relaxation due to tidal earth deformations. For resolving this problem we will use Wu (2004) approach [7]. As earth deforming potential V_1 is harmonic function, so it must satisfy to Laplace equation. Another hand, exactly resolving of the Laplace equation can be derived from the radial displacements u_r on the heterogeneity layer boundaries of the earth. The radial displacements can be detected by the finite element method [5]. The combined resolving of the problem is realizing during for iteration process:

1)firstly, the finite element problem is realizing for $V_1 = 0$, as a result, the radial displacements u_r are defined;

2)the meanings of the earth deforming potential V_1 are defining from the detected radial displacements u_r by the Wu formalism [7];

3)then, presented here procedure repeats with the calculation of the derived meanings of the potential V_1;

4)the iteration process is going on till of the convergence of the results of the resolving problem; as calculation shows, the convergence have been reached after 4 - 6 iterations.

3. The detection of the diurnal Love and Shida numbers with the calculation of the upper mantle heterogeneities

The diurnal Love and Shida numbers of the second order, following to the SOS (1980) approach [4], were detected by formulas:

$$k = k_0 + \frac{k_1\beta}{\gamma - \epsilon}; h = h_0 + \frac{h_1\beta}{\gamma - \epsilon}; l = l_0 + \frac{l_1\beta}{\gamma - \epsilon}; \quad (5)$$

$$\begin{aligned} \epsilon &= (1 - \frac{\kappa_c}{e}\frac{n}{\Omega})\epsilon_r + \frac{A}{A_f}\frac{n}{\Omega - n}\beta; \\ \beta &= \frac{A}{A_m}\frac{\Omega - n}{n - n_0}(1 - \frac{\gamma_c}{e} + \frac{\gamma_c - \kappa_c}{e}\frac{n}{\Omega})\epsilon_r; \\ \epsilon_r &= \frac{e\Omega}{\Omega - n}\gamma; n_0 = -\frac{A}{A_m}(e_f - \beta_c)\Omega; \end{aligned} \quad (6)$$

$$\begin{aligned} \kappa_c &= \frac{k_o R_e^5\Omega^2}{3GA}; \gamma_c = \frac{k_0^f R_f^5\Omega^2}{3GA_f}; \\ \zeta_c &= \frac{k_1 R_e^5\Omega^2}{3GA}; \beta_c = \frac{k_1^f R_f^5\Omega^2}{3GA_f}. \end{aligned} \quad (7)$$

Where G - gravitation constant; ϵ_r - polar motion radius for rigid earth; e, e_f - ellipticities of the earth and outer core respectively; A, A_m, A_f - the main moments of inertia of the earth, mantle and outer liquid core; n, n_o - the circle nutation frequency and resonance of the liquid core frequency; R_e, R_f - radiuses of the earth and liquid core respectively. The static parameters: k_0, k_o^f, h_0, l_0 - were defined on the base of above presented iteration procedure under the static conditions: $\gamma - \epsilon = 1, \beta = 0$, from the formulas: $k_0 = \tilde{\gamma}_1(R_e), k_0^f = \tilde{\gamma}_1(R_f), h_0 = \tilde{u}_R(R_e), l_0 = \tilde{u}_\tau(R_e)$. There $\tilde{\gamma}_1(R_e), \tilde{\gamma}_1(R_f)$ - are undimensioned amplitudes of the gravitating potential V_1 on the earth and liquid core surfaces respectively; $\tilde{u}_R(R_e), \tilde{u}_\tau(R_e)$ - are undimensioned amplitudes of the radial and tangential displacement on the earth surface respectively. The dynamic parameters: k_1, k_1^f, h_1, l_1 - were defined analogically under dynamic conditions: $\gamma - \epsilon = 0, \beta = 1$. Then compliances: $\kappa_c, \gamma_c, \zeta_c, \beta_c$ were detected by the formulas (7). At the detection iteration process ten layer earth model was used. For the counting out of modeling of the lateral upper mantle heterogeneities the spherical symmetric earth model PREM was chose. Following to conclusions of the work [1], it was made suggestion, that power of the continental lithosphere in the region of the horizontal anomaly reaches 250 km. As horizontal dimensions of the upper mantle anomaly, the values: 1000 and 3000 km were took. In the Table 2. the meanings of the Love and Shida numbers of the second order for the main diurnal tidal waves, obtained on the base of the PREM model and with calculation of considering above horizontal anomalies, are presented. The comparison of the table data shows, that maximum deviations of such upper mantle heterogeneities for the Love number k change from 0.07 percent for the waves, distant from the liquid core resonance frequency n_0, to 0.5 percent for the tidal wave Ψ_1, for the Love number h from 0.03 to 0.5 percent. For the Shida number l - the deviations don't depend from the tidal wave frequency and approximately equal 0.2 percent. For the evaluation of the accuracy of suggested above combined method, the comparison of the Love numbers k of the second order for the main tidal waves, obtained by this method on the base of PREM model, with respective results of Mathews, Buffet, Shapiro (1995) [2] and Dehant (1987) [3] are presented in the Table 1.

Table 1: The Love numbers k of the second order for main tidal waves, obtained by the combined method for the PREM model, also analogical results of Mathews, Buffet, Shapiro (1995) and Dehant (1987).

Tidal waves	Comb. method	MBS	Dehant
O_1	0.2951	0.2962	0.2958
M_1	0.2940	0.2950	0.2945
P_1	0.2841	0.2848	0.2850
S_1	0.2773	0.2766	0.2783
K_1	0.2535	0.2537	0.2547
Ψ_1	0.4667	0.4662	0.4667
OO_1	0.2977	0.2989	0.2985

Table 2: The Love and Shida numbers of the second order for main tidal waves, obtained by the combined method on the base of the spherical symmetric earth model PREM, also with calculation of the upper mantle heterogeneities of the order 1000 and 3000 km respectively.

Tidal waves	PREM	1000 km	3000 km
O_1			
k	0.295093	0.295160	0.295293
h	0.597420	0.597486	0.597619
l	0.0837803	0.0838468	0.0839797
P_1			
k	0.284098	0.284167	0.284305
h	0.575715	0.575783	0.575919
l	0.0844556	0.0845203	0.0846498
S_1			
k	0.277294	0.277367	0.277513
h	0.562282	0.562357	0.562507
l	0.0848735	0.0849370	0.0850640
165.545			
k	0.256157	0.256257	0.256457
h	0.520556	0.520679	0.520926
l	0.0861716	0.0862304	0.0863481
K_1			
k	0.253455	0.253560	0.253770
h	0.515221	0.515354	0.515619
l	0.0863376	0.0863957	0.0865120
165.565			
k	0.250363	0.250474	0.250696
h	0.509117	0.509261	0.509549
l	0.0865275	0.865848	0.0866995
Ψ_1			
k	0.466626	0.467409	0.468975
h	0.936045	0.937565	0.940606
l	0.0732452	0.0732927	0.0733877
Φ_1			
k	0.324854	0.324943	0.325122
h	0.656171	0.656289	0.656525
l	0.0819524	0.0820218	0.0821606
OO_1			
k	0.297658	0.297725	0.297859
h	0.602483	0.602550	0.602685
l	0.0836228	0.0836896	0.0838232

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