# ATLAS OF LIGHT CURVES OF FAINT MIRA-TYPE STARS. STATISTICAL RELATIONS BETWEEN THE CHARACTERISTICS OF SMOOTHED PHASE LIGHT CURVES 

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#### Abstract

We propose a set of the photometric parameters which could be useful for the classification of the pulsating Mira-type stars and related objects and determination of the EAGB and TPAGB stages of the stellar evolution.

To solve this problem, the light curves of faint Miratype stars and of the semi-regular variable V411 Sct were approximated using the program FDCN, which computes a trigonometric polynomial of a statistically optimal degree (I.L.Andronov, 1994, 2003). The atlas of statistically optimal fits of the phase curves of 34 long-period is presented, based on digitized data from the scanned "Atlas" by P. Maffei and G.Tosti (http://astro.fisica.unipg.it/atlasmaffei/main.htm).

Some statistical relations between the parameters of the trigonometrical polynomial approximation of the phase curve are analyzed. for an additional criterion of detailed classification of long-perodic variables, we used various parameters, e.g. "period", "amplitude", "asymmetry", "slope of the ascending branch", "characteristic time of brightening by 1 " . Discussion of the results is presented.


Key words: Variable stars: pulsating: Mira-type.

Earlier we had approximated light curves of 62 variable Mira-type stars based on the observations obtained by the members of AAVSO between 1974 and 1977 using the trigonometrical polynomial with optimum values of number of harmonics and the period (Kudashkina and Andronov, 1996). The atlas of smoothed curves was compiled.

On the basis of this research, for more detailed classification of the Mira-type stars and related semiregular variable red giants and supergiants of an asymptotic branch (for example, at stellar evolutionary stages EAGB and TPAGB), we have introduced three groups of parameters: tbasic (the period, amplitude, asymmetry of a light curve), additional (degree of a trigonomet-
rical polynom, amplitudes of harmonics, phase shifts in relation to the main wave of a light curve), slope parameters (an inclination of ascending and descending branches of a light curve, time interval of brightness increase by $1^{\mathrm{m}}$, difference of this parameter from a corresponding sinusoid). The correlation analysis of 25 parameters from listed above had been carried out. More than 60 dependences have a correlation coefficient of $\geq 0.6$ and above. All these dependences were presented in the papers: Andronov, Kudashkina and Rudnitskij (1989); Kudashkina and Andronov (1994, 1996, 1998ab); Andronov and Kudashkina (2006, 2008). In particular, it is noticed that stars RT Cyg, S UMi, T Cas, R Aql, most likely, are at the stage of first helium flash, or they are at the stage of the double shell source; and the stars X Aur, U CMi, possibly, only recently have come to AGB, and the stars T Cam and especially W And are for a long time already at the AGB stage and can undergo multiple flashes in a shell source (Kudashkina, 1999).
In the present work we continue similar research for 34 more stars using the observations of same authors - Paolo Maffei and Gino Tosti (http://astro.fisica.unipg.it/atlasmaffei/main.htm).
Also the atlas of smoothed light curves was compiled. Examples of light curves for stars representatives of two from three groups of the periods (less than 250 days, from 250 till 350 days and more than 450 days) are shown in Fig. 1,2. The parameters are listed in Table 1.

Contrary to previous papers, it should be noted that here the parameters of a slope of branches $\left(m_{i}=\right.$ $\left.(\mathrm{d} m / \mathrm{d} t)_{\text {asc.br. }}, t_{i}=(\mathrm{d} t / \mathrm{d} m)_{\text {asc.br. }}\right)$ are calculated in the middle between the maximum and minimum. In earlier researches where the same parameters were computed at a phase of the largest slope. Correlation coefficients between the pairs of key parameters are listed in the Table 2.
Thus, we don't see significant correlations among the calculated parameters of smoothed light curves.


Figure 1. Smoothed light curve of HI Ser.
Table 1: Characteristics of smoothed phase light curves of faint Mira-type stars

| Star | $P, \mathrm{~d}$ | $\Delta \mathrm{mag}$ | $f$ | $m_{i}$ | $t_{i}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| LL Ser | 250 | $1.68 \pm .12$ | $0.248 \pm .021$ | - | - |
| GI Ser | 254 | $3.93 \pm .09$ | $0.500 \pm .001$ | -0.049 | 20.6 |
| LP Ser | 258 | $1.79 \pm .06$ | $0.444 \pm .052$ | -0.027 | 36.4 |
| V 420 Sct | 261 | $2.67 \pm .07$ | $0.450 \pm .010$ | -0.011 | 90.6 |
| V 1970 Sgr | 272 | $2.10 \pm .06$ | $0.428 \pm .019$ | -0.029 | 34.0 |
| GH Ser | 272.5 | $1.56 \pm .06$ | $0.329 \pm .027$ | - | - |
| GN Ser | 273.5 | $2.47 \pm .04$ | $0.435 \pm .013$ | -0.034 | 29.8 |
| IZ Ser | 278 | $1.68 \pm .08$ | $0.482 \pm .016$ | -0.009 | 110.4 |
| V 404 Sct | 288 | $1.51 \pm .04$ | $0.372 \pm .013$ | -0.010 | 102.5 |
| V 383 Sct | 289 | $2.58 \pm .07$ | $0.420 \pm .014$ | -0.029 | 35.0 |
| V 3926 Sgr | 290 | $2.60 \pm .06$ | $0.398 \pm .0 .25$ | -0.030 | 33.4 |
| GP Ser | 292 | $3.38 \pm .11$ | $0.509 \pm .014$ | -0.008 | 127.3 |
| V 3939 Sgr | 292 | $1.62 \pm .05$ | $0.502 \pm .016$ | -0.008 | 120.6 |
| V 3938 Sgr | 297 | $2.59 \pm .06$ | $0.445 \pm .016$ | -0.028 | 35.6 |
| GW Ser | 298.8 | $2.97 \pm .11$ | $0.482 \pm .015$ | -0.024 | 41.2 |
| V 384 Sct | 454 | $3.15 \pm .06$ | $0.283 \pm .021$ | - | - |
| V 411 Sct | 457 | $1.31 \pm .06$ | $0.584 \pm .077$ | -0.011 | 94.1 |
| IY Ser | 463 | $3.75 \pm .12$ | $0.595 \pm .017$ | - | - |
| ET Ser | 463 | $2.03 \pm .06$ | $0.409 \pm .034$ | -0.026 | 38.4 |
| KM Ser | 469 | $4.18 \pm .11$ | $0.429 \pm .017$ | -0.052 | 19.2 |
| V 1977 Sgr | 469 | $4.75 \pm .11$ | $0.376 \pm .013$ | -0.041 | 24.4 |
| V 409 Sct | 469 | $3.86 \pm .08$ | $0.452 \pm .019$ | -0.026 | 38.5 |
| KU Ser | 472 | $3.68 \pm .10$ | $0.586 \pm .008$ | -0.007 | 151.0 |
| V 424 Sct | 474 | $4.50 \pm .11$ | $0.500 \pm .001$ | -0.030 | 33.6 |
| HH Ser | 476 | $3.25 \pm .08$ | $0.418 \pm .021$ | -0.034 | 29.5 |
| V 392 Sct | 480 | $2.49 \pm .05$ | $0.639 \pm .009$ | - | - |
| HI Ser | 121.2 | $2.76 \pm .06$ | $0.552 \pm .025$ | -0.032 | 31.5 |
| V 3925 Sgr | 151.5 | $3.09 \pm .13$ | $0.486 \pm .028$ | -0.057 | 17.4 |
| NSV 10266 | 168.5 | $2.21 \pm .07$ | $0.414 \pm .020$ | -0.040 | 25.1 |
| GV Ser | 230.5 | $1.47 \pm .05$ | $0.486 \pm .017$ | -0.008 | 118.5 |
| NSV 10251 | 236.5 | $1.85 \pm .05$ | $0.489 \pm .013$ | -0.008 | 126.5 |
| HO Ser | 237 | $1.18 \pm .07$ | $0.512 \pm .020$ | - | - |
| LR Ser | 239 | $1.67 \pm .05$ | $0.428 \pm .020$ | -0.021 | 48.0 |
| NSV 10642 | 243 | $1.77 \pm .08$ | $0.623 \pm .012$ | - | - |
|  |  |  |  |  |  |

Earlier used parameters of slope of branches, defined at phase of maximum slope correlate better with period and amplitude, than the ones at the middle between maximum and minimum.


Figure 2. Smoothed light curve of V404 Sct.

Table 2: Correlations between three groups of parameters: 1) "key": period, amplitude $\Delta$ mag and asymmetry f) for 34 weak Mira-type stars; 2) "key": period , amplitude $\Delta$ mag and asymmetry $f$ ) for all stars, including earlier investigated and 34 stars from Table 1.; 3) "parameters of a slope of branches" and amplitude.

| Pair of parameters | $\rho$ | $\sigma_{\rho}$ | $\rho / \sigma_{\rho}$ | N |
| :---: | :---: | :---: | :---: | :---: |
| $\Delta$ mag $-f$ | 0.058 | 0.176 | 0.33 | 34 |
| $P-\Delta$ mag | 0.508 | 0.152 | 3.34 | 34 |
| $P-f$ | 0.077 | 0.176 | 0.44 | 34 |
| $\Delta$ mag $-f$ | -0.278 | 0.099 | -2.80 | 96 |
| $P-\Delta$ mag | 0.183 | 0.102 | 1.780 | 95 |
| $P-f$ | -0.095 | 0.103 | -0.93 | 96 |
| $\Delta \mathrm{mag}-m_{i}$ | -0.275 | 0.011 | -2.49 | 78 |
| $\Delta \mathrm{mag}-t_{i}$ | 0.308 | 0.109 | 2.82 | 78 |

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