DETERMINATION OF THE SATELLITE'S ROTATION USING ITS LIGHT CURVE

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ABSTRACT. This article describes the method of determination of the Earth's artificial satellite's free (unplanned) rotation using its light curve and preliminary computed brightness charts (the spatial indicatrices of light scattering by the artificial satellite's surface), as well as provides examples of such solutions for simulated occurrences.

Introduction

In addition to the spinning with known parameters that is deliberately imparted to an artificial satellite, an unplanned rotation may occur. And generally, the characteristics of that rotation are undefined, yet the associated problems, such as inability to receive or transmit signals and energy shortage due to the failure in the satellite's positioning relative to the Sun, do not allow of estimating the parameters of such a rotation with only satellite's capabilities. In its turn, that may result in the loss of an expensive or unique satellite. However, it is possible to additionally support such an out-of-order satellite by the ground-based monitoring of changes in its light curve and interpreting them in terms of the rotation parameters. The rotation frequency can be determined by the brightness change intervals; and the orientation in space can be defined by non-uniformities in the light reflection [1]. It is natural that it is not all that simple, and there is a host of problems. For instance, the charts of characteristic features of the light reflecting off a satellite, allowing for all possible positions of that satellite and angles of the light reflection, should be created as early as the launch is performed. And it is reasonable that those charts can turn out to be useless when the specific characteristics of the brightness change later in space, for example, due to the damages caused by micrometeorites or incomplete unfolding of the solar panels. Besides, errors in both the brightness charts and the measurement of the satellite's brightness in orbit should not be neglected. With allowance for the above, the determined rotational parameters will often appear as a certain set of solutions with different probability rather than a single-valued result. Nevertheless, in most cases such investigation can provide useful and topical information on the satellite rotational parameters, not using the satellite's own capabilities at that.

Problem statement

It is required to determine the rotational parameters of the artificial satellite model by the passive remote photometric sensing method. Hence, the following will be known: two-line set of orbital elements (TLE), the model satellite light curve or curves, obtained by a single observer or by many observers - i.e., as a matter of fact, the photometric satellite model will be defined with already obtained brightness charts, which are the databases on the satellite brightness depending on its orientation relative to the light source and observer [2-3]. In the present study, the brightness charts were obtained for the model of axisymmetric satellite that allows of reduction the dimension of the four-dimensional array of brightness values; however, the artificial satellite model is planned to be replaced by a more complex one. The rotational parameters are supposed to be unaltered along the whole light curve.

Thus, the closed-loop model problem is considered: there are brightness charts available for the defined artificial satellite model, and one or several initial light curves, which act as the observed light curves, obtained either by a single or several observation sites for the real satellite orbit, are computed for that model.

The determination of the path or rotational path of, for instance, longitudinal axis of an artificial satellite (in this case – the axis of symmetry), for which the model light curve, as well as the array of the brightness values obtained by the corresponding charts will coincide with the initial observed light curve within the stipulated measurement accuracy, is deemed to be a solution of such a problem. As simple search of all possible rotational trajectories is a trivial method of solving the indicated problem, the omission of such a search to reduce the time and computer's capacity consumption to the maximum will be an additional requirement to the solution procedure.

Principle of the method

Three directions of the body's preferred axis (the axis of symmetry) in space at three different moments are necessary and sufficient to determine its rotational parameters. That allows of finding the small circle, the centre of which is the rotation pole; the small circle radius defines the tilt angle of the rotation axis to the body's preferred axis while the angular velocity gives the rotation period. As initially we do not know the symmetry axis direction, we should determine it by ourselves; so we use the light curve and the brightness charts to do that. The brightness charts are created in such a way that two other coordinates define the symmetry axis direction in space for the fixed value of the phase angle (the angle between the satellite orientations towards the light source and observer), and the body's brightness is assigned to that direction [2-3]. Thus, having known the artificial satellite brightness and the angle between the Sun and observer for the specified instant, the corresponding symmetry axis direction in space can be determined by the brightness charts at the given moment. The problem is that equal brightness values are typically common to a large set of the symmetry axis directions. In other words, for each moment of time we will have several possible symmetry axis directions in the brightness chart reference frame, which are characterised by the same brightness (within the specified measurement accuracy). For brevity, those points on the brightness charts will be called isophots. If we try to determine the rotational parameters by the set of three different isophots, the number of alternative solutions will equal to the product of the quantity of isophots for each of three moments. For a hundred of isophots for each moment of time we will eventually obtain 100x100x100=1000000 – a million of probable paths, and only one of them will be the true one. To find the true path, it would be necessary to plot a million of simulated light curves and compare them with the initial measured light curve for all instants. Although such operation is feasible, the computing time required would be out of proportion, the more so it is possible that the number of alternative paths is larger by an order of magnitude.

However, the number of implicit paths can be reduced. As we proceed from the assumption that the rotation frequency is constant, then such a path can be undoubtedly discarded as an infeasible one when the rotation frequency changes considerably at the time interval under test. We compute the rotation frequency within the interval between moments 1 and 2; subsequently, by a similar way we compute the frequency between moments 2 and 3, and if the frequency values differ by more than a certain value, the corresponding path is omitted. Thus, it is possible to considerably reduce the number of paths. At that, the stiffer constancy criterion of the rotation frequency is, the fewer alternative paths meet that criterion, and the higher is the probability to miss the true path.

The next step of reduction of false paths implies the consideration of alternative implicit paths determined by the above-described method, but this time for different sets of moments. Eventually, we will get several sets of alternative rotation paths. As the rotation frequency, the rotation axis direction and the initial phase (the initial direction of the symmetry axis) do not change with time, each set should contain one alternative solution with similar parameters of the path, and it is that solution that is the true rotation path.

It is clear that with a large number of false alternative paths it will become more likely that many false paths will be tested along with the true solution. That problem can be solved by statistically increasing the number of solutions; but taking into account the finiteness of the light curve, their quantity will also be finite.

To reduce the number of false paths, it is also possible to toughen the criterion, according to which either similarity or equality of the observed satellite brightness to the values, indicated in the corresponding brightness chart grid points, is defined. Consequently, it is not always the case even for the true path that an alternative path suitable by the similarity criterion is found. Eventually, with correctly selected similarity criterion we will obtain matching distribution of alternative solutions with none of the solutions 100% matched for all sets of moments while the true rotation path has the maximum number of matches. That path can be finally tested by simulating the relevant light curve and comparing it with the observed one.

Moreover, it is worth of testing other alternative solutions that have number of matches standing out among the others. It is not improbable that there can be several alternative rotation paths that exhibit the light curve similar to the observed one. In that case, it can be concluded that a single-valued solution is not feasible under given conditions; and to obtain a single-valued result, it is necessary to collectively or individually increase the following factors: the discrete brightness chart features, the observed light curve accuracy, the number of the observed passes, and the quantity of the observers that on being geographically dispersed simultaneously measure the satellite brightness.

Measure of inaccuracy

To noticeably reduce the number of false alternative paths not accidentally rejecting the true one in so doing, it is necessary to properly choose the similarity factor stiffness and other parameters. The best result can be obtained when the similarity factor is equal to the mean photometric error. The parameters with poorer accuracy in their determination should also be excluded from the list of criteria. For instance, such a parameter as the initial position of the symmetry axis accumulates errors rapidly; therefore, it can be substituted by the angle between the symmetry axis and the rotation axis.

The total error is to be determined by two components, namely the light curve inaccuracy and the brightness chart discreteness. In our particularly simulation experiment the light curve inaccuracy was considered to be equal to zero whilst the discreteness in the brightness charts was 1 degree. Therefore, instead of actual direction of the symmetry axis with the precise phase angle the isophot determined a three-dimensional cube formed by the brightness charts, within the limits of which the corresponding brightness value fell. It is evident that all values, derived by the positions of those isophot points, have deviation from the true ones; at that it was found that such an error decreased with increasing distance between the relevant isophot points.

Figure 1 shows the errors in determination of the rotation axis pole coordinates, the angle between the rotation axis and the longitudinal axis of the model, as well as the current rotation phase depending on the angular distance between the relevant isophots expressed in terms of period's portion. It can be seen that when the time elapsed between two measurements of isophots is close to the rotation phase 0.15, the accuracy in measurements will be optimal. Figure 2 shows the same errors in depends from the angle between the rotation axis and the axis of symmetry.

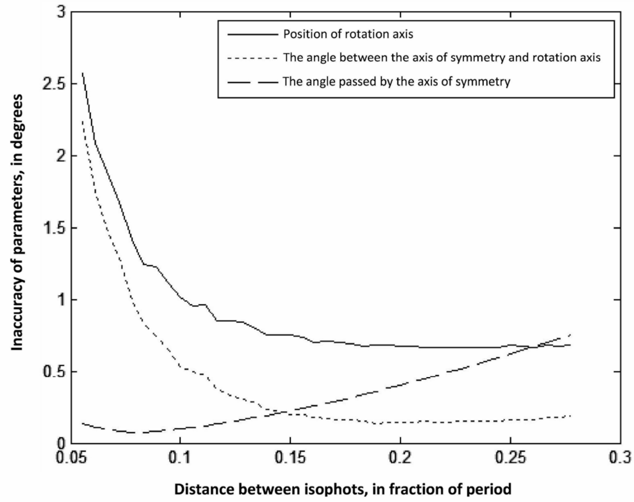

Figure 1. Errors in determination of the model rotational parameters against the angular distance between the relevant isophots.

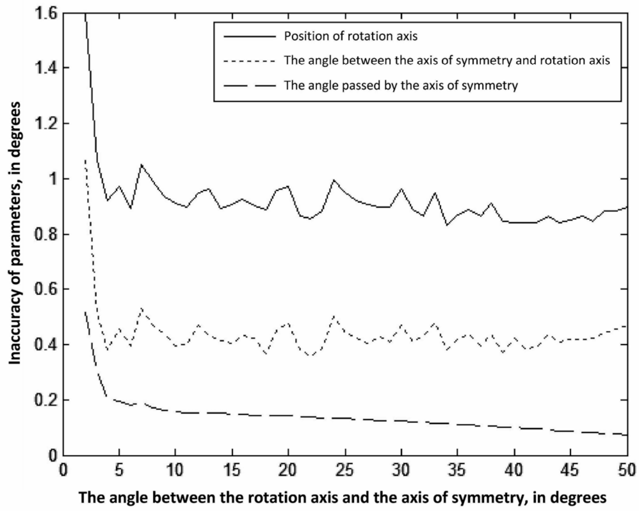

Figure 2. Errors in determination of the model rotational parameters against the angle between the rotation axis and the axis of symmetry.

One of the weaknesses of the described method of determination of rotational parameters of the artificial satellite model is that it is necessary to know an approximate value of the probable satellite angular velocity as early as at the first stage of analyzing probable rotational paths. Generally, the synodic rotation period is easily estimated when the apparent brightness change interval is available for the light curve. In case of great uncertainty, it is necessary to split up the range of probable rotation velocities into smaller ranges, for which the difference between the maximum and minimum velocities does not exceed 50%, and then, to find a general solution (including that for the sidereal angular velocity) for each of them individually.

The description of the solution and result

1. Determination of the satellite position in orbit using its orbital data in the TLE format for each discrete time point of the observed light curve.
2. Computation of the phase angle between the light source (the Sun) and observer for each time point of the light curve.
3. Creation of the matrix for transformation of vectors from the equatorial satellite-centric coordinate system to the brightness chart reference frame at a certain instant in time. For the latter reference frame the Z-axis coincides with the direction towards the light source whilst the XOZ-plane contains the direction towards the observer (the phase angle plane).
4. Determination of the estimated range of values for the satellite rotation period based on the characteristics of its observed light curve if possible.
5. Selection of certain three moments with the interval between them close to a ninth of the rotation period and readout of the observed brightness.
6. Determination of probable symmetry axis directions in the brightness chart reference frame for each instant. The coincidence of the "chart's brightness" for a certain direction with the brightness of the observed light curve, within the specified measurement accuracy, is considered to be the grounds to designate that this direction as an "isophot".
7. Picking of all possible combinations of three isophots for different moments. Elimination of those combinations, for which the rotation frequency is essentially different for various pairs of time instants.
8. By using each three directions (isophots), converted back to the equatorial coordinate system, we derive the equation of plane, therefore the normal to that plane is the axis of rotation. The rotational velocity is determined by division of the angle between the points by the elapsed time. Then we derive other characteristics. So, the sum of those characteristics is the probable decision for rotation's kind.
9. Performance of the similar procedure for a certain number of times for other sets of moments. Comparison of the solutions resulted from each operation. Recording how many times the determined path of the longitudinal axis of the model body coincided with paths computed by other sets within the specified measurement accuracy. It is very likely that the path with maximum number of coincidences will be the true one.
10. Check-up of the obtained solution with maximum number of coincidences by plotting a corresponding light curve according to the brightness charts and comparing it with the initial one.
11. It is obvious that the light curves are very similar, and therefore, the obtained path is true or indistinguishable from that with the specified inaccuracy and discreteness in initial data.

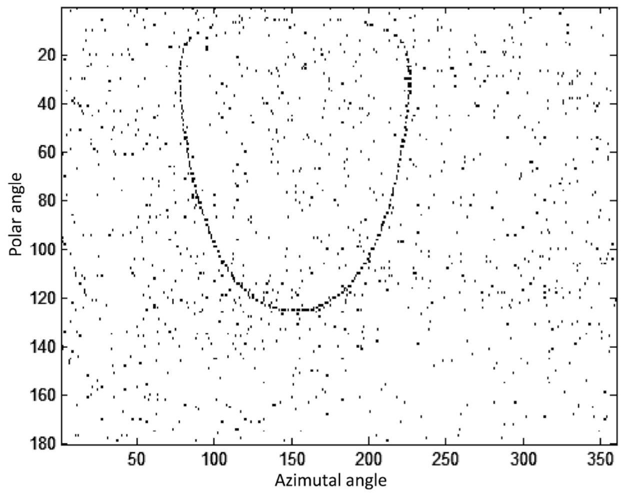

Figure 3. The summary isophots for a given time in a fixed satellite-centered coordinate system.

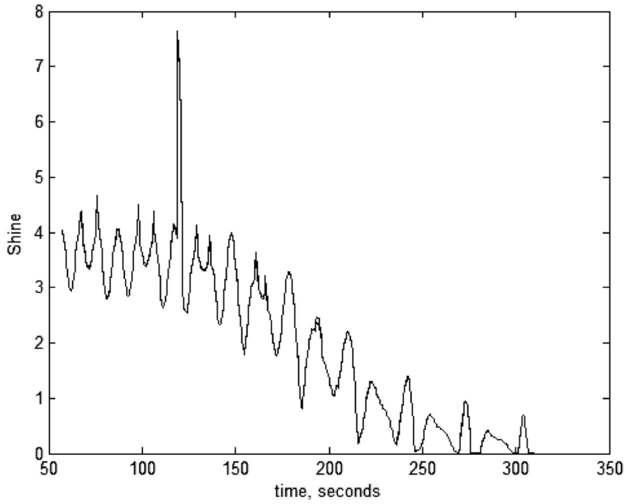

Figure 4. A comparison of the light curve, constructed for found rotation parameters, with the original light curve.

First we solve the direct problem – simulate the light curve low-orbit satellites. For example, consider the elements of the orbit of the Sich-2 [4].

1 37794U 11044G 12141.23295985.00000379 00000-0 88476-4 0 8582
2 37794 098.2290 218.2236 0014010 141.1597 219.0631 14.60067392 40408

Coordinates of the observer: latitude 46°20'40.08" longitude 39o45'20.32". We take one of the passages of the satellite and set start time of observation – May 17, 2012 in $20^h.1667$. We define rotation parameters – inclination of the axis of symmetry to rotation axis nk = 60°, the polar angle of the rotation axis pW = 64° and the right ascension αW = 150°, period of the one complete revolution T = 42 seconds (the angular velocity is 8.57 °/s). The error of shine values equal to 0.03 %. As a result, we obtain a quasi-periodic light curve.

Inverse task. On the resulting light curve is sometimes difficult to determine the rotation period of the satellite, but as the initial value will take T = 50 s.

To visualize the solution of the problem look at the distribution of isophot's points for discrete moments of time in the coordinate system of brightness map (Figure 3). In this case, we can easily see the oval chain of isophots. This is the true direction in space of the longitudinal axis of body at different times, and the center of the oval - the rotation pole. The other points of isophots are false directions in the space of the longitudinal axis, but have the same shine (within the error) as in the true orientation of the body.

Now look at the result of the algorithm test. Found 34 possible paths of rotation (some are shown in Table 1).

Table 1 shows the following rotating parameters – three projection of rotation axis, the angle between the rotation axis and the longitudinal axis of the body, the rotation angular velocity. The sixth column – the number of matches (repeating solutions). The maximum number of repeating is 131 from 593, the other values are not higher than 3. Consider the rotation parameters with maximum repeatability. Translating

Table 1. Results of the solution to the rotation parameters.

X(W)	Y(W)	Z(W)	NK	w	N
-0,6138	-0,48301	0,624454	95,24231	-8,38806	0
-0,35448	0,32879	0,875351	147,7485	-10,9361	1
-0,31798	0,368376	0,873606	150,6086	-12,1351	0
0,362565	0,911941	0,192118	134,8943	-9,93484	0
-0,46488	-0,88232	0,073479	60,92391	-8,30496	131
-0,3791	0,404378	0,832325	157,4329	-13,2543	0
-0,41075	-0,38042	0,828595	111,5165	-10,7619	2
-0,81511	0,052696	0,5769	113,7678	8,096023	0
-0,81951	-0,26247	0,509418	96,18076	7,09923	0
-0,82469	0,10628	0,555502	117,0159	8,223101	0
-0,86196	-0,17144	0,477105	138,6802	2,952431	3

the Cartesian coordinates of the rotation axis from the coordinate system of brightness map we obtain a spherical equatorial coordinate pW = 64°.18 and aW = 149°.9. The period is found 43.3 seconds. The period is a relatively inaccurate, but it can be refined using the most distant in time isophots belonging to this rotation. As a result, we get the period = 42.8 seconds.

Conclusion

As a result of the algorithm is better to define the rotation axis of the satellite's model, the slope of the longitudinal axis to the rotation axis has an inaccuracy about one degree and the period - about one second. A comparison of the light curve constructed with the found rotation parameters, with the original one shows that they are extremely similar, but slightly different in extremes (Figure 4).

References

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