STATISTICAL ANALYSIS OF THE MAGNETIC FIELD MEASUREMENTS

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ABSTRACT. Investigation of statistical features of the magnetic field fluctuations in bound regions of Earth's magnetosphere, on different time scales, is carried out in this work. Measurements of Cluster-II with frequency 22.5 Hz for 2004-2008 years have been used for the analysis. During these investigations we have studied the changes of shape and parameters of probability density function for magnetic field fluctuations for periods of presence of the satellite in magnetosheath, solar plasma wind and in region of magnetopause. We have considered the evolution of change of maximum of probability density function and investigated structure functions of different orders as characteristics of turbulent processes for different time scales. Investigation of structure functions of high orders helped us to determine the character of turbulent processes and study diffusion in the considered regions. We have found, that the highest intermittence is observed in the postshock region; for the middle magnetosheath the results of experimental data correspond to log-Poisson turbulent cascade model, and for the description of processes in SW plasma one can use the Iroshnikov-Kraichnan`s model.

Key words: turbulent model, intermittency, statistical properties of boundary layers, Earth's magnetosphere

1. Introduction

Investigation of the processes in the magnetosheath is significantly complicated by presence of the turbulence. Since the processes in boundary layers of space plasma is characterized by a great number of degrees of freedom, nonlinearly interacting modes, multi-scale structure and random fluctuations of velocities so that the methods of statistical physics and theory of probability are most suitable for its description. In order to describe an random process one have to determine the probability density function for plasma parameters and the moments of this probability function. The most often the probability distribution of fluctuation amplitudes corresponds to Gauss (normal) law with quickly decreasing correlations. There are other known distribution laws in the theory of probability which can describe processes with far correlations. The distribution probability functions for such processes are not always described by known mathematical functions and series, and for many types of random processes one knows only a method for fitting of their distribution functions. Investigation of statistical symmetries of turbulence allows us to obtain information about a character of dependence of structure functions (statistical moments of probability function) of different orders on time and space, without to resort to detailed consideration of individual conditions of its exciting.

Random pulsations in the medium under structure heterogeneity of the turbulent process (intermittency) have the distribution function different from the Gaussian distribution [Kozak et al., 2011, Kozak et al., 2012].

The research of the statistical properties of boundary layers allows to determine the role of turbulent processes in the interaction of plasma flows with the magnetic obstacles, whether these are fields of planets, stars, or laboratory traps, and to reveal the actual mechanisms of the energy transformation in collisionless plasma. In this study based on the satellite Cluster-II measurements the characteristic turbulent regions in the boundary layers of Earth's magnetosphere.

2. Used observational data

In order to analyze the features of turbulent processes in the transition regions of the Earth's magnetosphere twenty-five events of the magnetic field measurements obtained by Cluster-II mission for 2004-2008 years with frequency resolution 22.5 Hz were considered. The satellite moving from SW passed through the foreshock (FSH) the bow shock (BSH), the magnetosheath (MSH), crossed the magnetopause (MP), and came into the magnetosphere.

During the transition from SW to MP the rate of fluctuations of the field changed dramatically the dispersion of plasma variations normalized to a current mean value accounts for: SW – 0.02 – 0.05; FSH 0.2 – 0.3; PSH – 0.5; deep in the MSH region the rate of fluctuations drops to 0.1– 0.2.

3. Results of analysis

3.1. Features of the probability density functions of fluctuations

In order for investigations of features of the probability density functions of magnetic field fluctuations we analyzed the statistical properties of the absolute value of magnetic field variations $dB = B(t+\tau) - B(t)$ in the different regions of near earth space and for different time scales.

The dependence of a maximum of the probability density distribution of the fluctuations P_0 on the shift in time τ can be approximated by the power-law dependence $P_0(\tau) \sim \tau^{-S}$. For the case of Gaussian distribution parameter $S = 0.5$; in the general case (the Levi distribution) $S > 0.5$. For the turbulence with intermittency the presence of considerable fluctuations on distribution's wings appears due to abundance of energy of large-scale disturbances, which are generated by external source or boundaries of flows. The investigation of changes of a maximum of the probability density function of magnetic field fluctuations on various time scales was applied, for example, for studying the magnetic turbulence in the magnetospheric tail during the period of transverse current disruption [Consolini et al., 2005, Kozak et al., 2008].

The dependence of maximal value of probability density function of magnetic field fluctuations $P_0(\tau)$ on time shift τ being divisible to 0.0445 s (time step of initial data) for the event May 15, 2005 is shown in the Fig. 1. For short time scales from 0.00445 s to 1 s The values of the power s are presented in the Tab. 1.

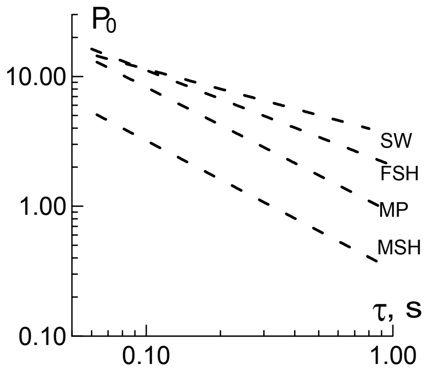

Figure 1: Logarithmic dependence of the probability density distribution function maximum of magnetic field fluctuations P(0) on the time step in the SW, FSH, MP, MSH. The experimental points are approximated by a straight line

Table 1: The values of the power s in the transition regions of the Earth's magnetosphere

	04/03 2004	15/05 2005	10/03 2006	05/04 2007	08/08 2008
SW	0.52	0.52	0.48	0.45	0.55
FSH	0.7	0.75	0.68	0.6	0.76
PSH	0.98	0.9	0.95	0.92	0.97
MSH	0.95	0.87	0.94	0.87	0.91
MP	0.65	0.8	0.8	0.67	0.73

Thus, during the time of satellite location in SW, the found value of S in the entire interval of studied scales is close to the Gaussian distribution (S = 0.5). For FSH, MSH, MP, TBL and cusp the distribution corresponds to Levi distribution (intermittency).

3.2. Kurtosis values comparison

For quantitative characterization of possible deviations of fluctuation statistics from the normal distribution, the kurtosis (excess) $K(\tau)$ is used, which is defined through moments of the second and fourth orders:

$$K(\tau) = S_4(\tau) / (S_2(\tau))^2,$$

where $S_q(\tau) = \left\langle \left| X(t+\tau) - X(t) \right|^q \right\rangle$ is the moment (structural function) of the q-th order [Benzi et al., 1993], angular brackets $(\langle\ \rangle)$ designate time averaging of studied parameter $X(t)$, τ is the time shift. For the normal distribution one must have $K(\tau) = 3$ [Zaks, 1976].

The value of kurtosis is one of parameters displaying the character of intermittency of the process. However, this parameter does not allow one to make a quantitative comparison of the degree and mechanism of intermittency. If remains constant on various time scales τ, this indicates to the absence of intermittency.

The values of kurtosis of magnetic field fluctuations on scaling parameter τ were constructed (fig.2). It is clearly seen in the plots, that for SW the $K(\tau)$ function value varies around 3. This feature indicates to the Gaussian distribution function, which confirms the results obtained above in analyzing function $P_0(\tau)$. For FSH and MSH regions the value of function $K(\tau)$ on small scales sharply grows (up to 23), and on large time scales (> 2 s) this value approaches 3.

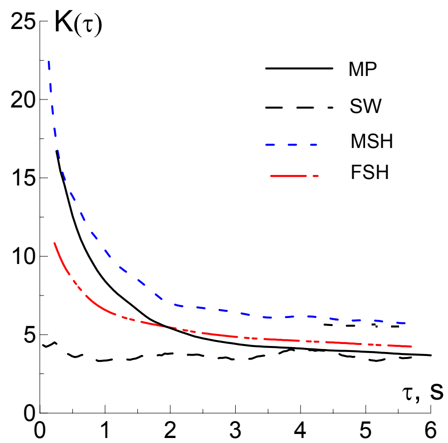

Figure 2: Dependence of the kurtosis value on the scale parameter τ for different magnetospheric regions and SW

3.3. ESS analysis results

In order to specify the type of turbulent processes the analysis of structure function features (moments of PDF) of different orders according to time interval for dataset $X(t)$ was performed in the study – Extended Self-Similarity analysis. Structure functions of high orders allow characterizing the properties of heterogeneity at the small scales of process. The dependence of structure function on the time shift τ assumes to be power law $S(\tau) \sim \tau^{\zeta(q)}$.

In a case of fully homogeneous isotropic Kolmogorov 3D (K41) turbulence the values of exponent are defined by a relationship $\zeta(q) = q/3$ [Frisch *et al.*, 1978], and for the

Iroshnikov-Kraichnan model (IK), which describes plasma turbulence in the strong magnetic field, $\zeta(q) = q/4$ [Kraichnan, 1965].

To describe the intermittent turbulence it is very often to use log-Poisson model which considers the stochastic multiplicative cascade [Dubrulle, 1994]:

$$\zeta(q) = (1-\Delta)q/3 + \Delta/(1-\beta)\left[1-\beta^{q/3}\right].$$

Index β characterizes the degree of intermittency (β=1 for the non-intermittent homogeneous fully developed turbulence, for instance, in K41 model), Δ – the parameter related to the geometry of dissipative structures and edge effects. For the model of Pulitano-Pukke (PP):

$$\zeta(q) = q/8 + 1 - (1/2)^{q/4}.$$

ESS analysis consists in the determining of a relative value of power index for the different structure function orders. In the general case, for q-th and p-th orders the following relationship is assumed:

$$S_q(\tau) \sim S_p(\tau)\ \tau^{\varsigma(q)/\varsigma(p)}.$$

The following regions were examined with ESS analysis: FSH, MSH, MP (fig. 3).

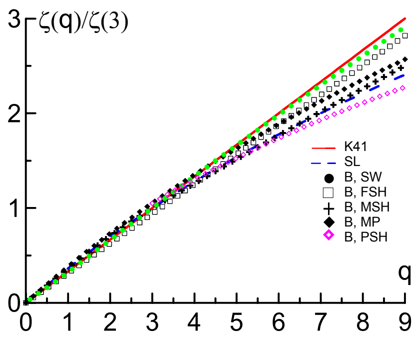

Figure 3a: Ratio of the power of the q-th order structural function to the third order function power on May 15, 2005. The experimental data for the magnetic field are marked with symbol; the dotted line corresponds to the value calculated using the formula in the log-Poisson cascade model for β = Δ = 2/3 (SL), and the solid line corresponds to the q/3 (K41)

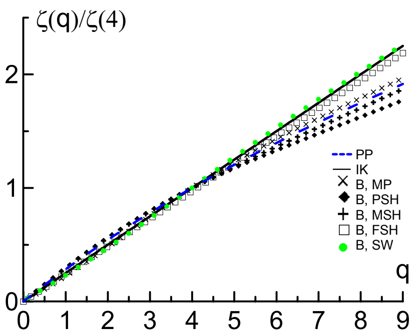

Figure 3b: Ratio of the power of the q-th order structural function to the fourth-order function power on May 15, 2005. The experimental data for the magnetic field are marked with symbol; the dotted line corresponds to the value calculated using the Pulitano-Pukke model (PP), and the solid line corresponds to the q/4 (IK)

ESS analysis demonstrates the intermittency of turbulent processes in MSH well described with log-Poisson SL cascade model, the values in the SW and FSH region are close to Iroshnikov-Kraichnan model of inhomogeneous anisotropic turbulence which describes plasma turbulence in the strong magnetic field.

4. Discussion and conclusions

The used set of techniques for determining the statistical properties of fluctuations has shown various properties of oscillations in FSH, PSH, MSH and MP.

Amplitude of fluctuations in MSH just after crossing BSH exceeds in a few time amplitude of fluctuations for non-perturbed SW or FSH.

The character of turbulent plasma flux in MSH is not associated directly with turbulence in SW but represents, to a considerable degree, the manifestation of intrinsic processes in MSH.

The use of the technique of probability density function for magnetic fluctuations has shown the following characteristic features of turbulence in the process of transition from SW through FSH deep into MSH. In SW the amplitude of fluctuations is minimal, the dependence of maximum of probability density function $P_0(\tau)$ corresponds to Gaussian distribution. In FSH, PSH, MSH and MP for small time scales the observed features are better described by the Levi distribution (intermittency).

ESS analysis demonstrates the intermittency of turbulent processes in MSH, PSH, FSH and MP also.

Moreover the turbulent processes in the SW can be described by Iroshnikov-Kraichnan model of turbulence, and the processes in MSH correspond to the log-Poisson cascade SL-model.

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References

Benzi R., Ciliberto S., Tripiccione R. et al.: 1993, *Phys. Rev. E.*, **48,** 29.
Consolini, G., Kretzschmar M., Lui A.T.Y. et al.: 2005, *J. Geophys. Res.*, **110,** A07202.
Dubrulle B.: 1994, *Phys. Rev. Lett.*, **73,** 959.
Frisch U., Sulem P., Nelkin M.: 1978, *Fluid Mech.,* **87,** 719.
Kozak L.V., Lui A.T.: 2008, *Kinem. i Fiz. Nebesnykh Tel*, **24,** 72.
Kozak L.V., Pilipenko V.A., Chugunova O.M. et al.: 2011, *Kosm. Issled.*, **49,** 202.
Kozak L.V., Savin S.P., Budaev V.P. et al.: 2012, *Geomagnetism and Aeronomy*, **52,** 445.
Kraichnan R.H.: 1965, **8,** 575.
She Z., Leveque E.: 1994, *Phys. Rev. Lett.*, 72, 336.
Zaks L.: 1976, Moscow: Statistics, 598 p.