MODELLING OF MOTION OF BODIES NEAR TRIANGULAR LAGRANGIAN POINTS

O.A.Bobrov
Department of Astronomy, Odessa National University,
Shevchenko Park, 65014, Odessa, Ukraine, *oleg_bobrov@ukr.net*

ABSTRACT. In this paper, we consider a system of three bodies connected by gravity, two of which are of comparable mass (the Sun and Jupiter), and the third is negligible and it is located in one of the triangular Lagrange points (restricted 3 – body problem). We used the equations of motion in a planar coordinate system that rotates together with massive bodies. Several programs have been written in the programming environment Pascal ABC, in order to build the trajectory of a small body, to indicate the osculating orbit around a massive body, to display equipotential surfaces.

Key words: Stars: binary – celestial mechanics – Solar system – asteroids

1. Introduction

Triangular Lagrangian points are important for planetary systems and planetary satellites. For example, asteroids are located close to this points of the Sun – Jupiter system. They move along the same orbit as Jupiter, in front and behind it, and are called, respectively, the Greeks and the Trojans. Similar asteroids were also found at Neptune, Uranus, Mars and Earth. There are very sparse clusters of interplanetary dust in the triangular points of the Earth-Moon system, so called Kordylewski clouds. Thus the study of the stability of the trajectories of small space objects near the triangular Lagrange points is interesting and relevant task.

2. Calculations

In the rotating coordinate system, the potential is:

$$P = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{w^2 r_3^2}{2} \quad (1),$$

where M_1, M_2 – the masses of massive bodies, r_1, r_2 – the distance from a point A to them, w – the angular velocity of the coordinate system r_3 – the distance from a point to the axis of rotation.

Here the first two terms correspond to the gravitational potential of all bodies, and the third – the potential of centrifugal force.

Let the first body be at the origin of the Cartesian coordinate system, the plane of rotation – in the plane XY, the second body – on the axis OX (in the positive part). Then X, Y – coordinates of the third body X', Y' – the velocity of the third body X'', Y'' – the acceleration of the third body, a – the distance between the massive bodies; R_1, R_2 – distance from the third body to the first and second bodies; *w*– the angular velocity of the system

$$w = \sqrt{\frac{G(M_1+M_2)}{a^3}} \quad (3)$$

Let the rotation system of two bodies directed conterclockwise at right-handed coordinate system.

In order to simplify computing, we scaled coordinates in units of a, velocity in units $a \cdot w$, acceleration in units of $a \cdot w^2$, mass in units of the total mass $M_1 + M_2$. Then $m_2 = 1 - m_1$. The equations of motion in projections (e.g. Subbotin, 1968, Andronov, 1990):

$$R_1 = \sqrt{X^2 + Y^2} \quad (4)$$

$$R_2 = \sqrt{(X-1)^2 + Y^2} \quad (5)$$

$$X'' = -\frac{Xm_1}{R_1^3} - \frac{(X-1)m_2}{R_2^3} + X - m_2 + 2Y' \quad (6)$$

$$Y'' = -\frac{Ym_1}{R_1^3} - \frac{Ym_2}{R_2^3} + Y - 2X' \quad (7)$$

First term is the gravity law for the first and second bodies. Other forces appear because the coordinate system rotates. The third term – the centrifugal force, and the fourth term – the Coriolis force. We integrated these equations using the Runge – Kutta method of 4th order.

Strict concept of stability motion is a complicated mathematical concept. But we used, an intuitive concept of stability. If a sufficiently long period of time the Trojan body will not leave the area near the Lagrange point, the motion in our understanding is considered stable. This area is considered as a region, where the potential energy of the Trojan body is higher than the one at the Lagrange point (located behind the second body). Using the dichotomy method, by this criterion we determined the maximum speed value in different directions.

Fig.3 shows the speed limit for Lagrange points L_4 and L_5, where the angle is measured from the axis OX in the rotation direction of the system. We can see that the two graphs are symmetrical and shifted relatively to each other. The bodies at different Lagrangian points, moving out in the same direction, are under the effect of Coriolis force directed the same. This force is directed in one case to the small body, and the other – from the small body, so we must introduce different directions of the reference angle. Then it was introduced the system of reference angle where the graphs for different points coincide. The direction of velocity 0° - direction of orbital motion at the Lagrangian point.The angle is measured at the point L_5 in the direction of rotation second body, and at the point L_4 - backwards. The body, which has a direction parallel to the direction at first body, has the greatest speed limit.

In opposite, a body, which has perpendicular direction of motion, has a lower limit speed.

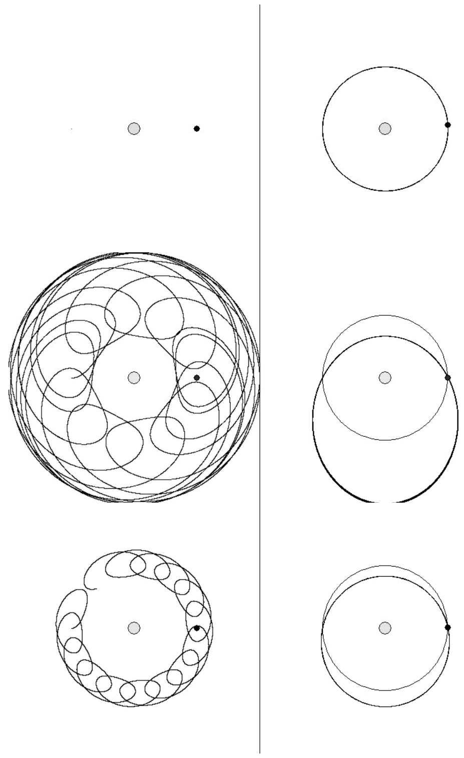


Figure 1: Example of motion in the rotating coordinate system without affecting the second body, a computer simulation. Left – the motion relative to the second body, right – in the inertial reference system.

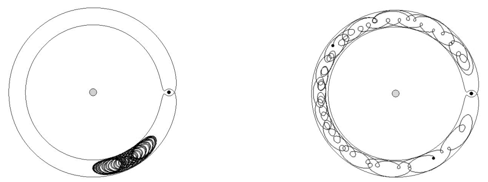


Figure 2: The greatest speed limit is the steady motion of a body around a Lagrange point, and the minimum – motion at all area.

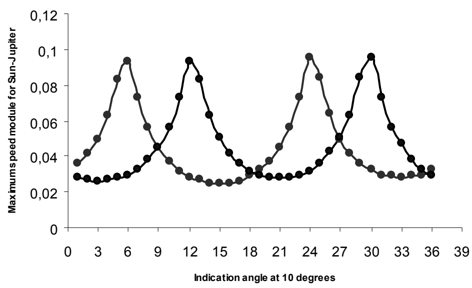
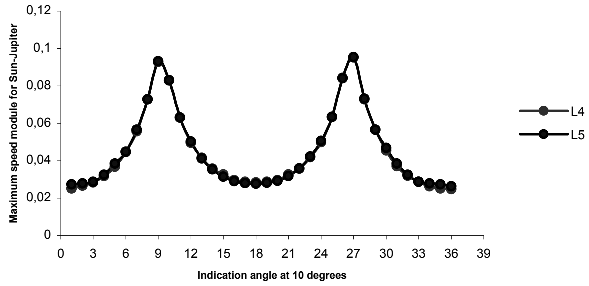
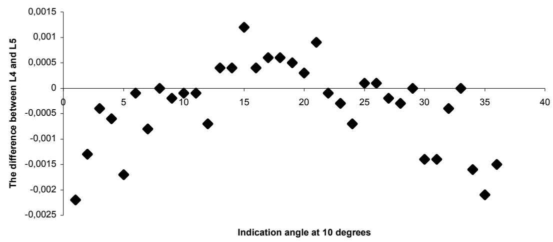


Figure 3: The dependence of the speed limit on flying of the Sun – Jupiter system (in the same frame of reference edges). The time of motion is 20 orbital periods.

3. Conclusions

We determined maximum speed, at which a small body leaves the neighborhood of the triangular Lagrangian points for the Sun-Jupiter system and the Earth-Moon system, as well as its dependence on the velocity direction of the body at the Lagrangian point. For different Lagrangian points, opposite directions for reference angle between the velocity vector and the line of centers of mass bodies were introduced (clockwise in one point and conterclockwise in another). With this choice of measuring direction angles, the graphs of limiting velocities almost coincided. This can be explained by the fact that the body in different Lagrange points with the same initial velocity vector have the same effect of the Coriolis force. However, at one point, this force will be directed to the second body (Jupiter), and another – from the second body.





Figures 4, 5: The dependence of the speed limit on the direction flying of Sun – Jupiter system (up) and the difference between the two graphs (bottom).

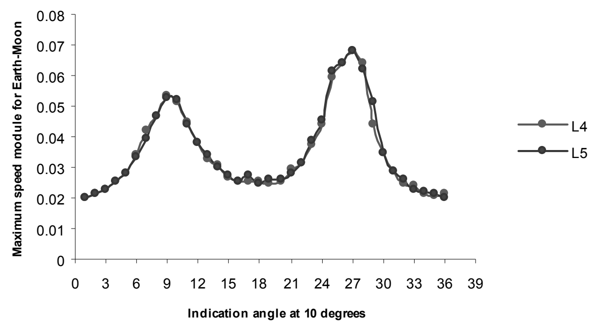


Figure 6: The dependence of the speed limit on the direction flying of Earth – Moon system.

Acknowledgements. We thank Prof. I.L. Andronov and Dr. V.I. Marsakova for helpful discussions.

References

Andronov I.L.: 1991, *Stroenie i Evoluciya Peremennykh Zvyozd* (in Russian), Odessa, 84 p.

Subbotin M.F.: 1968, *Vvedenie v Teoreticheskuyu Astronomiyu* (in Russian), Moscow, Nauka.