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THE FORMULATION OF NONLINEAR DEFORMATION AND FRACTURE OF HETEROGENEOUS 3D BODIES SUBJECT TO THE EMERGENCE AND SPREAD OF CRACKS UNDER DYNAMIC LOADING

Describes the main output parameters of the problems of fracture mechanics and existing calculation methods for inhomogeneous spatial bodies with cracks in terms of nonlinear dynamic effects.
Key words: J-integral, spread of crack, nonlinear deformation, dynamic, contact stresses.

Introduction

In previous works [4, 8, 9], devoted to the dynamics of destruction, the authors were limited to studies of spatial prismatic bodies and bodies of revolution with longitudinal cracks within the linear elastic deformations.

This article is about the creation of a new task, which greatly expands the class of objects is investigated, both in the geometric and physical characteristics.

To research selected objects, each of which has characteristic features, which requires both correction methods developed in previous works, and creating new ones.

One such object is a reference device, which is a cyclically symmetric body with the limiting case of heterogeneity (see Fig. 1), that is, the object contains cuts that break the axial symmetry of the form. In addition, as it was shown in the published works of Bazhenov, Guliar, Topor, Solodei, under quasi-static and dynamic loads at the boundaries of the compounds of the cylindrical part with the tabs having a zone of plastic flow.

If there are cracks in these areas, subject to dynamic loads, the application of traditional approaches to determining the fracture toughness of the object is impossible, because the task parameters do not meet the restrictions, which are imposed on the use of the SIF or the J-integral.

For example, studies of the dynamic deformation containment with a longitudinal crack should be analysis of the effectiveness of the new parameter fracture toughness, which is in contrast to the J-integral Cherepanov-Rice, not to have restrictions regarding the availability of the loads applied on the crack edges.

It is planned to develop new approaches for determination of fracture toughness parameters in the spatial bodies with dissimilar physical and mechanical properties on the base of SAFEM in the presence of growing cracks under the dynamic loading.

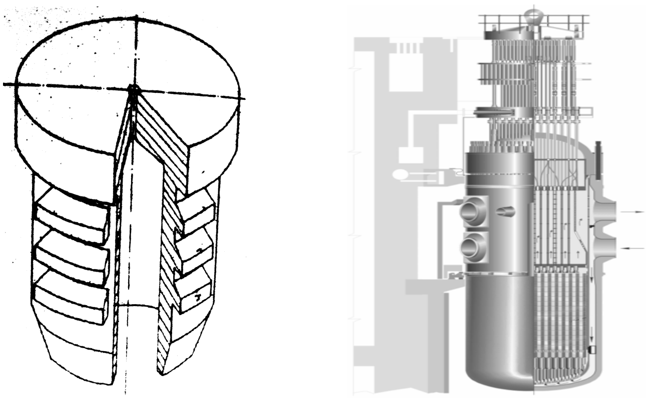

Fig. 1. The reference device and the protective shell of the reactor (General view)

The description of the mechanical and geometrical characteristics of objects, initial and boundary kinematic conditions, external loads is carried out in the basis of orthogonal circular cylindrical or coordinate systems of Descartes $Z^{i'}$.

It is believed, that anywhere in the body known relation between the baseline and the local coordinate systems, which is determined using forward and reverse coordinate transformation tensors:

$$z^{i'}_{,j} = \frac{\partial Z^{i'}}{\partial x^{j}}, \quad x^{i}_{,j'} = \frac{\partial x^{i}}{\partial Z^{j'}}. \qquad (1)$$

Here and in what follows, the indices, which are denoted by Latin letters, take the values 1, 2, 3; Greek - 1, 2; the comma before the index shows the operation of differentiation.

Covariant components of the metric tensor of the local coordinate system can be represented through covariant components of the base system:

$$g_{ij} = z^{m'}_{,i} z^{n'}_{,j} g_{m'n'}. \qquad (2)$$

Contravariant components:

$$g^{ij} = \frac{A(g_{ij})}{g}, \tag{3}$$

$A(g_{ij})$ - is the algebraic complement of the element g_{ij}, $g = \det[g_{ij}]$ - matrix determinant.

In the general case, the components of the deformation tensor are defined in the local coordinate system:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - u_k \Gamma_{ij}^k, \tag{4}$$

$u_{i,j} = \frac{\partial u_i}{\partial x^j}$, Γ_{ij}^k - the Kristoffels' symbols of the second kind, u_i - displacement in a local coordinate system.

For convenience, let us imagine a displacement and the Kristoffels' symbols their values in the base coordinate system:

$$u_k = z_{,k}^{m'} u_{m'} \tag{5}$$

$$\Gamma_{ij}^k = x_{r'}^k z_{,i}^{m'} \left(z_{,j}^{n'} \Gamma_{m'n'}^{r'} + \frac{\partial z_{,j}^{r'}}{\partial z^{m'}} \right) \tag{6}$$

$$z_{,k}^{s'} x_{,r'}^k = \delta_{r'}^{s'} \tag{7}$$

After substituting (5) - (7) in (4), we get the formula for the presentation component of the deformation tensor in the local coordinate system through the components of displacement on the base [5]:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{k',i} z_{,j}^{k'} + u_{k',j} z_{,i}^{k'} \right) - u_{k'} z_{,i}^{m'} z_{,j}^{n'} \Gamma_{m'n'}^{k'} . \tag{8}$$

Description of prismatic bodies and bodies of revolution with variable geometrical and physic-mechanical parameters of the most naturally done in the orthogonal cylindrical coordinate system:

$$g_{1'1'} = g_{2'2'} = 1,\ g_{3'3'} = (Z^{2'})^2,\ \Gamma_{3'3'}^{2'} = -Z^{2'},\ \Gamma_{3'2'}^{3'} = \Gamma_{2'3'}^{3'} = \frac{1}{Z^{2'}} \tag{9}$$

and coordinate system of Descartes:

$$g_{1'1'} = g_{2'2'} = g_{3'3'} = 1,\ \Gamma_{l'm'}^{k'} = 0 . \tag{10}$$

In this case, the components of the metric tensor in the local coordinate system are fed through the components in the base according to the formula:

$$g_{ij} = z_{,i}^{1'} z_{,j}^{1'} + z_{,i}^{2'} z_{,j}^{2'} + z_{,i}^{3'} z_{,j}^{3'} g_{3'3'} . \tag{11}$$

The relationship between the displacements and deformations of (8) can be written in the form:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{k',i} z_{,j}^{k'} + u_{k',j} z_{,i}^{k'} \right) - u_{2'} z_{,i}^{3'} z_{,j}^{3'} \Gamma_{3'3'}^{2'} - u_{3'} z_{,i}^{2'} z_{,j}^{3'} \Gamma_{2'3'}^{3'} - u_{3'} z_{,i}^{3'} z_{,j}^{2'} \Gamma_{3'2'}^{3'} . \tag{12}$$

An important special case, which is of practical importance, is the objects with the canonical form, for which the geometric equation (12) is much easier. This is primarily inhomogeneous circular body of revolution and rectilinear prismatic body with a variable cross-sectional area.

Due to the convergence x^3 i $Z^{3'}$, and their orthogonality to the plane of the cross-section in a cylindrical coordinate system ($0 \le x^3 \le 2\pi$):

$$z^{3'}_{,\alpha} = z^{\alpha'}_{,3} = 0 ,\; z^{3'}_{,3} = 1 \tag{13}$$

in Descartes ($0 \le x^3 \le 2$):

$$z^{3'}_{,\alpha} = z^{\alpha'}_{,3} = 0 ,\; z^{3'}_{,3} = a \tag{14}$$

a – is the half length of the body.

Considering (13) and (14) equation (12) takes the form in the orthogonal cylindrical coordinate system:

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(z^{\gamma'}_{,\alpha}u_{\gamma',\beta} + z^{\gamma'}_{,\beta}u_{\gamma',\alpha}\right);\; \varepsilon_{\alpha 3} = \frac{1}{2}\left(u_{3',\alpha} + z^{\gamma'}_{,\alpha}u_{\gamma',3} - \frac{2z^{2'}_{,\alpha}u_{3'}}{Z^{2'}}\right);$$

$$\varepsilon_{33} = u_{3',3} + Z^{2'}u_{2'} . \tag{15}$$

in Descartes:

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(z^{\gamma'}_{,\alpha}u_{\gamma',\beta} + z^{\gamma'}_{,\beta}u_{\gamma',\alpha}\right),$$

$$\varepsilon_{\alpha 3} = \frac{1}{2}\left(au_{3',\alpha} + z^{\gamma'}_{,\alpha}u_{\gamma',3}\right),$$

$$\varepsilon_{33} = au_{3',3} . \tag{16}$$

The components of the stress tensor in the local coordinate system are expressed through the components of the deformation tensor on the base of the generalized law of Hooke [5]:

$$\sigma^{ij} = d^{ijkl}\varepsilon_{kl} . \tag{17}$$

In an isotropic body, components of the tensor of elastic constants d^{ijkl} associated with the coefficients Liame λ and μ relationships [5]:

$$d^{ijkl} = \lambda g^{ij}g^{kl} + \mu(g^{jl}g^{ik} + g^{il}g^{jk}) , \tag{18}$$

$\lambda = \dfrac{E\nu}{(1-2\nu)(1+\nu)}$, $\mu = \dfrac{E}{2(1+\nu)}$, $E = E\left(Z^{i'}\right)$, $\nu = \nu\left(Z^{i'}\right)$ - are the values of modulus of elasticity and Poisson's ratio at the point of the body that is treated.

It is assumed, that in the process of the loading, in the body arise elastic ε^{e}_{ij} and instantaneous plastic ε^{p}_{ij} deformation. Description of plastic deformation of the material is based on the following General hypotheses and assumptions based on experimental data [6]:

1. The body material is homogeneous and isotropic, the change in its volume - linearly elastic:

$$\varepsilon^{p}_{kk} = 0 . \tag{19}$$

2. The components of the tensor of strain increment $d\varepsilon_{ij}$ derived from the increase in elastic $d\varepsilon^{e}_{ij}$ and plastic $d\varepsilon^{p}_{ij}$ components:

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + d\varepsilon^{p}_{ij} . \tag{20}$$

3. The reverse of the tensor strain increment is associated with the stress tensor and its growth:

$$d\varepsilon^{e}_{ij} = k_{ijkl} d\sigma^{kl} + dk_{ijkl}\sigma^{kl} . \tag{21}$$

4. The region of elastic deformation is limited by the surface flow, the equation of which in the space of stresses is:

$$f\left(\sigma^{ij}, \chi\right) = 0 , \tag{22}$$

χ - is the hardening parameter.

5. In accordance with the associative law of plastic flow plastic deformation developed normal to the yield surface:

$$d\varepsilon^{p}_{ij} = d\lambda \frac{\partial f}{\partial S_{ij}} = d\lambda S_{ij} . \tag{23}$$

For isotropic material, subject to Mises' yield conditions, the equation of the surface is:

$$f = \frac{1}{2} S_{ij} S^{ij} - \tau_s^2(\chi) , \tag{24}$$

τ_s - is the yield stress in pure shear, $\chi = \int \sqrt{\frac{2}{3} d\varepsilon^{p}_{ij} d\varepsilon^{ij}_{p}}$ - the parameter Adcwist, $S^{ij} = \sigma^{ij} - \sigma_0 g^{ij}$ - components of the deviator stress, as $\sigma_0 = \frac{1}{3}\sigma^{ij} g_{ij}$.

Movement inhomogeneous isotropic solids, with the volume V and surface S described by the equation, which is a consequence of the principle of D'alembert in the curvilinear coordinate system [5, 7]:

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\sqrt{g} z^{j'}_{,k} \sigma^{ki} \right) + f^{j'} = \rho \ddot{u}^{j'} . \tag{25}$$

The uniqueness of the solution (25) is provided with appropriate initial and boundary conditions.

The initial condition is well-known the displacements and velocities distribution in the body in time t_0, which is taken as the start of the time coordinates:

$$u(Z^{i'},t_0)=u_0(Z^{i'})\,,\ \dot{u}(Z^{i'},t_0)=\dot{u}_0(Z^{i'})\,,\ Z^{i'}\in V\,. \tag{26}$$

It is assumed that part of the surface S_u is set to kinematic boundary conditions:

$$u(Z^{i'},t)=\tilde{u}(Z^{i'},t)\,,\ Z^{i'}\in S_u \tag{27}$$

and on the surface S_p with normal $\vec{n}=n_j e^j$ - the system loads arbitrarily oriented in space and in time:

$$z^{k'}_{,i}\sigma^{ij}n_j=\tilde{p}\left(Z^{k'},t\right),\ Z^{k'}\in S_p\,. \tag{28}$$

The spatial bodies with longitudinal and transverse cracks, which are grown, are considered (Fig. 2).

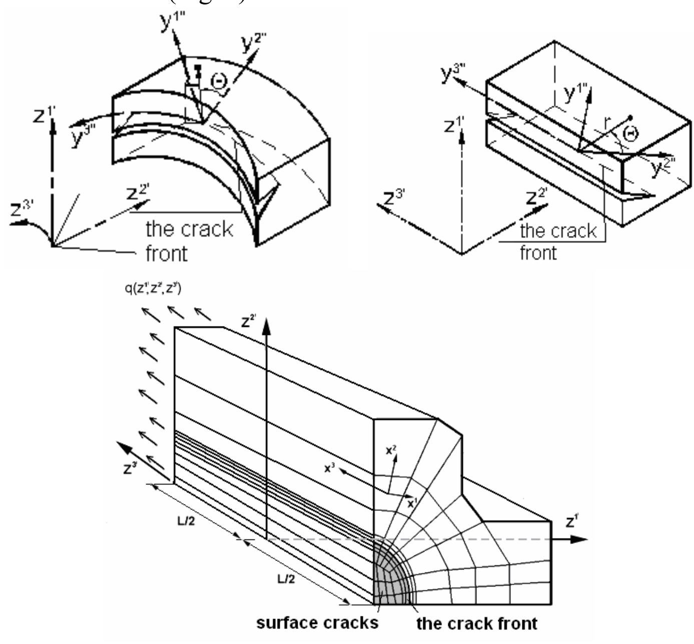

Fig. 2. Fragments of bodies with cracks

Application of J-integral Cherepanov-Rice (29), as the main parameter of the fracture toughness, in the study of stationary crack under static and dynamic loads within the elastic strains showed high efficiency and reliability.

$$J_k(t) = \frac{1}{\Delta}\int_S \left[(W+T)n_k - f_i \frac{\partial u_i}{\partial x^k} \right] dS \,, \tag{29}$$

$S = S_k + S_1 + S_2$ - the surface of integration, n_k - projection on the axis x^k unit external normal to the surface S, f_i - projection on the axis x^k vector of effort on the surface S, u - the displacement, W i T - potential and kinetic energy, respectively.

However, discussion of consideration of crack development subject to plastic flow and arbitrary load history needs to use another parameter fracture, which takes into account for the behavior of the body with a crack in the specified conditions. One such parameter is the T-integral expression for the entries of which contains an additional member in the form of the integral over the region, which takes into account the presence of mass forces and loads applied to the surfaces of the crack.

$$T^* = \int_{\Gamma_\varepsilon} ((W+T)n_1 - t_i \frac{\partial u_i}{\partial x_i}) d\Gamma \equiv \int_{\Gamma + S_{c\Gamma}} ((W+T)n_1 - t_i \frac{\partial u_i}{\partial x_1}) d\Gamma -$$

$$- \int_{V_\Gamma - V_\varepsilon} \left[(\frac{\partial W}{\partial x_1} - \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1}) + \rho(\dot{u}_i \frac{\partial \dot{u}_i}{\partial x_i} - \ddot{u}_i \frac{\partial u_i}{\partial x_i}) + f_i \frac{\partial u_i}{\partial x_i} \right] dv \,. \tag{30}$$

W is the total work stress in a material point, which is determined according to the formula (31)

$$W = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \,, \tag{31}$$

$\frac{\partial W}{\partial x_1}$ is calculated directly on the base of two points on an infinitely small distance from each other.

Another feature by considering bodies with cracks, especially fractures of the 1st mode is conditions for the penetration of the crack edges. This is achieved by applying a contact layer in additional coordinate system $y^{i''}$, associated with the configuration of the surfaces of bodies (Fig. 3).

In each moment, conditions of the penetration, friction is based on the Coulon's law and the absence of tensile stresses normal to the surface of the contact:

$$\sigma_t^{n''(n'')} \le 0 \,, \ \tau_t \le f_{fr} \sigma_t^{n''(n'')} \,, \tag{32}$$

f_{fr} - coefficient of friction, n'' - normal to the contact surface.

It is assumed that for the contact layer material density and Poisson's ratio equal to zero:

$$\rho_c = 0 \,, \ \nu_c = 0 \,. \tag{33}$$

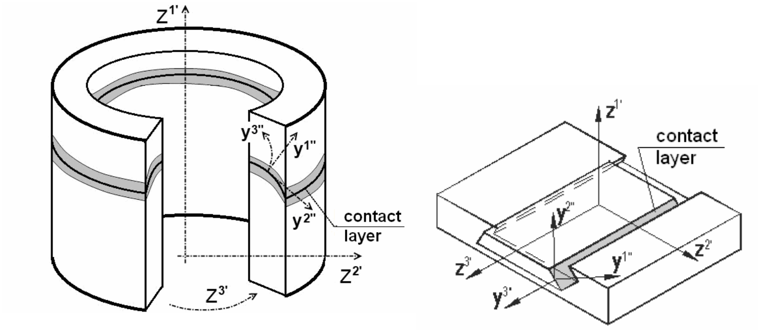

Fig. 3. Modeling of interaction of the bodies

Formulation (33) provides no-weight border and instantaneous transmission of forces from one body to another under dynamic loading.

For the numerical study of objects moving crack on the base of the finite element method, typically used stationary [2] or moving grid [1].

The first difference consists in the transfer of the crack tip from one node to another without violating the topology of the finite element mesh.

$$\int_V (\sigma_{ij}\delta\varepsilon_{ij} + \rho\ddot{u}_i\delta u_i)dV - \int_{S_\sigma} \overline{T}_i\delta u_i dS - \int_{CD} \delta(T_2 u_2)dS = 0 . \qquad (34)$$

In case of using the second approach for crack growth is changing or the whole grid, or moving grid only a small region surrounding the crack tip.

Since the application of the stationary grid is limited by problems of elastic fracture, for research identified in stat objects will be used by the mobile grid in combination with the use of rolling singular (special) finite element at the crack tip.

Conclusions

Analysis of existing approaches to the solution of problems of fracture mechanics for inhomogeneous bodies with cracks developing in the conditions of dynamic load indicates the need for development of numerical methods for solving a certain class of problems. Thus, further development of the application of SAFE to calculate the dynamic parameters of fracture mechanics in terms of crack growth is relevant.

REFERENCES

1. *Atluri S.N., Nishioka T.* Hybrid methods of analysis. — Unification of Finite Element Methods, Ed. N. Kardestuncer. — Amsterdam: North-Holland, 1984, p. 65—96.
2. *Yagama G., Sakai Y., Ando Y.* Analysis of a rapidly propagating crack using finite elements. - Fast Fracture and Crack Arrest, Eds. G. T. Hahn,. M. F. Kanninen, ASTM STP 627, 1977.
3. *Atluri S.* Computational Methods in the Mechanics of Fracture., Mir, Moscow, Russia, 1990 (Russian).

4. *Bazhenov V.A., Vabishchevych M.O., Hulyar O.I., Solodey I.I.* "Special features calculate stress intensity factors under dynamic loading", Strength of Materials and Theory of Structures, vol.82, KNUBA, Kyiv, Ukraine, 2008 (Ukrainian).
5. *Blokh V.Y.* "Theory of Elasticity", Publish. Kharkov University, Kharkov, USSR, 1964 (Russian).
6. *Korotkikh Y.G., Belevich S.M.* Fundamental equations of thermoplasticity under complex loading. - Methods for solving problems of elasticity and plasticity., Gorkiy, Russia, 1969 (Russian).
7. The finite element method in mechanics of solids / [Saharov A.S., Kislookiy V.N., Kirichevskiy V.V. and others] Kyiv, 1982 (Russian).
8. *Solodey I.I., Vabishchevych M.O., Hulyar O.I. Saharov A.S.* "Calculation of stress intensity factor in non-stationary problems of the dynamics of 3d bodies based on the energy approach", Strength of Materials and Theory of Structures, vol.83, KNUBA, Kyiv, Ukraine, 2009 (Ukrainian).
9. *Solodey I.I., Vabishchevych M.O., Hulyar O.I.* "Solutions of nonstationary problems of fracture mechanics based on the approximation of cracks by special finite elements", Strength of Materials and Theory of Structures, vol.85, KNUBA, Kyiv, Ukraine, 2009 (Ukrainian).

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ПОСТАНОВКА ЗАДАЧІ НЕЛІНІЙНОГО ДЕФОРМУВАННЯ ТА РУЙНУВАННЯ НЕОДНОРІДНИХ ПРОСТОРОВИХ ТІЛ З УРАХУВАННЯМ ПОЯВИ ТА РОЗПОВСЮДЖЕННЯ ТРІЩИНИ В УМОВАХ ДИНАМІЧНОГО НАВАНТАЖЕННЯ

Розглянуті основні вихідні параметри задач механіки руйнування та існуючі методики розрахунку для неоднорідних просторових тіл з тріщинами в умовах нелінійних динамічних впливів.

Ключові слова: J-інтеграл, розповсюдження тріщини, нелінійне деформування, динаміка, контактні напруження.

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ПОСТАНОВКА ЗАДАЧИ НЕЛИНЕЙНОГО ДЕФОРМИРОВАНИЯ И РАЗРУШЕНИЯ НЕОДНОРОДНЫХ ПРОСТРАНСТВЕННЫХ ТЕЛ С УЧЕТОМ ПОЯВЛЕНИЯ И РАСПРОСТРАНЕНИЯ ТРЕЩИНЫ В УСЛОВИЯХ ДИНАМИЧЕСКОГО НАГРУЖЕНИЯ

Рассмотрены основные исходные параметры задач механики разрушения и существующие методики расчета для неоднородных пространственных тел с трещинами в условиях нелинейных динамических влияний.

Ключевые слова: J-интеграл, распространение трещины, нелинейное деформирование, динамика, контактные напряжения.